Nuclear beta decay

- Introduction
- Energy release
- Fermi theory of β-decay
- Shape of β spectrum (Kurie plot)
- Total decay rate
- Selection rules
- Neutrino mass
- Summary

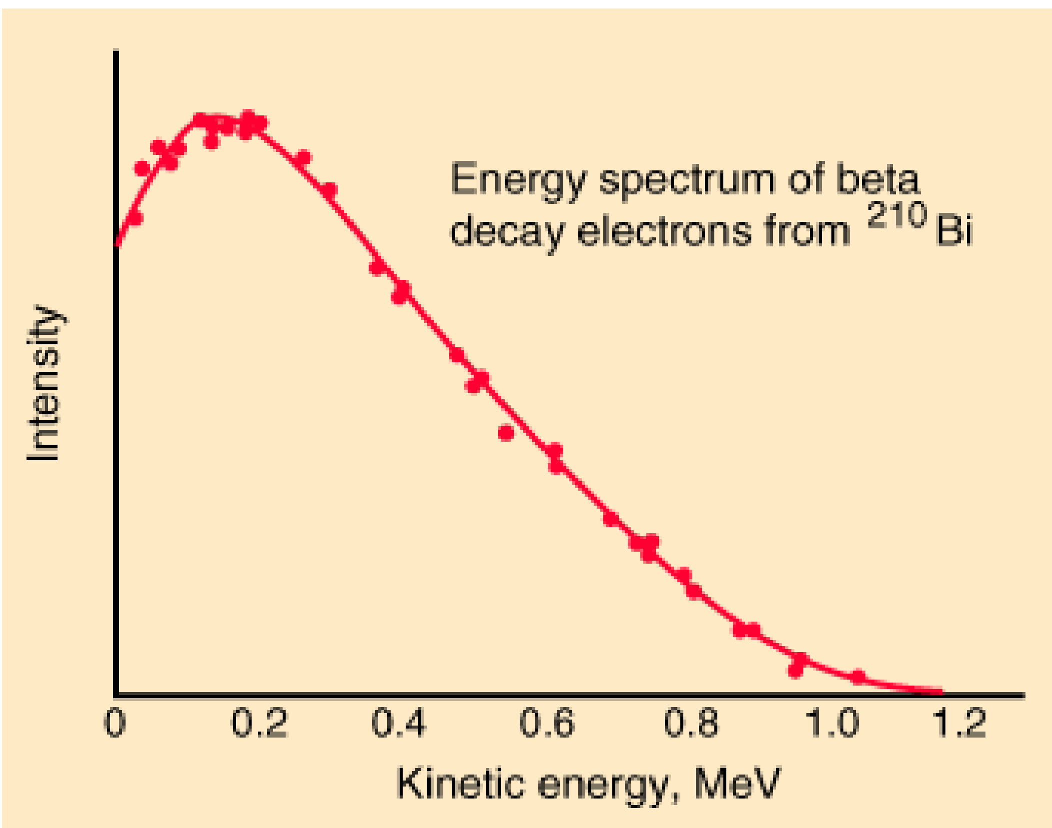
Introduction

Basic process: conversion of a proton to a neutron or of a neutron to a proton.

Both Z and N change by one unit: $Z \rightarrow Z \pm 1$, $N \rightarrow N \mp 1$, so that $A = Z+N$ remains constant $\Rightarrow$ slide down the mass parabola.

- β^-: emission of an e^-
- β^+: emission of a e^+
- EC: capture of an orbital electron (in competition with β^+)

In case of β emission, continuous energy spectrum:



Energy release

- β^-

  $M^*(Z,A)+Zm_e=(M^*(Z+1,A)+(Z+1)m_e)+E_0$

  $M(Z,A)=M(Z+1,A)+E_0$

  $Q_{\beta^-}=M(Z,A)-M(Z+1,A)=E_0$

- β^+

  $M^*(Z,A)=(M^*(Z-1,A)+m_e)+E_0$

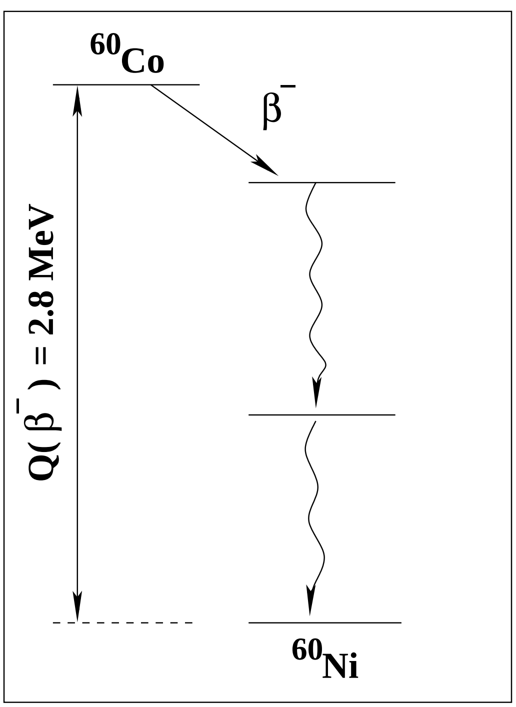
  $M^*(Z,A)+Zm_e=M^*(Z-1,A)+(Z-1)m_e+2m_e)+E_0$

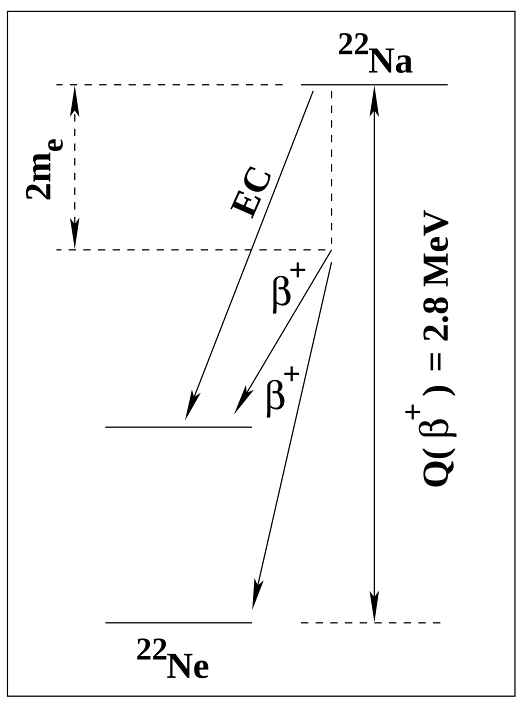
  $M(Z,A)=M(Z-1,A)+2m_e+E_0$

  $Q_{\beta+}=M(Z,A)-M(Z-1,A)=2m_e+E_0$

- Electron capture

  $Q_{EC}=M(Z,A)-M(Z+1,A)=E_0+B_i$





Fermi theory of β-decay

Treat the decay-causing interaction as a weak perturbation, find a relationship for the transition rate: Fermi's Golden Rule

$$\lambda = \frac{2\pi}{\hbar} |V_{fi}|^2 \rho(E_f)$$

- λ : total decay rate
- $V_{fi} = g \int [\psi_f^* \phi_e^* \phi_\nu^*] \; \hat{O} \, \psi_i dV \; = g M_{fi}$
- $\rho(E_f) = \frac{dn}{dE_f}$: density of final states

Taking the e^- and ν wave functions as plane waves, expanding the exponentials and keeping the first term gives the **allowed approximation** ($e^{i\vec{p}\cdot\vec{r}} \approx 1 + i\vec{p}\cdot\vec{r} + ...$).
Taking into account the nuclear Coulomb field and higher order terms in the exponential expansion yields the partial decay rate:

$$d\lambda = N(p) \propto \underbrace{p^2 (Q - T_e)^2}_{statistical factor} \underbrace{F(Z', p)}_{Fermi function} |M_{fi}|^2 \underbrace{S(p, q)}_{forb.\ corr.}$$

Including phase space yields (density of final states):

