Fermi matrix element with isospin breaking

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Prompted by the level of accuracy now being achieved in tests of the unitarity of the CKM matrix, we consider the possible modification of the Fermi matrix element for the β-decay of a neutron, including possible in-medium and isospin violating corrections. While the nuclear modifications lead to very small corrections once the Behrends-Sirlin-Ademollo-Gatto theorem is respected, the effect of the $u-d$ mass difference on the conclusion concerning V_{ud} is no longer insignificant. Indeed, we suggest that the correction to the value of $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$ is at the level of 10^{-4}.

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I. INTRODUCTION

The unitarity of the Cabbibo-Kobayashi-Maskawa (CKM) matrix, $\mathcal{U}$, is a fundamental aspect of the Standard Model and any deviation would represent clear evidence of new physics. The most accurate test of unitarity involves the (11) element of $\mathcal{U}^\dagger\mathcal{U}$ [1–4], namely

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.99995 \pm 0.00061 \,. \tag{1}$$

This remarkable result is primarily a triumph of decades of precise studies of super-allowed nuclear β-decay, which at the present time yields a more accurate value of V_{ud}, namely $V_{ud} = 0.97425 \pm 0.00022$, than studies of the free neutron.

In the light of the remarkable level of accuracy that has been obtained, as well as the improvements anticipated in the near future, we feel that it is timely to address once more the issue of two corrections which are little discussed. The first issue concerns the effect of the mean scalar field in the nucleus. In Ref. [5] it was shown that the difference between the radius of a neutron and a proton would grow with density and it was then claimed that this would lead to a violation of the Behrends-Sirlin-Ademollo-Gatto [6, 7] (BSAG) theorem. This result, which was a consequence of an incorrect application of the constraint equations in the bag model, is wrong. Indeed, suppose that the u and d masses are equal in a nucleon immersed in an *isoscalar* scalar field. As m_u and m_d move apart the BSAG theorem must apply whatever the scalar field and one must not obtain a correction to the Fermi matrix element linear in $m_u - m_d$. Of course, the coefficient of the term in $(m_u - m_d)^2$ *may* change and we estimate this effect here. It is very small.

On the other hand, when we calculate the change in the Fermi matrix element in a manner consistent with BSAG, it turns out that at the physical value of $m_u - m_d$ it is no longer totally negligible in comparison with the current errors on V_{ud}. The reduction in the overlap of the u and d wave functions turns out to be of order 2×10^{-5}. Finally, we combine this calculation with an earlier estimate of the effect of pion mass differences carried out using chiral perturbation theory, in order to obtain a total correction. When applied to the unitarity test in Eq. (1), the total correction amounts to about 1.2×10^{-4}.

As a final note on the possibility of a genuine correction associated with the nucleon decaying in-medium, we point out that the relatively small isovector scalar potential (in the QMC model associated with the δ meson) expected in nuclei with $N \neq Z$ will serve to effectively increase the value of $m_d - m_u$ by an amount V_δ. The latter may be as large as 4 or 5 MeV in a heavy nucleus and this could produce a decrease in the apparent value of V_{ud} as large as 2×10^{-5}, depending on the neutron-proton asymmetry of the particular nucleus. While for the present this is below the level at which it could be detected it is a systematic error which may need to be accounted for in the near future.

II. EXPLICIT CALCULATION

In terms of the quark field isodoublet $\psi = (\psi_u, \psi_d)$, the Fermi operator for β decay is

$$\mathcal{F}^\pm = \int d\vec{r} \psi^\dagger \tau^\pm \psi$$

It is related to the generators ($T^\alpha, \alpha = 1, 2, 3$) of the isospin transformations

$$T^\alpha = \int d\vec{r}\psi^\dagger \frac{\tau^\alpha}{2}\psi,$$

which are conserved by strong interactions in the isospin symmetry limit. In the following we assume that the quark mass difference $\delta m = m_d - m_u$ is the only source of isospin breaking. So we write the exact Hamiltonian of the strong interaction as

$$H = H_0 + v = H_0 + \frac{\delta m}{2}\int d\vec{r}\bar{\psi}\tau^3\psi,$$

where H_0 is the isospin symmetric part: $[H_0, T^\alpha] = 0$. Even in the presence of the symmetry breaking term v the isospin generators satisfy the current algebra relations

$$[T^\alpha, T^\beta] = i\epsilon^{\alpha\beta\gamma}T^\gamma$$

and combined with the fact that $[H, T^\alpha] \sim \delta m$ this leads to the BSAG theorem for the Fermi matrix element:

$$F^+ =< p|\mathcal{F}^+|n >= 1 + O(\delta m^2)\,.$$

We wish to estimate how the overlap defect $\Delta = 1 - F^+$ changes when the neutron is in the nuclear medium and the challenge is to do that in a way consistent with BSAG. When we naively use the bag model to compute F^+, as originally done in the QMC model, we have the problem that the bag radius depends on the flavor content. As it changes during the $n \to p$ transition, there is a lack of orthogonality between the eigenmodes of the initial and final bag and this leads to an overlap defect of order δm. Clearly this is an artefact of the static bag model which, by definition, cannot handle a radius which changes with time, as is the case in a β decay.

