Scaling functions and superscaling in medium and heavy nuclei

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The scaling function $f(\psi')$ for medium and heavy nuclei with $Z \neq N$ for which the proton and neutron densities are not similar is constructed within the coherent density fluctuation model (CDFM) as a sum of the proton and neutron scaling functions. The latter are calculated in the cases of ^{62}Ni, ^{82}Kr, ^{118}Sn, and ^{197}Au nuclei on the basis of the corresponding proton and neutron density distributions, which are obtained in the deformed self-consistent mean-field Skyrme HF+BCS method. The results are in reasonable agreement with the empirical data from the inclusive electron scattering from nuclei showing superscaling for negative values of ψ', including those smaller than -1. This is an improvement over the relativistic Fermi gas model predictions where $f(\psi')$ becomes abruptly zero for $\psi' \leqslant -1$. It is also an improvement over the CDFM calculations made in the past for nuclei with $Z \neq N$ assuming that the neutron density is equal to the proton one and using only the phenomenological charge density.

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Studies of the scaling phenomenon observed in inclusive electron scattering from nuclei make it possible to gain information about basic nuclear characteristics such as the local density $\rho(r)$ and momentum distribution $n(k)$ in nuclei. This concerns primarily the y scaling (e.g., Refs. [1–5]). It was found also that a properly defined function $f(\psi')$ of another scaling variable (the ψ' variable) has a superscaling behavior. The latter observation means that for $\psi' < 0$ this function is independent of the transfer momentum q (at $q > 500$ MeV/c) and of the mass number for a wide range of nuclei from ^{4}He to ^{197}Au. This was first considered within the framework of the relativistic Fermi gas (RFG) model (e.g., Refs. [6–10]). As pointed out in Ref. [8], however, the actual nuclear dynamical content of the superscaling is more complex than that provided by the RFG model. It was observed that the experimental data have a superscaling behavior for large negative values of ψ' (up to $\psi' \approx -2$), whereas the predictions of the RFG model are for $f(\psi') = 0$ at $\psi' \leqslant -1$. This imposes the consideration of the superscaling in realistic finite systems. Such works were performed [11,12] in the CDFM [13–15], which is related to the δ-function limit of the generator-coordinate method [11,16]. The calculated CDFM scaling function $f(\psi')$ agrees with the available experimental data from the inclusive electron scattering for ^{4}He, ^{12}C, ^{27}Al, ^{56}Fe, and, approximately, for ^{197}Au for various values of the transfer momentum $q = 500$, 1000, 1650 MeV/c [11] and 1560 MeV/c [12], showing superscaling for negative values of ψ' including also those smaller than -1 (in contrast to the RFG model result). It was shown in Refs. [11,12] that the superscaling in nuclei can be explained quantitatively on the basis of the similar behavior of the high-momentum components of the nucleon momentum distributions in light, medium, and heavy nuclei. It is known that the latter is related to the effects of the short-range and tensor nucleon-nucleon correlations in nuclei (see, e.g., Ref. [13]). Our scaling function was obtained starting from that in the RFG model [6–8] in two equivalent ways, on the basis of the local density distribution and of the nucleon momentum distribution. This gives a good opportunity to study simultaneously the role of the nucleon-nucleon correlations included in $\rho(r)$ and $n(k)$ in superscaling phenomenon.

Here we would like to emphasize, however, that in Refs. [11,12] we encountered some difficulties in describing within the CDFM the superscaling in the case of ^{197}Au, which was the heaviest nucleus considered. We related this to the particular A dependence of $n(k)$ in the model, which does not lead to realistic high-momentum components of the momentum distribution in the heaviest nuclei. We followed in Refs. [11,12] a somewhat artificial way of "improving" the high-momentum tail of $n(k)$ in ^{197}Au by taking the value of the diffuseness parameter b in the Fermi-type charge density distribution of this nucleus to be $b = 1$ fm instead of the value $b = 0.449$ fm (as obtained from electron elastic scattering experiments; see Ref. [17]). In such a case the high-momentum tail of $n(k)$ for ^{197}Au in CDFM becomes similar to those of ^{4}He, ^{12}C, ^{27}Al, and ^{56}Fe nuclei and this leads to good agreement of the scaling function $f(\psi')$ with the data also for ^{197}Au. Discussing this in Ref. [11] we pointed out, however, that all the nucleons may contribute to $f(\psi')$ for the transverse electron scattering and this could reflect on the diffuseness of the matter density for a nucleus such as ^{197}Au whose value can be different from that of the charge density used in our previous works [11,12].

The aim of the present work is to apply the CDFM by using both proton and neutron densities for medium and heavy nuclei (for which $Z \neq N$), in contrast to our previous approach, in which we assumed that the neutron density was equal to that of protons and we used only the phenomenological charge density [17]. In our work now the total scaling function $f(\psi')$ will be a sum of two scaling functions, those for protons and neutrons.