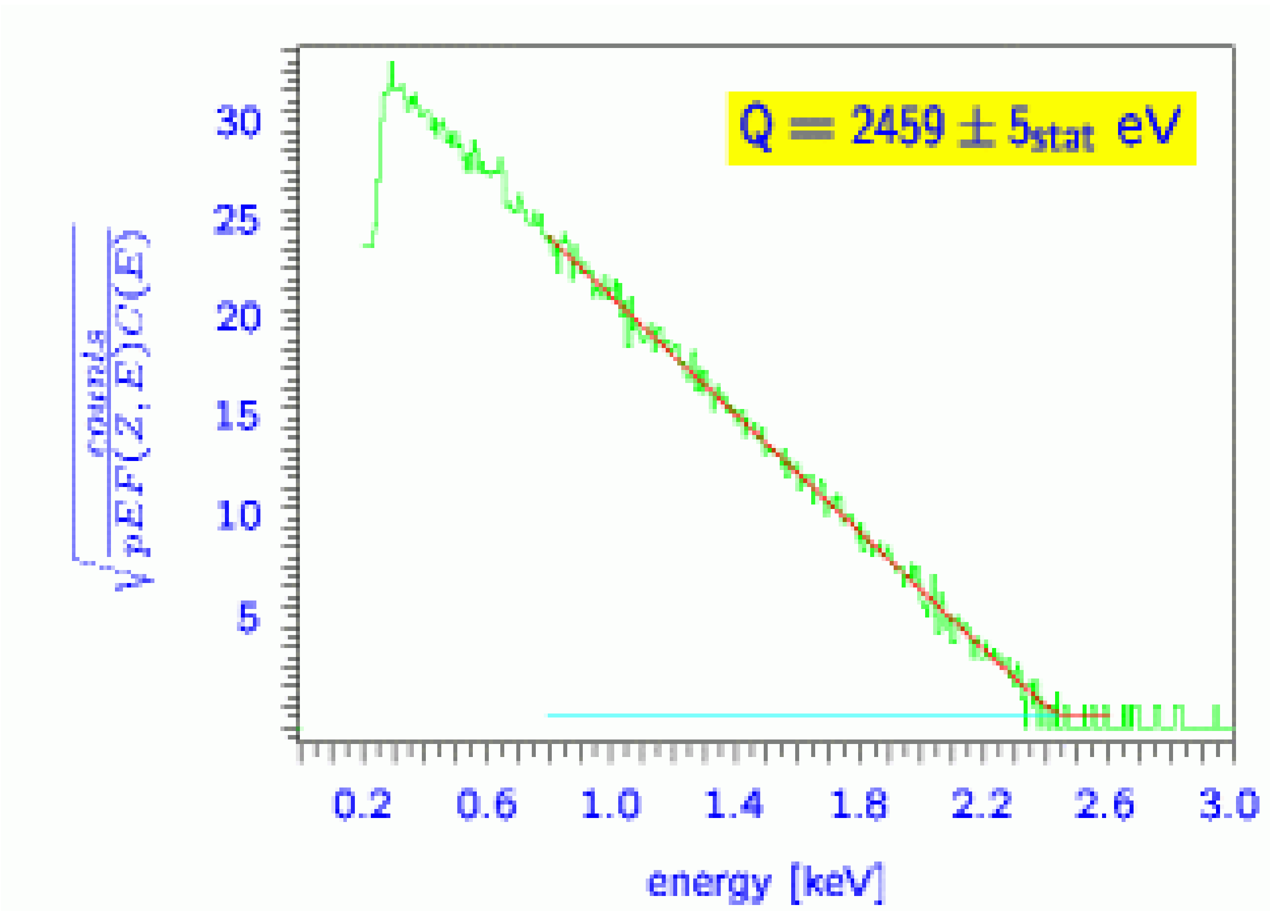
$$d\lambda = \frac{g^2 \, |M_{fi}|^2}{2\pi \hbar^7 c^3} F(Z', p) \, p^2 \, (Q - T_e)^2 \, dp$$

Shape of β spectrum (Kurie plot)

In the **allowed approximation**, we can rewrite:

$$Q - T_e \propto \sqrt{\frac{N(p)}{p^2 F(Z',p)}}$$

Plotting $\sqrt{\frac{N(p)}{p^2 F(Z',p)}}$ against T_e gives a straight line intercepting the x-axis at the decay energy Q ⇒ Kurie (or Fermi)-plot.