Since this overlap artefact arises from the small mass difference δm the correct approch is to start with the isospin symmetry limit and use the bag model only to compute the deviation from unity. We then find that this deviation is explicitly of order $(\delta m)^2$ – as it should be. Higher order deviations can also be computed in this framework but we stop at order $(\delta m)^2$, which is quite sufficient for our purpose.

We denote as $|N_\alpha, m_\tau = \pm 1/2>$, the eigenstates of H_0 with isospin 1/2.

$$H_0|N_\alpha, m_\tau = \pm 1/2 >= M_\alpha|N_\alpha, m_\tau = \pm 1/2>$$

The nucleon corresponds to $\alpha = 0$. The exact proton and neutron states are $|N, m_\tau>$ such that

$$H|N, m_\tau >= M(m_\tau)|N, m_\tau>$$

Let P be the projector onto $|N_0, m_\tau = \pm 1/2>$ and $Q = 1 - P$. We have

$$|N, m_\tau >= \sqrt{Z}\left(1 + \frac{Q}{M_0 - H_0}v + \ldots\right)|N_0, m_\tau>$$

with

$$Z = 1 - < N_0|v\frac{Q}{(M_0 - H_0)^2}v|N_0 > + O(v^3)$$

independent of m_τ.

Using $\mathcal{F}^+|N_0, -1/2 >= |N_0, 1/2>$ we have, up to $O(v^3)$ contributions:

$$\begin{aligned} F^+ &= < N, \frac{1}{2}|\mathcal{F}^+|N, -\frac{1}{2}> \\ &= Z + \sum_{\alpha\neq 0} < N_0, \frac{1}{2}|v\frac{1}{(M_0 - M_\alpha)^2}|N_\alpha, \frac{1}{2} >< N_\alpha, -\frac{1}{2}|v|N_0, -\frac{1}{2}> \end{aligned}$$

The terms linear in v vanish because $\mathcal{F}^+$ commutes with H_0 and hence with Q. From the isospin structure of the pertubation we have $< N_\alpha, -\frac{1}{2}|v|N_0, -\frac{1}{2} >= - < N_\alpha, \frac{1}{2}|v|N_0, \frac{1}{2}>$. So the contribution from the excited states adds to that in Z and we end with

$$F^+ = 1 - 2 < N_0|v\frac{Q}{(M_0 - H_0)^2}v|N_0>$$

Since the overlap defect, $1 - F^+$, is explicitly of order $(\delta m)^2$, we can use the (isospin symmetric) bag to finish the calculation. If we denote as ω_α the energies with the same quantum numbers as the nucleon we get

$$F^+ = 1 - \frac{3\delta m^2}{2} \sum_{\alpha \neq 0} \left(\frac{< \alpha|\gamma_0|0 >}{\omega_\alpha - \omega_0} \right)^2$$

with

$$< \alpha|\gamma_0|0 > = \int^R d\vec{r}\, \phi_\alpha^\dagger \gamma_0 \phi_0 \,,$$

where ϕ_α are the normalized eigenmodes of the cavity of radius R with quark mass $m^* = \bar{m} - g_\sigma^q \sigma$ and σ is the mean scalar field in the medium, as calculated for example in the QMC model [8–10]. In practice we can set $\bar{m} = 0$. Note that the nuclear vector fields simply shift the energy scale and cannot change F^+.

If we denote by Ω_α / R the energy (positive or negative) of the mode α, the eigenmode is

$$\phi_\alpha = \begin{pmatrix} f_\alpha(r) \\ i\vec{\sigma}.\hat{r} g_\alpha(r) \end{pmatrix} \frac{\chi}{\sqrt{4\pi}}$$

with

$$\begin{aligned} f_\alpha(r) &= \mathcal{N}_\alpha \frac{1}{r} \sin\left[\sqrt{\Omega_\alpha^2 - (m^* R)^2}\, \frac{r}{R}\right] \\ g_\alpha(r) &= \mathcal{N}_\alpha \frac{R}{(\Omega_\alpha + m^* R) r} \\ &\left(\frac{1}{r} \sin\left[\sqrt{\Omega_\alpha^2 - (m^* R)^2}\, \frac{r}{R}\right] - \frac{\sqrt{\Omega_\alpha^2 - (m^* R)^2}}{R} \cos\left[\sqrt{\Omega_\alpha^2 - (m^* R)^2}\, \frac{r}{R}\right] \right) \end{aligned}$$

and $\mathcal{N}$ the normalization constant such that

$$\int_0^R r^2 dr \left(f_\alpha^2 + g_\alpha^2 \right) = 1 \,.$$

The energy is determined by the boundary condition $f(R) = g(R)$, that is

$$\begin{aligned} (\Omega_\alpha + m^* R) \sin\left[\sqrt{\Omega_\alpha^2 - (m^* R)^2}\right] &= \sin\left[\sqrt{\Omega_\alpha^2 - (m^* R)^2}\right] \\ &\quad - \sqrt{\Omega_\alpha^2 - (m^* R)^2} \cos\left[\sqrt{\Omega_\alpha^2 - (m^* R)^2}\right] \,. \end{aligned}$$

