Gamow-Teller strength and nuclear deformation

Motivation :

- Spin-isospin nuclear properties.
- Exotic nuclei : Beta-decay + Charge Exchange Reactions
- Limits of applicability of well established microscopic models

Theoretical approach :

- Deformed HF+BCS+QRPA formalism with Skyrme forces and residual interactions in both ph and pp channels

Results :

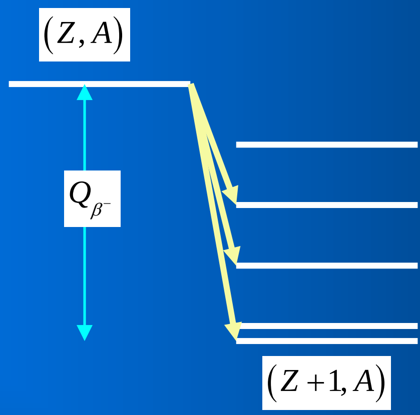
- **Nuclear structure**: GT strength distributions (Fe, Kr, Pb, Xe isotopes)
- **Nuclear astrophysics**: Half-lives of waiting point nuclei in rp-processes
- **Particle Physics**: Two-neutrino double beta-decay

β-decay

$$\beta^- : (Z,A) \rightarrow (Z+1,A) + e^- + \bar{\nu}_e$$

$$\beta^+ : (Z,A) \rightarrow (Z-1,A) + e^+ + \nu_e$$

$$EC : (Z,A) + e^- \rightarrow (Z-1,A) + \nu_e$$



$$Q_{\beta^-} = M_{parent} - M_{daughter} - m_e \quad \left(= T_e + E_{\bar{\nu}_e}\right)$$

Decay rate: Fermi's golden rule

$$\lambda\left(=\frac{\ln 2}{t}\right) = \frac{2\pi}{\hbar}\left|V_{fi}\right|^2 \frac{dN_F}{dW_0}$$

Density of final states

Matrix elements of the transition

Fermi integral

$$\frac{dN_F}{dW_0} = C\int_1^{W_0} pW\left(W_0 - W\right)^2 \lambda(Z,W)\,dW \equiv f\left(Z,W_0\right)$$

λ(Z,W): Fermi function: Influence of nuclear Coulomb field on electrons

β-decay

Matrix elements of the transition

$$V_{fi} = g\int \left[\psi_f \phi_e \phi_\nu\right]^* \Theta \psi_i dV \simeq g\int \psi_f \Theta \psi_i dV = M_{fi}$$

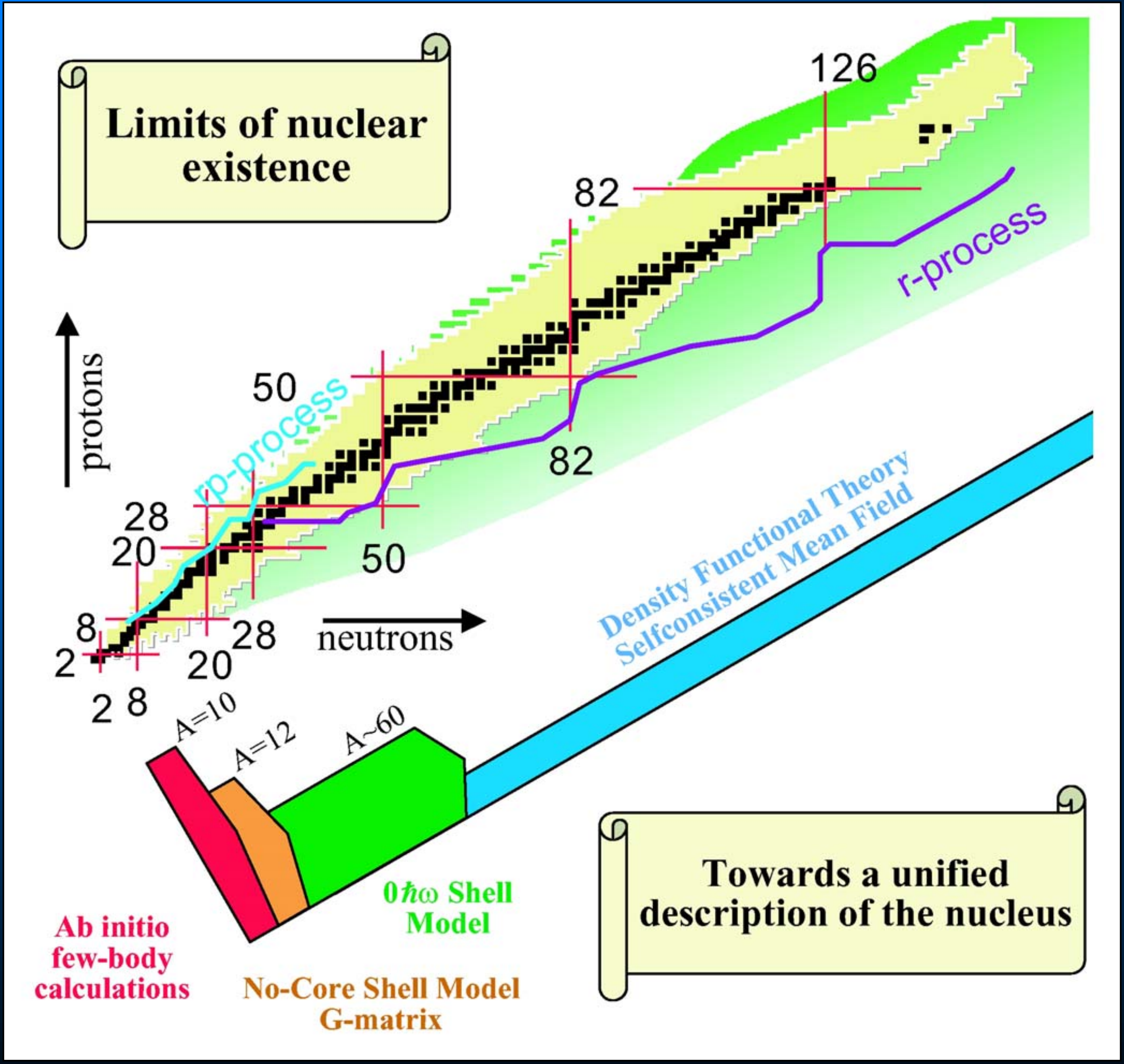
Allowed approximation

$$\phi_{e,\nu}\left(\vec{r}\right) = \frac{1}{\sqrt{V}} e^{i\vec{p}\cdot\vec{r}/\hbar} = 1 + i\vec{p}\cdot\vec{r}/\hbar + \cdots \simeq 1$$

Selection rules

- Allowed approximation $L = 0$, $\Delta\pi = \text{no}$
  - FERMI: $\Delta J = 0$ — e, ν $\downarrow\uparrow$ $(S = 0)$, $(\sum_i^A \tau_i^\pm)$
  - GAMOW TELLER: $\Delta J = 0, 1$ (no $0^+ \to 0^+$) — e, ν $\uparrow\uparrow$ $(S = 1)$, $(\sum_i^A \sigma_i \tau_i^\pm)$
- First Forbidden $L = 1$, $\Delta\pi = \text{yes}$
  - FERMI $(S = 0)$, $\Delta J = 0, 1$
  - GAMOW TELLER $(S = 1)$, $\Delta J = 0, 1, 2$

Nuclear Structure models



Gamow-Teller strength: different approaches

Nuclear matrix elements (β^+): $M_{fi} \sim \langle n \,|\{F, GT\}^+|\, p\rangle$

Gross theory: Statistical model (corrected for sp and pairing effects)
Takahashi et al. 1973–1990

Microscopic models

- Large scale shell model calculations: Madrid, Strasbourg, Aarhus — 1995 –
- QRPA calculations

Hamamoto ...	Spherical mean field + pairing + residual (RPA)	∼1970
Möller et al.	Nilsson (WS) + BCS + QRPA (χ_{ph}). Systematics	1985 –
Klapdor et al.	Nilsson (WS) + BCS + QRPA ($\chi_{ph} + \kappa_{pp}$)	1990 –
Hamamoto et al.	Def HF + BCS + TDA (χ_{ph}). A∼70	1995 –
Nazarewicz et al.	Sph HFB + QRPA ($\chi_{ph} + \kappa_{pp}$) Sph n-rich nuclei	1999 –
This work	Def HF + BCS + QRPA $(\chi_{ph} + \kappa_{pp})_{self}$	