In [12] the CDFM scaling function $f(\psi')$ was given in two equivalent ways: (1) by means of the density distribution

$$f(\psi') = \int_0^{\alpha/(k_F|\psi'|)} dR|F(R)|^2 f_{\mathrm{RFG}}(R,\psi'), \tag{1}$$

where

$$|F(R)|^2 = -\frac{1}{\rho_0(R)}\left.\frac{d\rho(r)}{dr}\right|_{r=R}, \tag{2}$$

$$\rho_0(R) = \frac{3A}{4\pi R^3}, \quad \alpha = \left(\frac{9\pi A}{8}\right)^{1/3} \simeq 1.52A^{1/3}, \tag{3}$$

$$f_{\mathrm{RFG}}(R,\psi') = \frac{3}{4}\left[1-\left(\frac{k_F R|\psi'|}{\alpha}\right)^2\right]\left\{1+\left(\frac{Rm_N}{\alpha}\right)^2 \times\left(\frac{k_F R|\psi'|}{\alpha}\right)^2\left[2+\left(\frac{\alpha}{Rm_N}\right)^2 -2\sqrt{1+\left(\frac{\alpha}{Rm_N}\right)^2}\right]\right\}, \tag{4}$$

with m_N being the nucleon mass, and (2), by means of the momentum distribution

$$f(\psi') = \int_{k_F|\psi'|}^{\infty} d\overline{k}_F|G(\overline{k}_F)|^2 f_{\mathrm{RFG}}(\overline{k}_F,\psi'), \tag{5}$$

where

$$|G(\overline{k}_F)|^2 = -\frac{1}{n_0(\overline{k}_F)}\left.\frac{dn(p)}{dp}\right|_{p=\overline{k}_F} \tag{6}$$

and

$$n_0(\overline{k}_F) = \frac{3A}{4\pi\overline{k}_F^{\,3}}. \tag{7}$$

In Eq. (5) the RFG scaling function $f_{\mathrm{RFG}}(\overline{k}_F,\psi')$ can be obtained from $f_{\mathrm{RFG}}(R,\psi')$ [Eq. (4)] by changing α/R to $\overline{k}_F$. In Eqs. (1), (4), and (5) the Fermi momentum k_F is not a free fitting parameter for different nuclei as in the RFG model, but it is calculated in the CDFM for each nucleus using the corresponding expressions

$$k_F = \int_0^\infty dR k_F(R)|F(R)|^2 = \int_0^\infty dR\frac{\alpha}{R}|F(R)|^2 = \frac{4\pi(9\pi)^{1/3}}{3A^{2/3}}\int_0^\infty dR\rho(R)R \tag{8}$$

when

$$\lim_{R\to\infty}[\rho(R)R^2] = 0 \tag{9}$$

is fulfilled and

$$k_F = \frac{16\pi}{3A}\int_0^\infty d\overline{k}_F n(\overline{k}_F)\overline{k}_F^{\,3} \tag{10}$$

when

$$\lim_{\overline{k}_F\to\infty}\left[n(\overline{k}_F)\overline{k}_F^{\,4}\right] = 0 \tag{11}$$

is fulfilled.

In Refs. [11,12] we used the charge density distributions to determine the weight function $|F(R)|^2$ in calculations of $f(\psi')$ from Eqs. (1)–(4) and (8). In the present work we assume that the reason why the CDFM does not work properly in the case of ^{197}Au is that we use in Refs. [11,12] only the charge density, whereas this nucleus has many more neutrons than protons ($N = 118$ and $Z = 79$), and therefore proton and neutron densities may differ considerably. In this case the proton and neutron scaling functions, $f_p(\psi')$, and $f_n(\psi')$, respectively, will be given by the contributions of the proton and neutron densities, $\rho_p(r)$ and $\rho_n(r)$, correspondingly:

$$f_{p(n)}(\psi') = \int_0^{\alpha_{p(n)}/(k_F^{p(n)}|\psi'|)} dR|F_{p(n)}(R)|^2 f_{\mathrm{RFG}}^{p(n)}(R,\psi'), \tag{12}$$

where the proton and neutron weight functions are obtained from the corresponding proton and neutron densities

$$|F_{p(n)}(R)|^2 = -\frac{4\pi R^3}{3Z(N)}\left.\frac{d\rho_{p(n)}(r)}{dr}\right|_{r=R}, \tag{13}$$

$$\alpha_{p(n)} = \left(\frac{9\pi Z(N)}{4}\right)^{1/3}, \tag{14}$$

$$\int_0^\infty \rho_{p(n)}(\vec{r})d\vec{r} = Z(N), \tag{15}$$

and the Fermi momentum for the protons and neutrons is given by

$$k_F^{p(n)} = \alpha_{p(n)}\int_0^\infty dR\frac{1}{R}|F_{p(n)}(R)|^2. \tag{16}$$

The RFG proton and neutron scaling functions $f_{\mathrm{RFG}}^{p(n)}(R,\psi')$ have the form of Eq. (4), where α and k_F are changed to $\alpha_{p(n)}$ from Eq. (14) and $k_F^{p(n)}$ from Eq. (16), correspondingly. The normalizations of the functions are as follows:

$$\int_0^\infty |F_{p(n)}(R)|^2 dR = 1, \tag{17}$$

$$\int_{-\infty}^\infty f_{p(n)}(\psi')d\psi' = 1. \tag{18}$$

Then the total scaling function can be expressed by means of both proton and neutron scaling functions as

$$f(\psi') = \frac{1}{A}[Zf_p(\psi') + Nf_n(\psi')] \tag{19}$$

and is normalized to unity.