In the following we assume that $m^* R$ is larger than the critical value $-3/2$ at which $\Omega_0 = 3/2$. This is not a restriction in normal nuclei. The scalar matrix element is then

$$< \alpha|\gamma_0|0 > = \int_0^R r^2 \left(f_\alpha f_0 - g_\alpha g_0 \right) dr \,.$$

We show the resultant overlap defect as a function of density for nuclear parameters [9] in Fig. 1. The $u - d$ mass difference has been set to 5 MeV, which is appropriate to the MIT bag [11]. The overlap defect changes by only a few times 10^{-6} as the density varies from zero to nuclear matter density.

III. DISCUSSION

Although the variation of the Fermi matrix element with density is small, we note from Fig. 1 that the intercept at zero density is around 1.94×10^{-5}, which is only a factor of 10 smaller than the current error quoted for V_{ud}. While such a deviation from unity for the hadronic form factor is carefully handled in the $s \to u$ transition, it seems to receive little attention in the literature in the $d \to u$ transition. Indeed, we found only the discussion by Kaiser[12], in the framework of chiral perturbation theory. That work reported a total reduction from the mass difference between charged and neutral pions of about 4×10^{-5}. The additional contribution from the difference between the $\pi^0 nn$ and $\pi^0 pp$ coupling constants, which is of order 0.4% [13], produces a negligible defect, well below 10^{-6}. The pionic

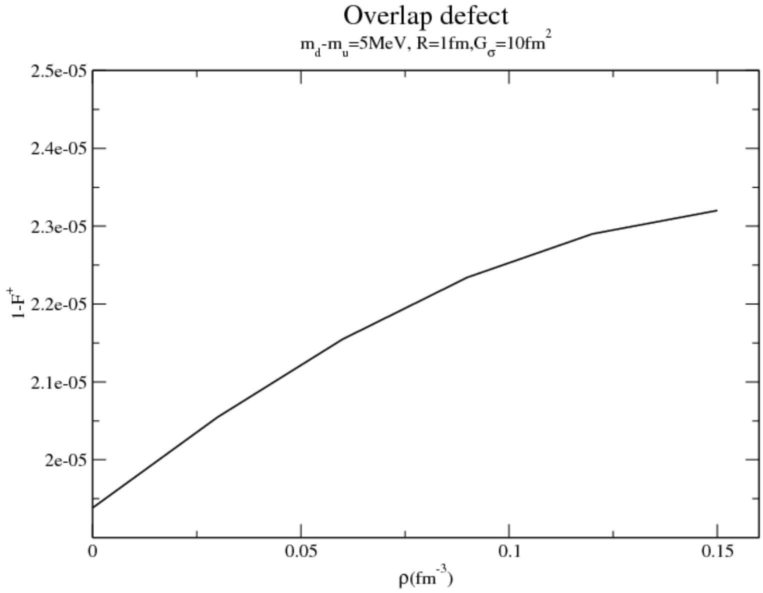

FIG. 1: Deviation below unity of the vector matrix element for the transition neutron to proton, as a function of the density of isoscalar nuclear matter.

correction of Kaiser and the correction from the valence quark overlap calculated here are independent and hence the total correction is $\delta g_V = 6\times10^{-5}$. Thus one should increase the value of V_{ud} deduced from the analysis of super-allowed Fermi β-decay by this amount. As noted earlier, this amounts to an increase in the value of $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$ by around 1.2×10^{-4}, which is only a factor of 5 below the error quoted in Eq. (1).

While the absence of a sizeable correction to g_V in-medium is reassuring, we note that there is a genuine correction which has so far been ignored, involving the isovector mean scalar field. This is often ignored because the coupling is so much smaller than that for the isoscalar case, but if the mean isovector scalar field felt by the quark is $\frac{\tau_z}{2}V_\delta$, the isospin breaking term will be proportional to $(m_d - m_u + V_\delta)^2$. This could potentially double the deviation associated with the quark mass difference, depending on the neutron excess and structure of the particular nucleus under consideration. It will be interesting to explore this effect quantitatively in future as well as to investigate the model dependence of isospin breaking term calculated here within the MIT bag model.

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