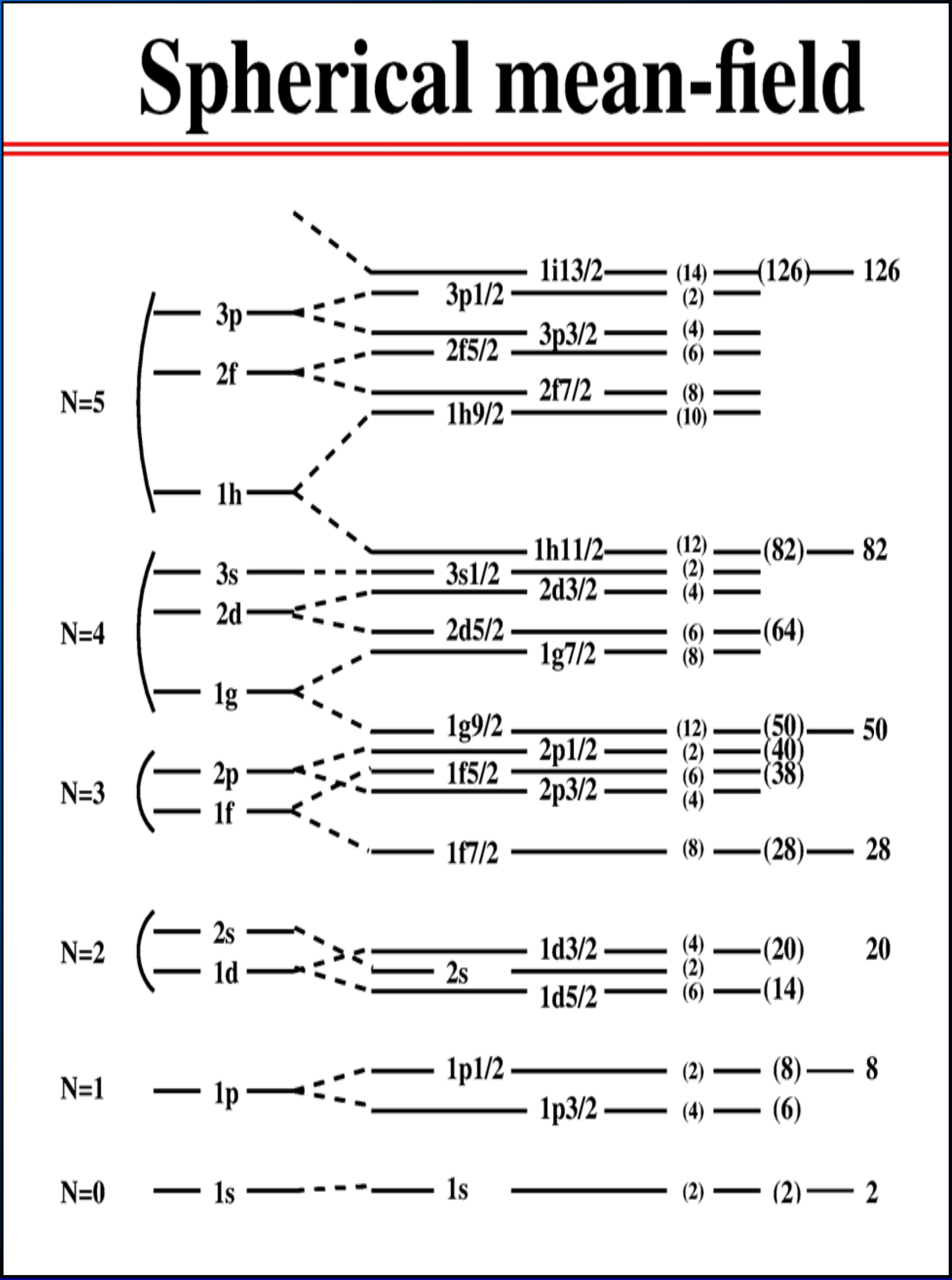
Spherical potentials

Empirical construction based in an harmonic oscillator potential plus a spin-orbit term to reproduce the magic numbers (M. Goeppert-Mayer and H. Jensen)

$$\boldsymbol{U}(r) = \frac{1}{2} m\omega^2 r^2 + D\boldsymbol{l}^2 + \boldsymbol{l}\cdot\boldsymbol{s}$$

$$\varepsilon_{nlj} = \hbar\omega[2(n-1)+l+3/2)] + Dl(l+1) + C \begin{cases} l+1 & j = l-1/2 \\ -l & j = l+1/2 \end{cases}$$

$$\hbar\omega = \frac{41}{A^{1/3}}\ \mathrm{MeV}$$



Deformed potentials

Anisotropic harmonic oscillator

$$H_0 = -\frac{\hbar^2}{2m}\Delta + \frac{m}{2}\left(\omega_x^2 + \omega_y^2 + \omega_z^2\right)$$

$$\omega_x\omega_y\omega_z = \omega_0^3$$

Axially symmetric shapes

$$\omega_\perp^2 = \omega_x^2 = \omega_y^2 = \omega_0^2(\delta)(1 + \frac{2}{3}\delta)$$

$$\omega_z^2 = \omega_0^2(\delta)(1 - \frac{4}{3}\delta)$$

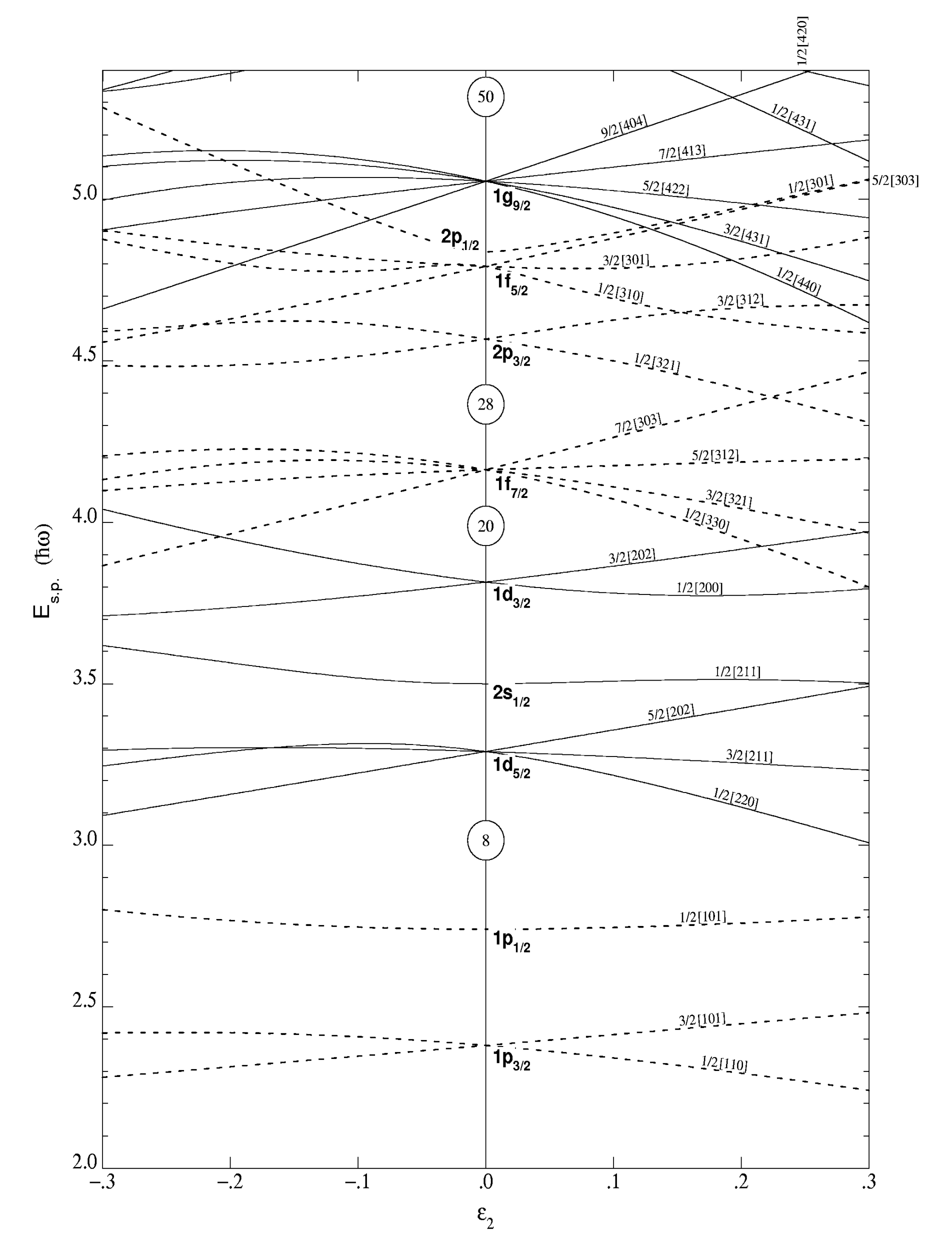
$$\Omega\pi[Nn_zm_\ell],$$

$$N = n_z + 2n_\rho + m_\ell$$

Nilsson model

$$\begin{aligned} H &= -\frac{\hbar^2}{2m}\Delta + \frac{m}{2}\omega_\perp^2(x^2+y^2) + \frac{m}{2}\omega_z^2 z^2 + C\vec{\ell}\cdot\vec{s} + D\vec{\ell}^2 \\ &= \hbar\omega_0(\delta)\left[-\frac{1}{2}\Delta' + \frac{1}{2}r'^2 - \beta r'^2 Y_{20}\right] - \kappa\hbar\omega_0(2\vec{\ell}\cdot\vec{s} + \mu\vec{\ell}^2) \end{aligned}$$

$$|k\rangle = \sum_\alpha D_{\alpha k}|\alpha\rangle, \qquad \alpha = \{N\ell j\Omega\}, \quad \{N\ell m_\ell m_s\}$$



Hartree-Fock method

How to extract a single-particle potential U(k) out of the sum of two-body interactions W(k,l)

$$\boldsymbol{H}\Psi(1,2,\ldots,A) = \left[\sum_{k=1}^{A} T(k) + \sum_{k<l=1}^{A} W(k,l)\right]\Psi(1,2,\ldots,A) = E\Psi(1,2,\ldots,A)$$

$$\boldsymbol{H} = \sum_{k=1}^{A}[T(k)+U(k)] + \left[\sum_{k<l=1}^{A} W(k,l) - \sum_{k=1}^{A} U(k)\right] = \boldsymbol{H}_0 + \boldsymbol{V}_{\mathrm{res}}$$

Hartree-Fock theory provides method to derive single-particle potential. The criterium is to search for the "best" A-particle slater determinant such us the value of $\boldsymbol{H}$ is minimum. Next, one assumes that the resulting residual interaction is small and that:

$$\Psi(1,2,\ldots,A) = \Phi_{a_1 a_2 \ldots a_A}(1,2,\ldots,A)$$

Variational principle

Effective interactions

Skyrme interactions: zero range, density-dependent

originally

$$V = \sum_{i<j} V(i,j) + \sum_{i<j<k} V(i,j,k)$$

$$V(i,j,k) = t_3 \delta(\mathbf{r}_i - \mathbf{r}_j)\delta(\mathbf{r}_j - \mathbf{r}_k)$$

3-body contact term equivalent to 2-body density-dependent

$$V(1,2) = \frac{1}{6} t_3 (1 + P_\sigma)\delta(\mathbf{r}_1 - \mathbf{r}_2)\rho\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right)$$

$$\begin{aligned} V &= t_0(1 + x_0 P_\sigma)\delta(\mathbf{r}_i - \mathbf{r}_j) + \frac{1}{2} t_1 \delta(\mathbf{r}_i - \mathbf{r}_j)(1 + x_1 P_\sigma)(\mathbf{k}^2 + \mathbf{k}'^2) + \\ &\quad + t_2(1 + x_2 P_\sigma)\mathbf{k}' \cdot \delta(\mathbf{r}_i - \mathbf{r}_j)\mathbf{k} + \mathrm{i}W_0(\sigma_i + \sigma_j) \cdot \mathbf{k}' \times \delta(\mathbf{r}_i - \mathbf{r}_j)\mathbf{k} + \\ &\quad + \frac{1}{6} t_3 (1 + x_3 P_\sigma)\delta(\mathbf{r}_1 - \mathbf{r}_2)\rho^\alpha\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right) \end{aligned}$$

	t_0	t_1	t_2	t_3	W	x_0	x_1	x_2	x_3	$1/\alpha$
Sk3	−1128.75	395.0	−95.0	14000.0	120.0	0.45	0.0	0.0	1.0	1.0
SG2	−2645.0	340.0	−41.9	15595.0	105.0	0.09	−0.0588	1.425	0.06044	6.0
SLy4	−2488.91	486.8	−546.4	137777.0	0.83	−0.340	−1.000	1.350	123.0	6.0