The same consideration can be performed equivalently on the basis of the nucleon momentum distributions for protons $n^p(k)$ and neutrons $n^n(k)$, presenting $f(\psi')$ by the sum of proton and neutron scaling functions (19) calculated similarly to Eqs. (12)–(19) [and to Eqs. (5), (6), (10), and (11)]:

$$f_{p(n)}(\psi') = \int_{k_F^{p(n)}|\psi'|}^{\infty} d\overline{k}_F |G_{p(n)}(\overline{k}_F)|^2 f_{\mathrm{RFG}}^{p(n)}(\overline{k}_F, \psi'), \tag{20}$$

where

$$|G_{p(n)}(\overline{k}_F)|^2 = -\frac{4\pi \overline{k}_F^{\,3}}{3Z(N)} \left.\frac{dn^{p(n)}(k)}{dk}\right|_{k=\overline{k}_F}, \tag{21}$$

with $f_{\mathrm{RFG}}^{p(n)}(\overline{k}_F, \psi')$ containing $\alpha_{p(n)}$ from Eq. (14) and $k_F^{p(n)}$ calculated by

$$k_F^{p(n)} = \int_0^{\infty} d\overline{k}_F \overline{k}_F |G_{p(n)}(\overline{k}_F)|^2. \tag{22}$$

We calculate the scaling function for several examples; for the medium stable nuclei ^{62}Ni and ^{82}Kr and for the heavy nuclei ^{118}Sn and ^{197}Au following Eqs. (12)–(19) and using the corresponding proton and neutron densities obtained in deformed self-consistent mean-field (HF + BCS) calculations with density-dependent Skyrme effective interaction (SG2) and a large harmonic-oscillator basis with 11 major shells [18,19].

The results of the calculations of $f(\psi')$ for ^{62}Ni, ^{82}Kr, ^{118}Sn, and ^{197}Au for $q = 1000$ MeV/c are given in Fig. 1 and are compared with the experimental data (presented by a gray area and taken from [8]) obtained for ^{4}He, ^{12}C, ^{27}Al, ^{56}Fe, and ^{197}Au. The scaling functions are in reasonable agreement with the data, which was not the case for ^{197}Au calculated in Ref. [11] by using the experimental Fermi-type charge density only with parameter values $b = 0.449$ fm and $R = 6.419$ fm from [17] (see also Fig. 2 of Ref. [11]). It must be stressed that the theoretical proton densities used in the present calculations are close to the experimental charge densities. At the same time we note also the improvement in comparison with the RFG model result in which $f(\psi') = 0$ for $\psi' \leqslant -1$. As an example, the proton and neutron scaling functions for ^{62}Ni are also given in Fig. 1, which clearly illustrates the different tails of the proton and neutron scaling functions when $Z \neq N$, as well as their role in building up the observed scaling function.

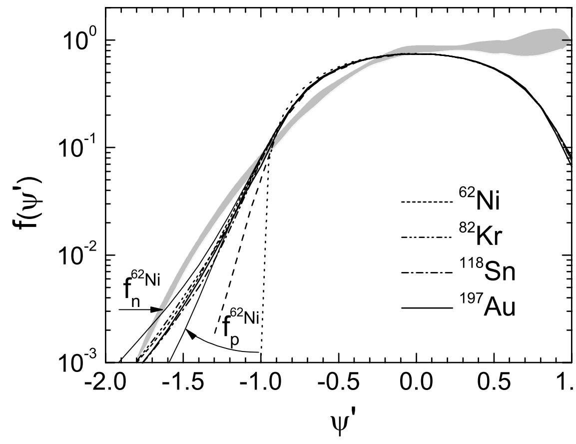

FIG. 1. Scaling function $f(\psi')$ calculated in the CDFM for ^{62}Ni, ^{82}Kr, ^{118}Sn, and ^{197}Au. The results are obtained using Eqs. (12)–(19). The experimental data for ^{4}He, ^{12}C, ^{27}Al, ^{56}Fe, and ^{197}Au at $q =$ 1000 MeV/c taken from [8] are shown by the shaded area. The RFG result is presented by the dotted line. The results of the calculations for ^{197}Au [11] by means of Eqs. (1)–(4) using the Fermi-type charge density [17] are shown by the dashed line.

In conclusion, we point out that the scaling function $f(\psi')$ for nuclei with $Z \neq N$ for which the proton and neutron densities are not similar has to be expressed by the sum of the proton and neutron scaling functions. The latter can be calculated within the CDFM on the basis of the knowledge (obtained theoretically and/or experimentally) of the corresponding proton and neutron local density distributions or momentum distributions. We should also point out that the agreement with experiment is quite reasonable given that no adjustable parameter at all has been used in the present calculations.

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