In the case of forbidden decays, the Kurie plot can be linearized by the correction factor $S(p, q)$.

Total decay rate

To find the total decay rate, integrate over all values of the electron momentum p.

$$\lambda = \frac{g^2\,|M_{fi}|^2}{2\pi\hbar^7 c^3} \int_0^{p_{max}} F(Z', p)\, p^2\, (Q - T_e)^2\, dp$$

$$= \frac{g^2\,|M_{fi}|^2}{2\pi\hbar^7 c^3} f(Z', E_0) \quad \text{and}$$

$$f(Z', E_0) \propto \int_0^{p_{max}} F(Z', p)\, p^2\, (E_0 - E_e)^2\, dp$$

where $f(Z', E_0)$ only depends on Z' and the maximum electron total energy E_0 and is called the Fermi integral. It is tabulated in *Feenberg and Trigg* diagrams.
We can also write it as:

$$ft_{1/2} = 0.693\, \frac{2\,\pi^3\,\hbar^7}{g^2\, m_e^5\, c^4\, |M_{fi}|^2}$$

$ft_{1/2}$ is called the Comparative half-life or ft value. This quantity gives an indication on the matrix element $|M_{fi}|^2$ responsible for the transition. Because the range in $ft_{1/2}$ is very large, what is usually quoted is the value of $log_{10}(ft)$ with t given in seconds.
Depending on the selection rules one obtains typical ft-values.

Selection rules

- Allowed and superallowed transitions:
  $\Delta\pi$ = NO, ΔJ = 0 Fermi (singulet)
  $\Delta\pi$ = NO, ΔJ = 0, $\pm$1, (no 0 $\rightarrow$ 0) Gamov-Teller (triplet)
  superallowed: $log_{10}\ ft \approx 3.5$ mirror decays
  allowed : $log_{10}\ ft \approx 4 - 7$
  Example: $n \rightarrow p$, ^{14}O $\rightarrow ^{14}$N
- Once forbidden (unique if $\Delta J = \pm 2$)
  $\Delta\pi$ = YES, ΔJ = 0, $\pm$ 1, $\pm$ 2
  $log_{10}\ ft \approx 6 - 10$
  Example: ^{17}N $\rightarrow ^{17}$O ($\frac{1}{2}^- \rightarrow \frac{5}{2}^+$)
- Twice forbidden (unique)
  $\Delta\pi$ = NO, ΔJ = $\pm$ 2, $\pm$ 3
  $log_{10}\ ft \approx 12 - 14$
  Example: ^{137}Cs $\rightarrow ^{137}$Ba ($\frac{7}{2}^+ \rightarrow \frac{3}{2}^+$)