Skyrme Hartree-Fock

Schroedinger equation

$$\left[-\vec{\nabla}\cdot\frac{\hbar^2}{2m^\star(\vec{r})}\vec{\nabla}+U_q(\vec{r})+\vec{W}_q(\vec{r})\cdot(-i)(\vec{\nabla}\times\vec{\sigma})\right]\phi_i=e_i\phi_i$$

$$\frac{\hbar^2}{2m_q^\star(\mathbf{r})}=\frac{\hbar^2}{2m}+\frac{1}{4}(t_1+t_2)\rho+\frac{1}{8}(t_2-t_1)\rho_q$$

$$\vec{W}_q(\vec{r})=\frac{1}{2}W_0(\vec{\nabla}\rho+\vec{\nabla}\rho_q)+\frac{1}{8}(t_1-t_2)\vec{J}_q(\vec{r})$$

$$\begin{aligned}U_q(\vec{r}) \;=\;& t_0\left[(1+\frac{x_0}{2}\rho-(x_0+\frac{1}{2})\rho_q\right]+\frac{1}{4}t_3(\rho^2-\rho_q^2)\\ &-\frac{1}{8}(3t_1-t_2)\nabla^2\rho+\frac{1}{16}(3t_1+t_2)\nabla^2\rho_q+\frac{1}{4}(t_1+t_2)\tau\\ &+\frac{1}{8}(t_2-t_1)\tau_q-\frac{1}{2}W_0(\vec{\nabla}\cdot\vec{J}+\vec{\nabla}\cdot\vec{J}_q)+\delta_{q,+1/2}V_C(\vec{r})\end{aligned}$$

Algebraic combinations of the densities

$$\rho_{st}(\mathbf{r})=\sum_i|\phi_i(\mathbf{r},s,t)|^2$$

$$\tau_{st}(\mathbf{r})=\sum_i|\boldsymbol{\nabla}\phi_i(\mathbf{r},s,t)|^2$$

$$\mathbf{J}_{st}(\mathbf{r})=\sum_{i,s'}\phi_i^*(\mathbf{r},s',t)(-i\boldsymbol{\nabla}\times\boldsymbol{\sigma})\phi_i(\mathbf{r},s,t)$$

$$\rho_t=\sum_s\rho_{st}\,,\qquad \rho=\sum_{t=p,n}\rho_t,$$

Deformed Hartree-Fock

Single particle states (axially symmetric even-even nuclei)

Expansion into eigenfunctions of deformed harmonic oscillator

$$V(r,z) = \frac{1}{2}M\omega_\perp^2 r^2 + \frac{1}{2}M\omega_z^2 z^2$$

$$\phi_\alpha(\vec{R},\sigma) = \psi_{n_r}^{\Lambda}(r)\psi_{n_z}(z)\frac{e^{i\Lambda\phi}}{\sqrt{2\pi}}\chi_\Sigma(\sigma)$$

$$E_\alpha = (2n_r + |\Lambda| + 1)\hbar\omega_\perp + (n_z + 1/2)\hbar\omega_z$$

$$\psi_{n_r}^{\Lambda}(r) = \mathcal{N}_{n_r}^{\Lambda}\beta_\perp\sqrt{2}\eta^{\Lambda/2}\mathrm{e}^{-\eta/2}L_{n_r}^{\Lambda}(\eta)$$

$$\psi_{n_z}(z) = \mathcal{N}_{n_z}\beta_z^{\frac{1}{2}}\mathrm{e}^{-\xi^2/2}H_{n_z}(\xi)$$

$$\beta_\perp = (M\omega_\perp)^{\frac{1}{2}} \qquad \beta_z = (M\omega_z)^{\frac{1}{2}}$$

$$\mathcal{N}_{n_r}^{\Lambda} = \left(\frac{n_r!}{(n_r+\Lambda)!}\right)^{\frac{1}{2}} \qquad \mathcal{N}_{n_z} = \left(\frac{1}{\sqrt{\pi}2^{n_z}n_z!}\right)^{\frac{1}{2}}$$

Optimal basis to minimize truncation effects

N= 12 major shells

$$\beta_0 = \left[M(\omega_\perp^2\omega_z)^{\frac{1}{3}}\right]^{\frac{1}{2}} = (\beta_\perp^2\beta_z)^{\frac{1}{3}}$$

$$q = \frac{\omega_\perp}{\omega_z} = \left(\frac{\beta_\perp}{\beta_z}\right)^2$$

$$\Phi_i(\vec{R},\sigma,q) = \chi_{q_i}\sum_\alpha C_\alpha^i\phi_\alpha(\vec{R},\sigma)$$

$$\alpha = \{n_r, n_z, \Lambda, \Sigma\}$$

Pairing correlations in BCS approximations

Pairing interaction G

$$H = \sum_{k>0} \epsilon_k (a_k^+ a_k + a_{\bar{k}}^+ a_{\bar{k}}) - G \sum_{kk'>0} a_k^+ a_{\bar{k}}^+ a_{\bar{k}'} a_{k'}$$

BCS ground state

$$|\varphi_{\mathrm{BCS}}\rangle = \prod_{i>0} (u_i + v_i a_i^+ a_{\bar{i}}^+)|0\rangle$$

Expectation value of particle number

$$\delta \langle \varphi_{BCS} | H - \lambda \hat{N} | \varphi_{BCS} \rangle = 0$$

$$\langle \varphi_{BCS} | \hat{N} | \varphi_{BCS} \rangle = 2 \sum_{k>0} v_k^2 = N$$

BCS eqs.

Number eq.

$$2 \sum_i v_i^2 = N$$

Gap eq.

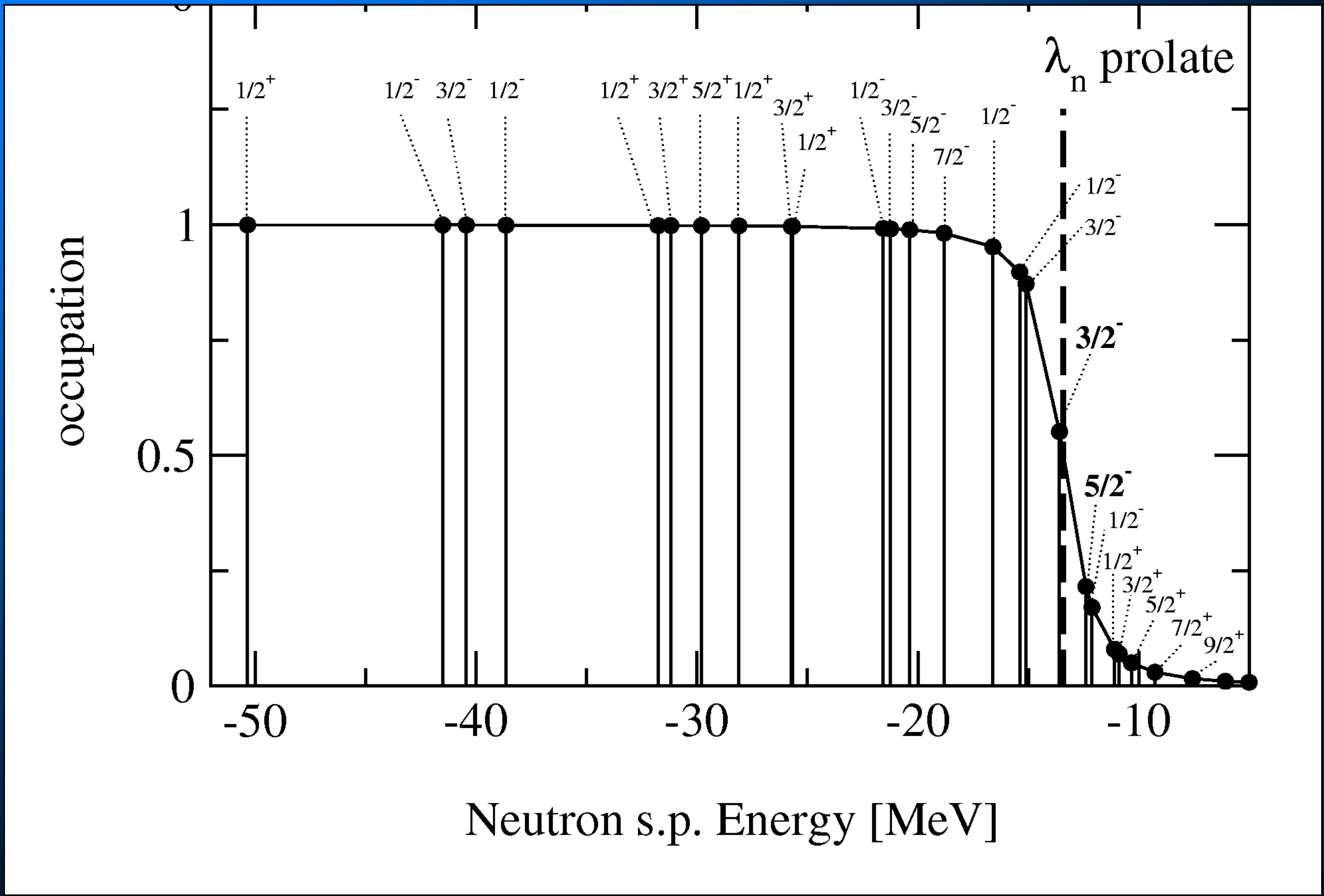
$$\Delta = G \sum_{k>0} u_k v_k$$

$$v_i^2 = \frac{1}{2}\left[1 - \frac{e_i - \lambda}{E_i}\right]; \qquad E_i = \sqrt{(e_i - \lambda)^2 + \Delta^2};$$

Fixed gaps taken from phenomenology

$$\Delta_n = \frac{1}{8}[B(N-2,Z) - 4B(N-1,Z) + 6B(N,Z) - 4B(N+1,Z) + B(N+2,Z)]$$

Occupation probability vs. energy



Constrained HF+BCS

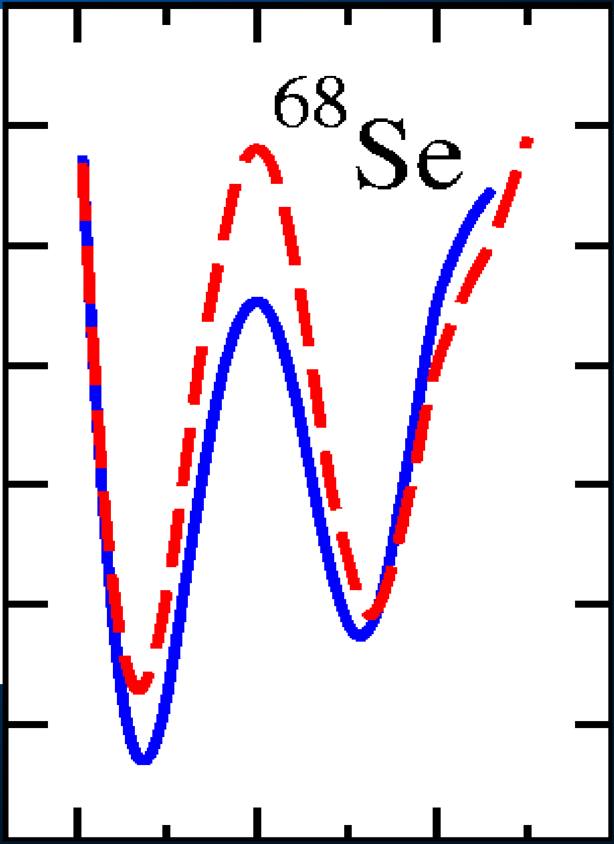
HF : single solution (Slater determinant of lowest energy)

Constrained HF : minimization of the HF energy under the constraint of holding the nuclear deformation fixed

Energy as a function of deformation:

more than one local minimum at close energy → shape coexistence

Energy vs. deformation



Theoretical approach: Residual interactions

Particle-hole residual interaction consistent with the HF mean field

$$V_{ph} = \frac{1}{16}\sum_{sts't'}\left[1+(-1)^{s-s'}\vec{\sigma}_1\cdot\vec{\sigma}_2\right]\left[1+(-1)^{t-t'}\vec{\tau}_1\cdot\vec{\tau}_2\right]\frac{\delta^2 E}{\delta\rho_{st}\left(\vec{r}_1\right)\delta\rho_{s't'}\left(\vec{r}_2\right)}\delta(\vec{r}_1-\vec{r}_2)$$

$$\begin{aligned} V_{ph}^{\sigma\tau} &= \frac{1}{16}\sum_{sts't'}(-1)^{s-s'}(-1)^{t-t'}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{\delta^2 E(\rho)}{\delta\rho_{st}(\mathbf{r}_1)\delta\rho_{s't'}(\mathbf{r}_2)}\delta(\mathbf{r}_1-\mathbf{r}_2) = \\ &= \frac{1}{16}\left[-4t_0-2t_1k_F^2+2t_2k_F^2-\frac{2}{3}t_3\rho^\alpha\right]\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,\delta(\mathbf{r}_1-\mathbf{r}_2). \end{aligned}$$

Average over nuclear volume → **Separable forces**

$$V_{GT}^{ph} = 2\chi_{GT}^{ph}\sum_{K=0,\pm1}(-1)^K\beta_K^+\beta_{-K}^-, \qquad \beta_K^+ = \sigma_K t^+ = \sum_{\pi\nu}\langle\nu|\sigma_K|\pi\rangle\, a_\nu^+ a_\pi$$

$$\chi_{GT}^{ph} = -\frac{3}{8\pi R^3}\left\{t_0+\frac{1}{2}k_F^2\left(t_1-t_2\right)+\frac{1}{6}t_3\rho^\alpha\right\}$$

$$V_{GT}^{pp} = -2\kappa_{GT}^{pp}\sum_K(-1)^K P_K^+ P_K, \qquad P_K^+ = \sum_{\pi\nu}\left\langle\nu\left|(\sigma_K)^+\right|\pi\right\rangle a_\nu^+ a_{\bar{\pi}}^+$$

RPA equations

$$H|\nu\rangle = E_\nu|\nu\rangle, \qquad |\nu\rangle = Q_\nu^+|0\rangle, \qquad Q_\nu|0\rangle = 0$$

Equation of motion

$$[H, Q_\nu^+]|0\rangle = \omega_{\nu 0} Q_\nu^+|0\rangle, \qquad \langle 0|[\delta Q, [H, Q_\nu^+]]|0\rangle = \omega_{\nu 0}\langle 0|[\delta Q, Q_\nu^+]|0\rangle$$

RPA: Phonon operator

$$Q_\nu^+ = \sum_{mi}\left(X_{mi}^\nu a_m^+ a_i - Y_{mi}^\nu a_i^+ a_m\right)$$

Creates (X) and destroys (Y)

ph pairs: g.s.correlations

$$\begin{aligned}\langle RPA|[a_i^+ a_m, [H, Q_\nu^+]]|RPA\rangle &= \omega_{\nu 0}\langle RPA|[a_i^+ a_m, Q_\nu^+]|RPA\rangle \\ \langle RPA|[a_m^+ a_i, [H, Q_\nu^+]]|RPA\rangle &= \omega_{\nu 0}\langle RPA|[a_m^+ a_i, Q_\nu^+]|RPA\rangle\end{aligned}$$

Quasiboson approximation

$$\begin{aligned}\langle RPA|[a_i^+ a_m, a_n^+ a_j]|RPA\rangle &= \delta_{ij}\delta_{mn} - \delta_{mn}\langle RPA|a_j a_i^+|RPA\rangle - \\ &\quad -\delta_{ij}\langle RPA|a_n^+ a_m|RPA\rangle \\ &\simeq \langle HF|[a_i^+ a_m, a_n^+ a_j]|HF\rangle = \delta_{ij}\delta_{mn}\end{aligned}$$

RPA equations

$$H = \sum_k \epsilon_k a_k^+ a_k + \frac{1}{4}\sum_{kk'll'} \bar{v}_{kk'll'} a_k^+ a_{k'}^+ a_{l'} a_l$$

$$\sum_{mi}\left(|X_{mi}^\nu|^2 - |Y_{mi}^\nu|^2\right) = 1$$

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}\begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \omega_{\nu 0}\begin{pmatrix} X^\nu \\ -Y^\nu \end{pmatrix}$$

$$\begin{aligned}A_{minj} &= \langle HF|[a_i^+ a_m [H, a_n^+ a_j]]|HF\rangle = (\epsilon_m - \epsilon_i)\delta_{mn}\delta_{ij} + \bar{v}_{mjin}, \\ B_{minj} &= -\langle HF|[a_i^+ a_m [H, a_j^+ a_n]]|HF\rangle = \bar{v}_{mnij}\end{aligned}$$

pnQRPA with separable forces

$$V_{GT} = 2\chi_{GT} \sum_K (-1)^K \ \beta_K^+ \beta_{-K}^-$$

$$\beta_K^+ \ = \ \sum_{np} \langle n | \sigma_K | p \rangle \, a_n^+ a_p = \sum_{np} \langle n | \sigma_K | p \rangle \, \{ u_n v_p \alpha_n^+ \alpha_{\bar{p}}^+ + v_n u_p \alpha_{\bar{n}} \alpha_p + u_n u_p \alpha_n^+ \alpha_p + v_n v_p \alpha_{\bar{n}} \alpha_{\bar{p}}^+ \}$$

Phonon operator

$$\Gamma_{\omega_K}^+ = \sum_{\gamma_K} \left[X_{\gamma_K}^{\omega_K} A_{\gamma_K}^+ - Y_{\gamma_K}^{\omega_K} A_{\bar{\gamma}_K} \right], \qquad A_{\gamma_K}^+ = \alpha_n^+ \alpha_{\bar{p}}^+$$

$$\Gamma_{\omega_K} \, |0\rangle = 0 \, ; \qquad \Gamma_{\omega_K}^+ \, |0\rangle = |\omega_K\rangle$$

pnQRPA equations

$$\langle \phi_0 | A_{\gamma_K} \, [H, \Gamma_{\omega_K}^+] | \phi_0 \rangle = \omega_K \, \langle \phi_0 | A_{\gamma_K} \Gamma_{\omega_K}^+ | \phi_0 \rangle$$

$$\langle \phi_0 | [H, \Gamma_{\omega_K}^+] \, A_{\bar{\gamma}_K}^+ | \phi_0 \rangle = \omega_K \, \langle \phi_0 | \Gamma_{\omega_K}^+ A_{\bar{\gamma}_K}^+ | \phi_0 \rangle$$

Transition amplitudes

$$\langle \omega_K | \beta_K^{\pm} | 0 \rangle = \mp M_{\pm}^{\omega_K}$$

$$X_{\gamma_K}^{\omega_K} = \frac{2\chi_{GT}}{\omega_K - \mathcal{E}_{\gamma_K}} \left(a_{\gamma_K} M_+^{\omega_K} + b_{\gamma_K} M_-^{\omega_K} \right)$$

$$Y_{\gamma_K}^{\omega_K} = \frac{-2\chi_{GT}}{\omega_K + \mathcal{E}_{\gamma_K}} \left(b_{\gamma_K} M_+^{\omega_K} + a_{\gamma_K} M_-^{\omega_K} \right)$$

$$M_+^{\omega_K} = \sum_{\gamma_K} \left(a_{\gamma_K} X_{\gamma_K}^{\omega_K} + b_{\gamma_K} Y_{\gamma_K}^{\omega_K} \right)$$

$$M_-^{\omega_K} = \sum_{\gamma_K} \left(b_{\gamma_K} X_{\gamma_K}^{\omega_K} + a_{\gamma_K} Y_{\gamma_K}^{\omega_K} \right)$$

$$a_{\gamma_K} = u_n v_p \Sigma_K^{np}, \qquad b_{\gamma_K} = v_n u_p \Sigma_K^{np}, \qquad \Sigma_K^{np} = \langle n | \sigma_K | p \rangle$$