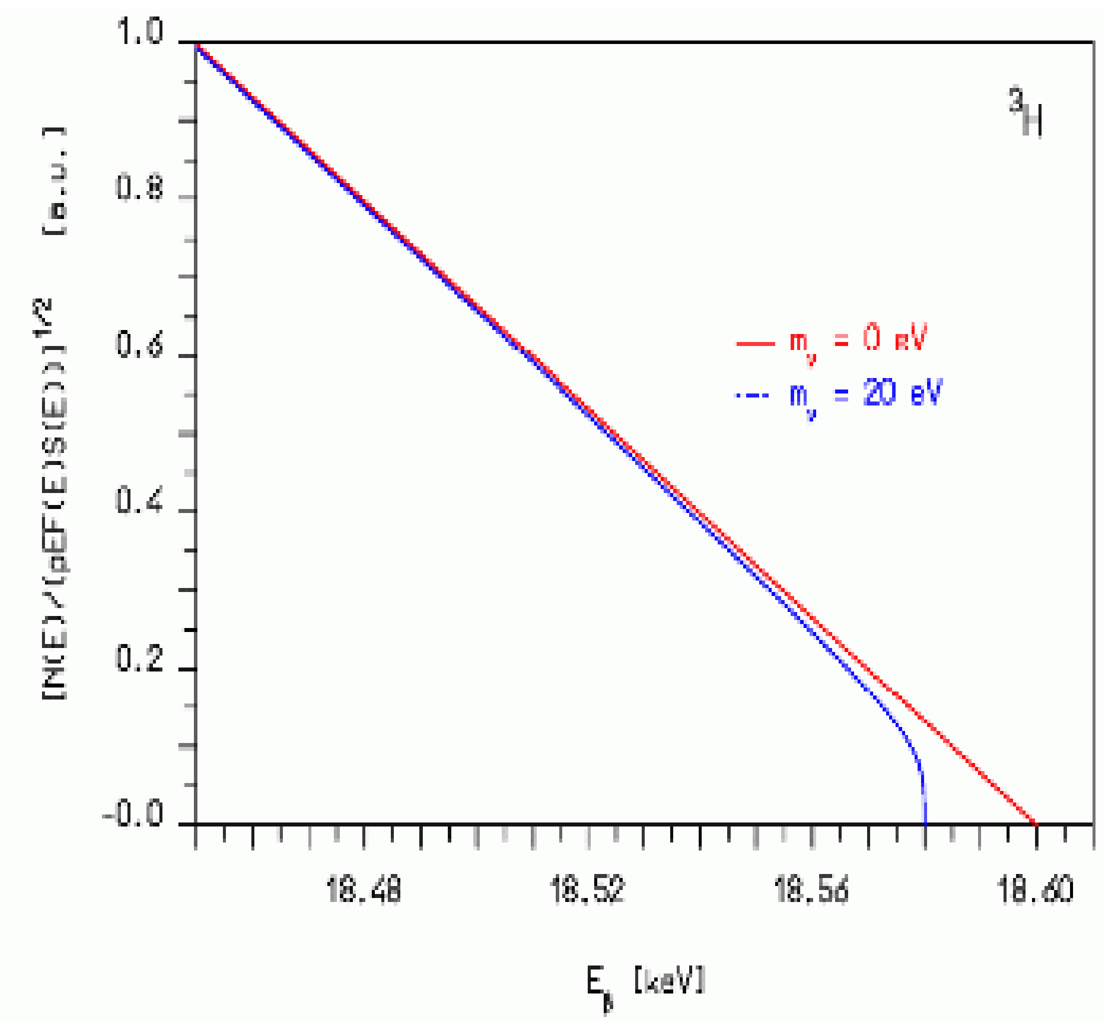
Forbidden decays are less probable because they contain an orbital angular momentum change.
Example:
E_0 = 1 MeV, $p_{max}(e^-)$ = 1.4 MeV/c, R = 3 fm
$\Rightarrow$ pR = 4.2 fm MeV/c = 0.2 $\hbar < 1\ \hbar$

Neutrino mass

Looking at the endpoint energy of the β-spectrum is a method to determine the neutrino mass (upper limit is now 17 eV). Taking the neutrino mass into account yields for the decay rate:

$$d\lambda = \frac{g^2 \, |M_{fi}|^2}{2\pi\hbar^7 c^3} F(Z', p) \, p^2 \, (Q - T_e)^2 \sqrt{1 - \frac{m_\nu^2 c^4}{(Q-T_e)^2}} \, dp$$

At the endpoint ($Q - T_e$ = 0), if m_ν=0 then $\frac{d\lambda}{dp} \to$ 0. If $m_\nu \neq$ 0 then $\frac{d\lambda}{dp} \to \infty$.



Summary

- Based on Fermi's Golden Rule (perturbation theory)

- β-spectrum is a continuous spectrum (3 body process)

- $log_{10}(ft)$-values allows one to predict the spin and parity of nuclear states

- Powerfull spectroscopic tool (but complicated)