B(GT) in the laboratory system

$$B(GT) = \sum_{\mu M_f} |\langle I_f M_f K_f | \sigma_\mu | I_i M_i K_i \rangle|^2$$

$$|IMK> = \sqrt{\frac{2I+1}{16\pi^2(1+\delta_{K,0})}} \{ D^{+I}_{MK} \phi_K + (-1)^{I-K} D^{+I}_{M-K} \phi_{\bar{K}} \}$$

$$\langle I_f M_f K_f | \sigma_\mu | I_i M_i K_i \rangle \;=\; \int d\vec{r} \left[\sum_\rho D^{+1}_{\rho\mu} \sigma^{1,\rho} \right] \left[\frac{1}{\sqrt{8\pi}} D^{+0}_{00} \phi_0 \right]$$

$$\begin{cases} [(2I_f+1)/16\pi^2]^{1/2} \left[D^{I_f}_{M_f K_f} \phi^*_{K_f} + (-1)^{I_f - K_f} D^{I_f}_{M_f - K_f} \phi^*_{\bar{K}_f} \right] \\ [(2I_f+1)/8\pi^2]^{1/2} D^{I_f}_{M_f 0} \phi^*_0 \end{cases}$$

$$\langle I_f M_f K_f | \sigma_\mu | 000 \rangle = \frac{1}{\sqrt{(2I_f+1)}} < 001\mu | I_f M_f > \begin{cases} < 0010 | I_f 0 >< \phi_0 | \sigma_0 | \phi_0 > \qquad (K_f = 0) \\ \sqrt{2} \sum_\rho < 001\rho | I_f K_f >< \phi_{K_f} | \sigma_\rho | \phi_0 > \; (K_f > 0) \end{cases}$$

In terms of intrinsic matrix elements

$$B(GT) = \delta_{K_f,0} < \phi_0 | \sigma_0 | \phi_0 >^2 + 2\delta_{K_f,1} < \phi_{K_f} | \sigma_{+1} | \phi_0 >^2$$

Intrinsic matrix elements

GT operator

$$\hat{\Theta} = \sum_{ij} < i|\vec{\sigma}|j > a^{+}_{i\nu} a_{j\pi}$$

$$a_i = u_i \alpha_i + v_i \alpha^{+}_{\bar{i}} \qquad a_{\bar{i}} = u_i \alpha_{\bar{i}} - v_i \alpha^{+}_{i}$$

$$\hat{\Theta}^0 = \sum_{ij}{}' < i|\sigma_0|j > \; [\; u^{\nu}_i u^{\pi}_j \alpha^{+\nu}_i \alpha^{\pi}_j + v^{\nu}_i v^{\pi}_j \alpha^{\nu}_{\bar{i}} \alpha^{+\pi}_{\bar{j}} + u^{\nu}_i v^{\pi}_j \alpha^{+\nu}_i \alpha^{+\pi}_{\bar{j}} + v^{\nu}_i u^{\pi}_j \alpha^{\nu}_{\bar{i}} \alpha^{\pi}_j$$
$$- u^{\nu}_j u^{\pi}_i \alpha^{+\nu}_{\bar{j}} \alpha^{\pi}_{\bar{i}} - v^{\nu}_j v^{\pi}_i \alpha^{\nu}_j \alpha^{+\pi}_i + u^{\nu}_j v^{\pi}_i \alpha^{+\nu}_{\bar{j}} \alpha^{+\pi}_i + v^{\nu}_j u^{\pi}_i \alpha^{\nu}_j \alpha^{\pi}_{\bar{i}}]$$

$$\hat{\Theta}^0 |\phi_0 > = \sum_{ij}{}' < i|\sigma^0|j > \left[+ u^{\nu}_i v^{\pi}_j \alpha^{+\nu}_i \alpha^{+\pi}_{\bar{j}} + u^{\nu}_j v^{\pi}_i \alpha^{+\nu}_{\bar{j}} \alpha^{+\pi}_i \right] |\phi_0 >$$

$$< \alpha\beta|\hat{\Theta}^{\mu}|\phi_0 > \qquad |\alpha\beta > = \alpha^{+\nu}_{\alpha} \alpha^{+\pi}_{\beta} |\phi_0 >$$

$$\mu = 0 \qquad K_i = K_f = 0 \qquad \alpha\bar{\beta} \quad \bar{\alpha}\beta$$

$$\begin{aligned} < \bar{\alpha}\beta|\hat{\Theta}^0|\phi_0 > &= \sum_{ij}{}' < i|\sigma_0|j >< \phi_0|\alpha^{\pi}_{\beta}\alpha^{\nu}_{\bar{\alpha}}[+u^{\nu}_i v^{\pi}_j \alpha^{+\nu}_i \alpha^{+\pi}_{\bar{j}} + u^{\nu}_j v^{\pi}_i \alpha^{+\nu}_{\bar{j}} \alpha^{+\pi}_i]|\phi_0 > \\ &= \sum_{ij}{}' < i|\sigma_0|j >< \phi_0|u^{\nu}_j v^{\pi}_i \delta_{\beta i}\delta_{\bar{\alpha}\bar{j}}|\phi_0 > \\ &= u^{\nu}_{\alpha} v^{\pi}_{\beta} < \beta|\sigma_0|\alpha > \end{aligned}$$

Intrinsic matrix elements

$0^+0 \rightarrow 1^+0$

$$< \bar{\alpha}\beta|\hat{\Theta}^0|\phi_0 >= u^\nu_\alpha v^\pi_\beta < \beta|\sigma_0|\alpha > \qquad (K_\alpha = K_\beta > 0)$$
$$< \alpha\bar{\beta}|\hat{\Theta}^0|\phi_0 >= u^\nu_\alpha v^\pi_\beta < \alpha|\sigma_0|\beta > \qquad (K_\alpha = K_\beta > 0)$$

$0^+0 \rightarrow 1^+1$

$$< \alpha\beta|\hat{\Theta}^{+1}|\phi_0 >= -u^\nu_\alpha v^\pi_\beta < \alpha|\sigma_{+1}|\bar{\beta} > \qquad (K_\alpha = K_\beta = 1/2)$$
$$< \bar{\alpha}\beta|\hat{\Theta}^{+1}|\phi_0 >= u^\nu_\alpha v^\pi_\beta < \beta|\sigma_{+1}|\alpha > \qquad (K_\beta = K_\alpha + 1)$$
$$< \alpha\bar{\beta}|\hat{\Theta}^{+1}|\phi_0 >= u^\nu_\alpha v^\pi_\beta < \alpha|\sigma_{+1}|\beta > \qquad (K_\alpha = K_\beta + 1)$$

$$\sigma_\rho|sm_s >= \sqrt{1+|\rho|}\,(2m_s)\,|s\,m_s+\rho >$$

$$< N'n'_z m'_\ell m'_s|\sigma_\rho|N n_z m_\ell m_s >= \delta_{NN'}\delta_{n_z n'_z}\delta_{m_\ell m'_\ell}\delta_{\rho+m_s,m'_s}\,(2m_s)\,\sqrt{1+|\rho|}$$

$$|\alpha >= \sum_{N n_z m_\ell} C^\alpha_{N n_z m_\ell m_s}|N n_z m_\ell m_s >$$

$$< \alpha|\sigma_\rho|\beta >= \sum_{N n_z m_\ell} C^\alpha_{N n_z m_\ell m'_s} C^\beta_{N n_z m_\ell m_s}\delta_{m'_s,\rho+m_s}(2m_s)\sqrt{1+|\rho|}$$
$$< \alpha|\sigma_\rho|\bar{\beta} >= \sum_{N n_z m_\ell} C^\alpha_{N n_z m_\ell m'_s} C^\beta_{N n_z m_\ell m_s}\delta_{0,m_\ell}\delta_{m'_s,m_s}\delta_{1/2,m_s}(-\sqrt{2})$$

$$B(GT) = \delta_{K_f,0} < \phi_0|\sigma_0|\phi_0 >^2 + 2\delta_{K_f,1} < \phi_{K_f}|\sigma_{+1}|\phi_0 >^2$$

Half-lives

$$T_{1/2}^{-1} = \frac{G^2}{D} \sum_{I_f} f^{\beta^+/EC} |\langle I_f \| \beta^+ \| I_i \rangle|^2$$

$$D = 6200 \text{ s}$$

$$G^2 = [(g_A/g_V)_{\text{eff}}]^2 = [0{,}77 (g_A/g_V)_{\text{free}}]^2 = 0{,}90$$

$$f^{\beta^\pm}(Z, W_0) = \int_1^{W_0} pW(W_0 - W)^2 \lambda^\pm(Z, W) \mathrm{d}W$$

phase space — Coulomb effect

$$\lambda^\pm(Z, W) = 2(1+\gamma)(2pR)^{-2(1-\gamma)} e^{\mp\pi y} \frac{|\Gamma(\gamma + iy)|^2}{[\Gamma(2\gamma+1)]^2}$$

$$\gamma = \sqrt{1 - (\alpha Z)^2} \qquad y = \alpha Z W / p$$

$$f^{EC} = \frac{\pi}{2} \left[q_K^2 g_K^2 B_K + q_{L_1}^2 g_{L_1}^2 B_{L_1} + q_{L_2}^2 g_{L_2}^2 B_{L_2} \right]$$

Neutrino energy

Exchange and overlap corrections

Radial components of the bound state electron wf at the origin

Limiting cases

B(GT) (2qp) V_{res} =0

B(GT) (QTDA) Y^{ω}=0 $1/\left(\omega_K + \mathcal{E}_{i_K}\right) \to 0$

Ikeda sum rule

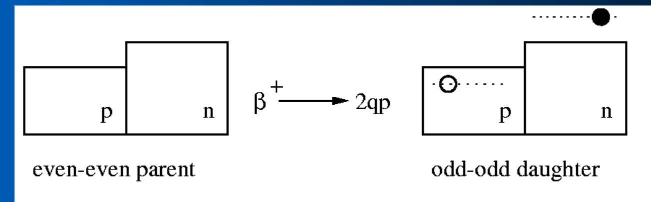
$$S_{GT}^{-} - S_{GT}^{+} = \sum_{K=0,\pm} \sum_{\omega_K} \left[\left| \langle \omega_K | \beta^- | \phi_0 \rangle \right|^2 - \left| \langle \omega_K | \beta^+ | \phi_0 \rangle \right|^2 \right] = 3\,(N - Z)$$

Type of GT transitions

Even-even nuclei $0^+0 \rightarrow 1^+K$

- (0qp→2qp)

$$\langle \omega_K | \beta_K^{\pm} | 0 \rangle = \mp Z_{\pm}^{\omega_K}$$

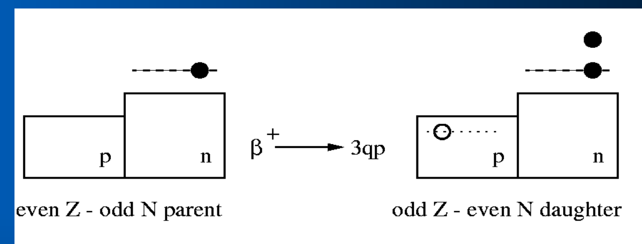


$E_{ex,2qp} = \omega - E_{p0} - E_{n0}$

Odd-A nuclei $I_i^{\pi} K_i \rightarrow I_f^{\pi} K_f$

- Phonon excitations : (1qp→3qp)
  Odd nucleon acts as a spectator

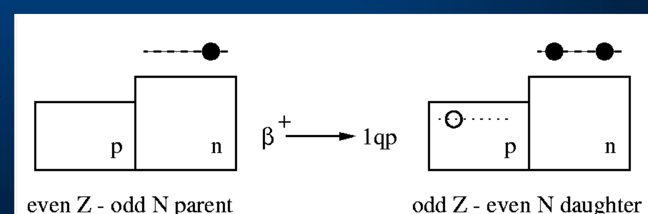
$$\left\langle f \left| \beta_K^{+} \right| i \right\rangle_{3qp} = \left\langle \omega_K, 1qp \left| \beta_K^{\pm} \right| 0, 1qp \right\rangle$$



$E_{ex,3qp} = \omega + E_{n,\,spect} - E_{p0} > 2\Delta$

- Transitions involving the odd nucleon state : (1qp→1qp)

$$\left\langle f \left| \beta_K^{+} \right| i \right\rangle_{1qp} = \left\langle \pi_{corr} \left| \beta_K^{+} \right| \nu_{corr} \right\rangle$$

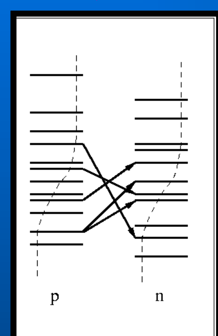


$E_{ex,1qp} = E_p - E_{p0}$

Qualitative behaviour of Matrix Elements

$$v_{\pi} u_{\nu} \ \langle \nu \,|GT|\, \pi \rangle$$

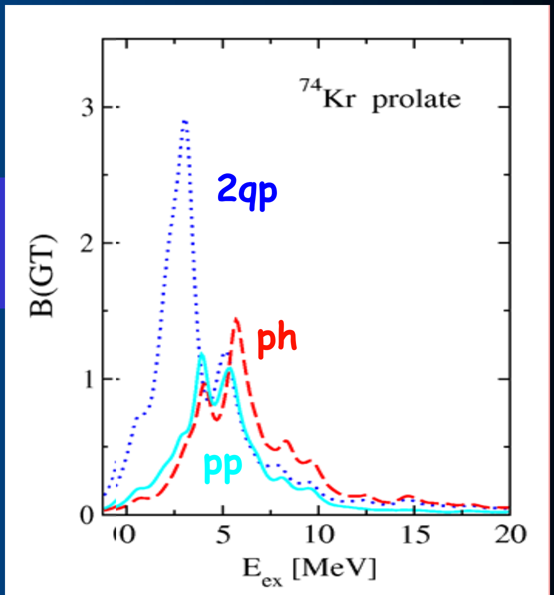
Skyrme HF | **+ def** | **+ BCS** | **+ QRPA (ph,pp)**

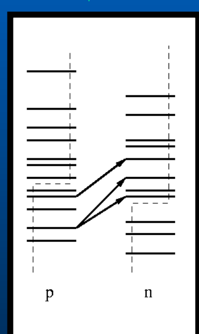


Smooth occupations

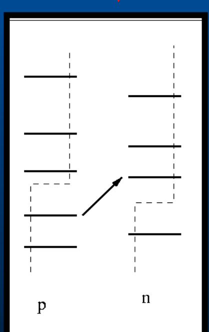
New excitations

Redistribution and reduction of the strength



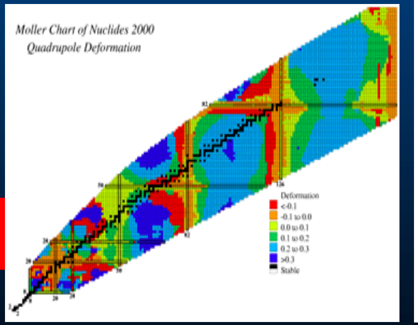


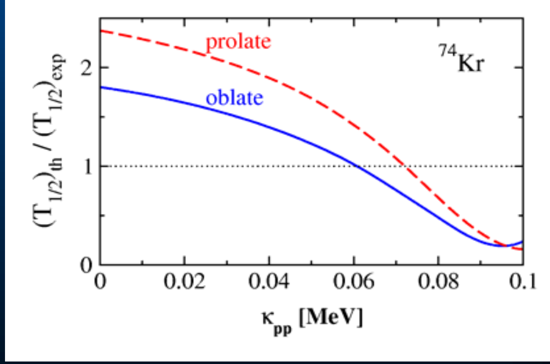
Fragmentation



Selfconsistent mean field with Skyrme forces

Suitable for extrapolations to unknown regions





GT strength in the lab frame

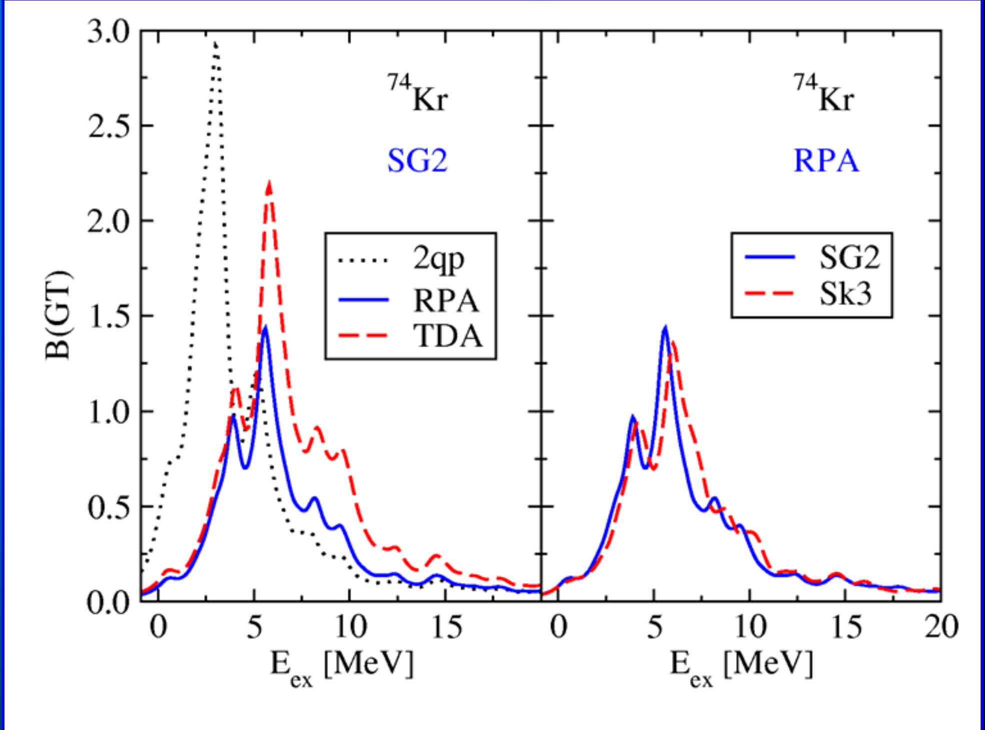
$$B_{GT}^{\pm} = \frac{g_A^2}{4\pi} \sum_{M_i, M_f, \mu} \left| \left\langle I_f M_f \left| \beta_\mu^{\pm} \right| I_i M_i \right\rangle \right|^2$$

$$|IMK\rangle = \sqrt{\frac{2I+1}{16\pi^2(1+\delta_{K0})}} \left\{ D_{KM}^{+I}(\Omega) |\phi_K\rangle + (-1)^{I-K} D_{-KM}^{+I}(\Omega) |\phi_{\bar{K}}\rangle \right\}$$

even-even nuclei

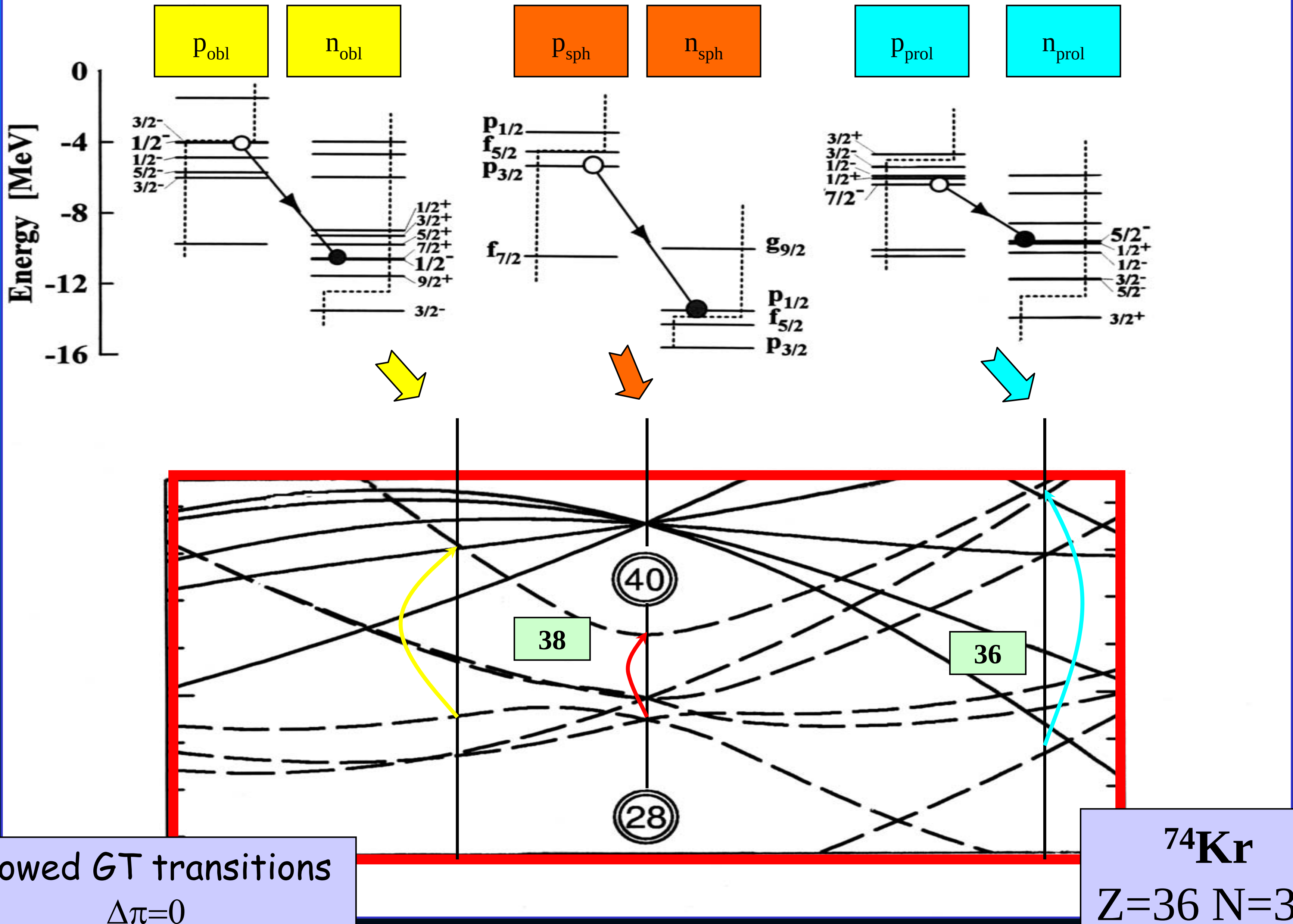
$$B_{GT}^{\pm} = \frac{g_A^2}{4\pi} \left\{ \delta_{K_f,0} \left\langle \phi_{K_f} \left| \beta_0^{\pm} \right| \phi_0 \right\rangle^2 + 2\delta_{K_f,1} \left\langle \phi_{K_f} \left| \beta_1^{\pm} \right| \phi_0 \right\rangle^2 \right\}$$

2qp / TDA / RPA	Skyrme force



TDA : (Y →0); 2qp : (V_{res}→0)

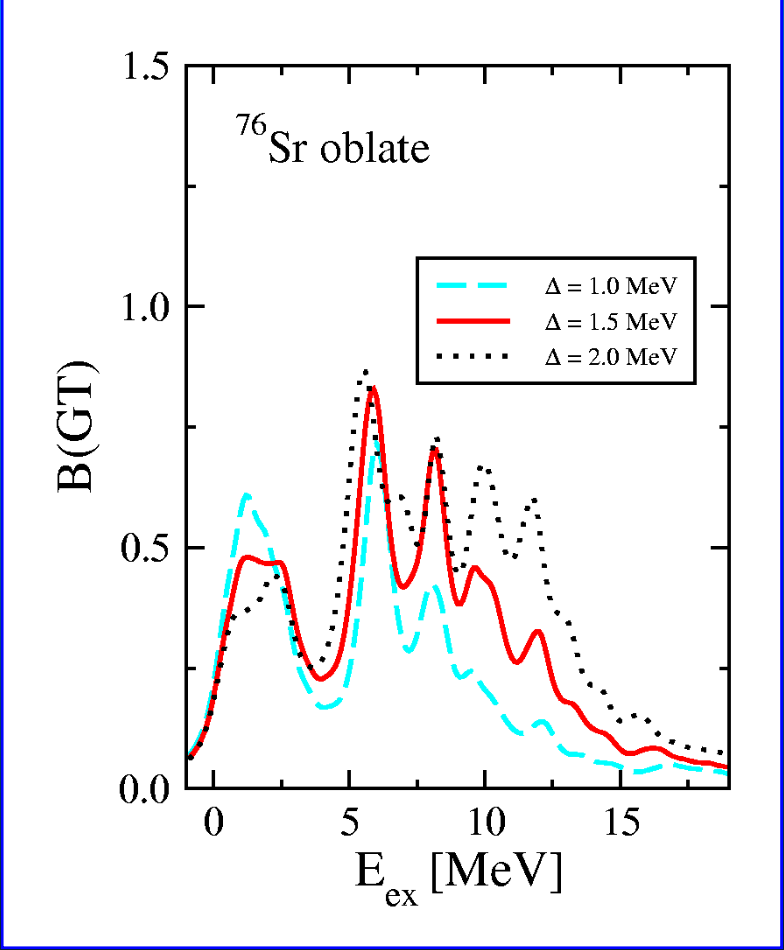
Role of deformation

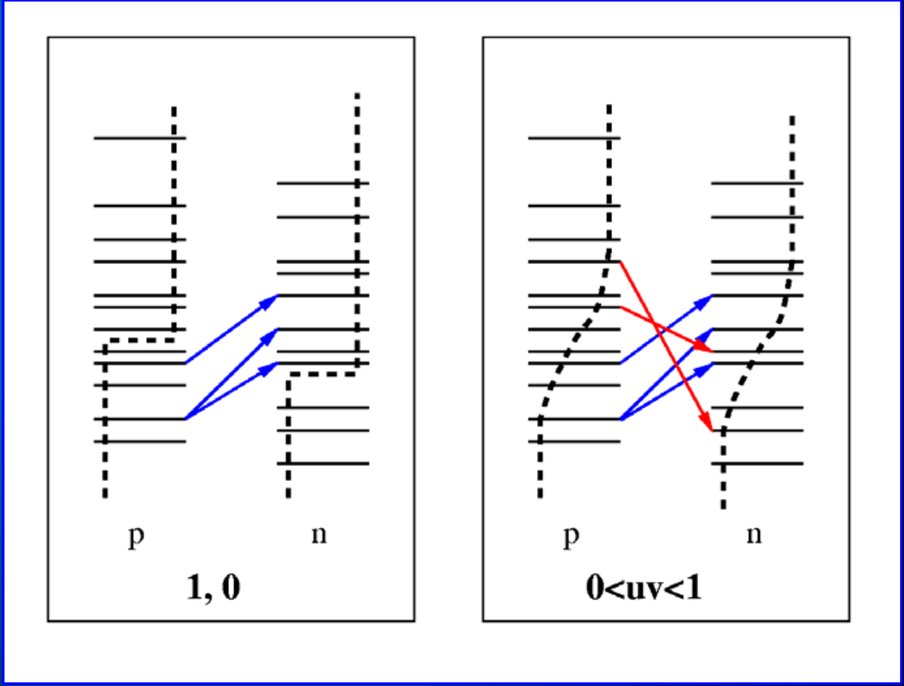


Allowed GT transitions
$\Delta\pi=0$

^{74}Kr
Z=36 N=38

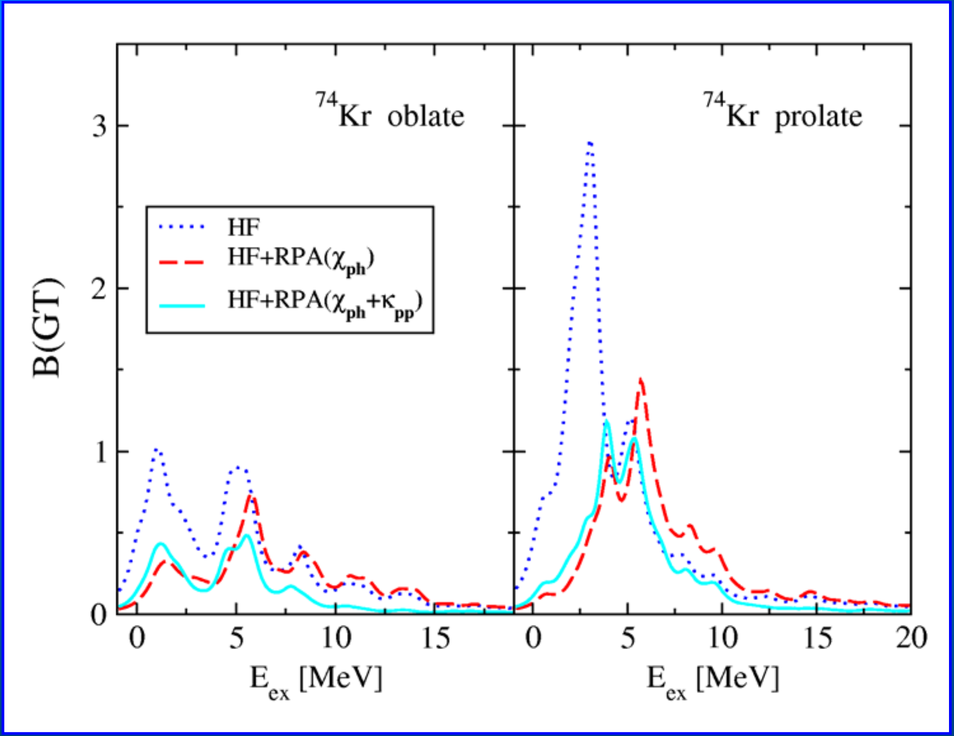
Role of pairing

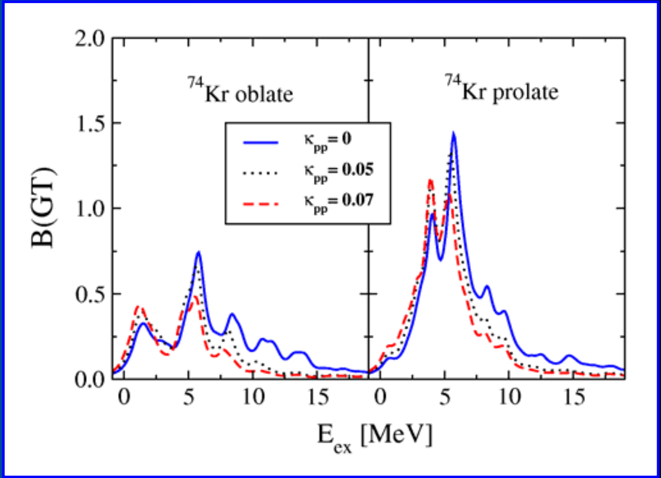


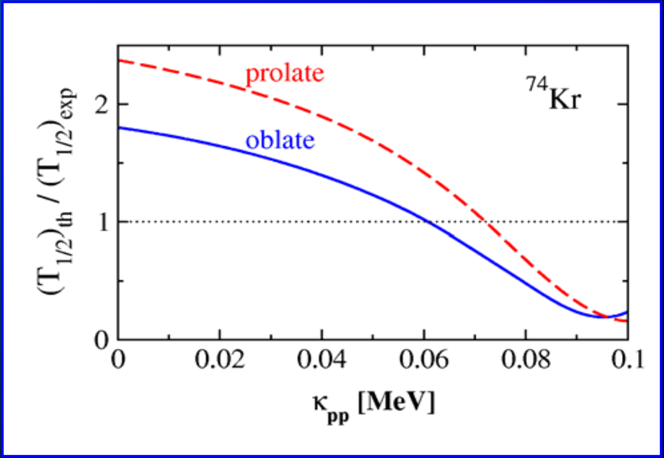


- Existing peaks: decrease with increasing Δ (at low energy)
- New peaks appear: increase with increasing Δ (at high energy)

Role of residual interactions

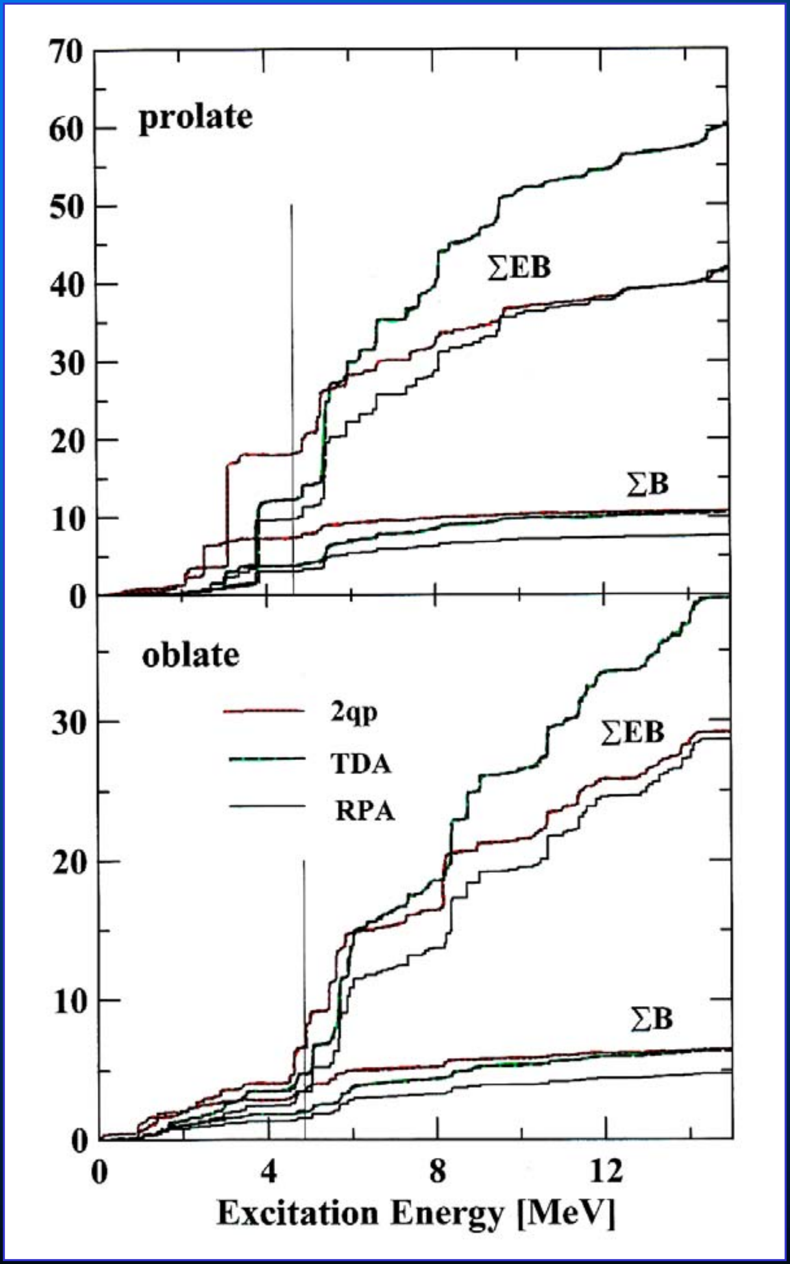






- Redistribution of the strength
  - ➢ ph : shift to higher energies
  - ➢ pp : shift to lower energies
- Reduction of the strength

Sum rules


70
60
50
40
30
20
10
0
prolate
ΣEB
ΣB
oblate
2qp
TDA
RPA
30
20
10
0
ΣEB
ΣB
0
4
8
12
Excitation Energy [MeV]

Nuclear Structure

Stable nuclei in Fe-Ni mass region

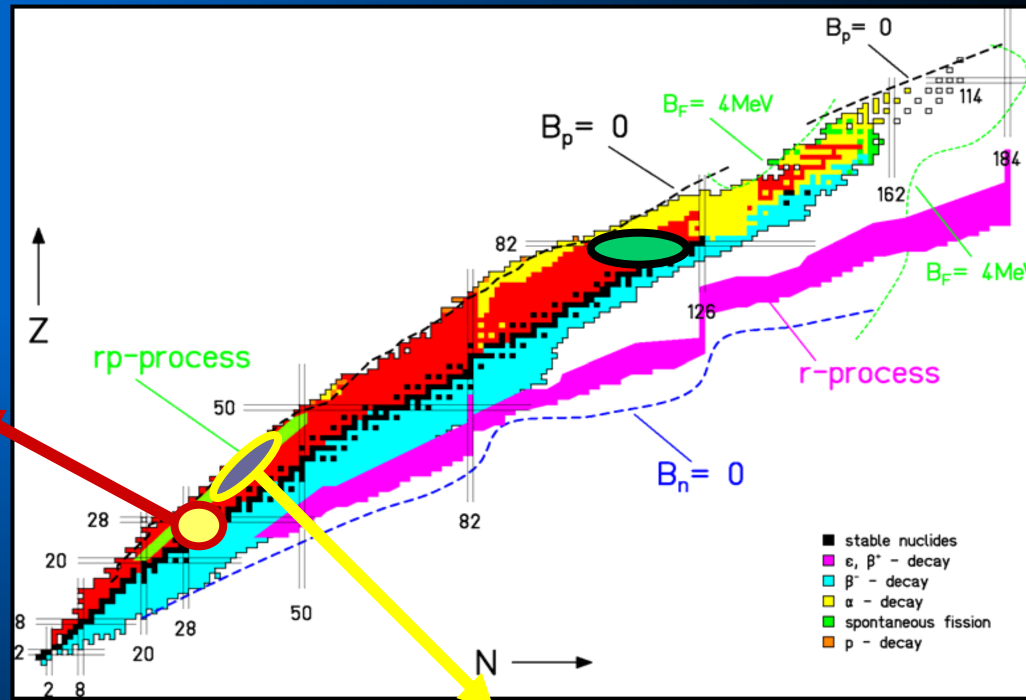
Medium-mass proton-rich nuclei : Ge, Se, Kr, Sr

Main constituents of stellar core in presupernovae

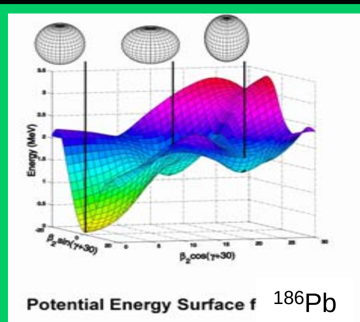
GT properties. Test of QRPA

Comparison with :

- exp. (n,p), (p,n)
- SM calculations



Neutron deficient Pb-Hg Isotopes



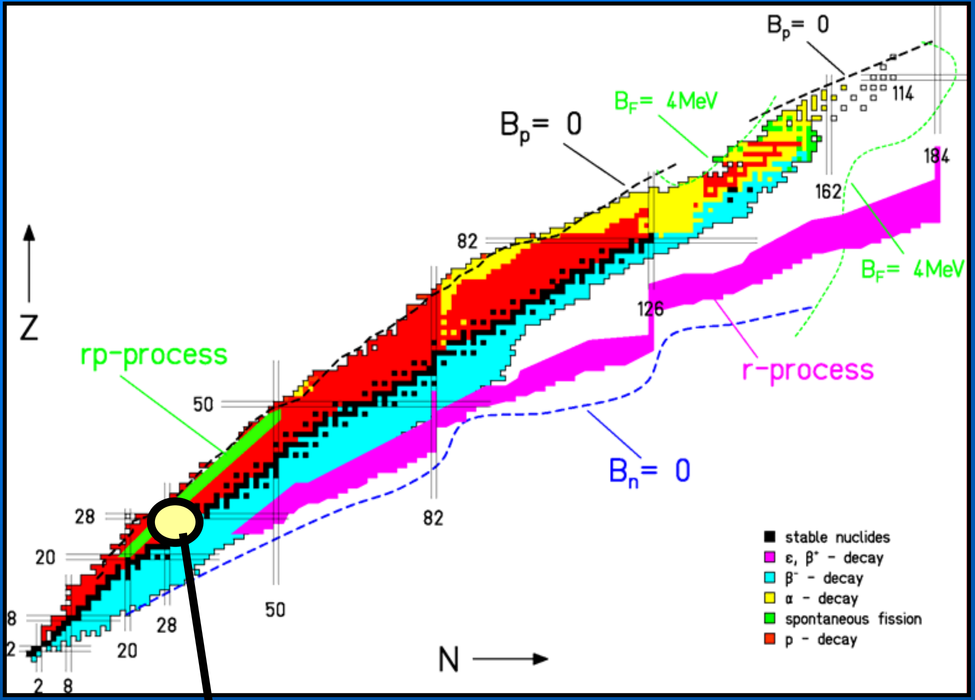
Waiting points in rp process

Shape coexistence

Isotopic chains approaching drip lines

Large Q values

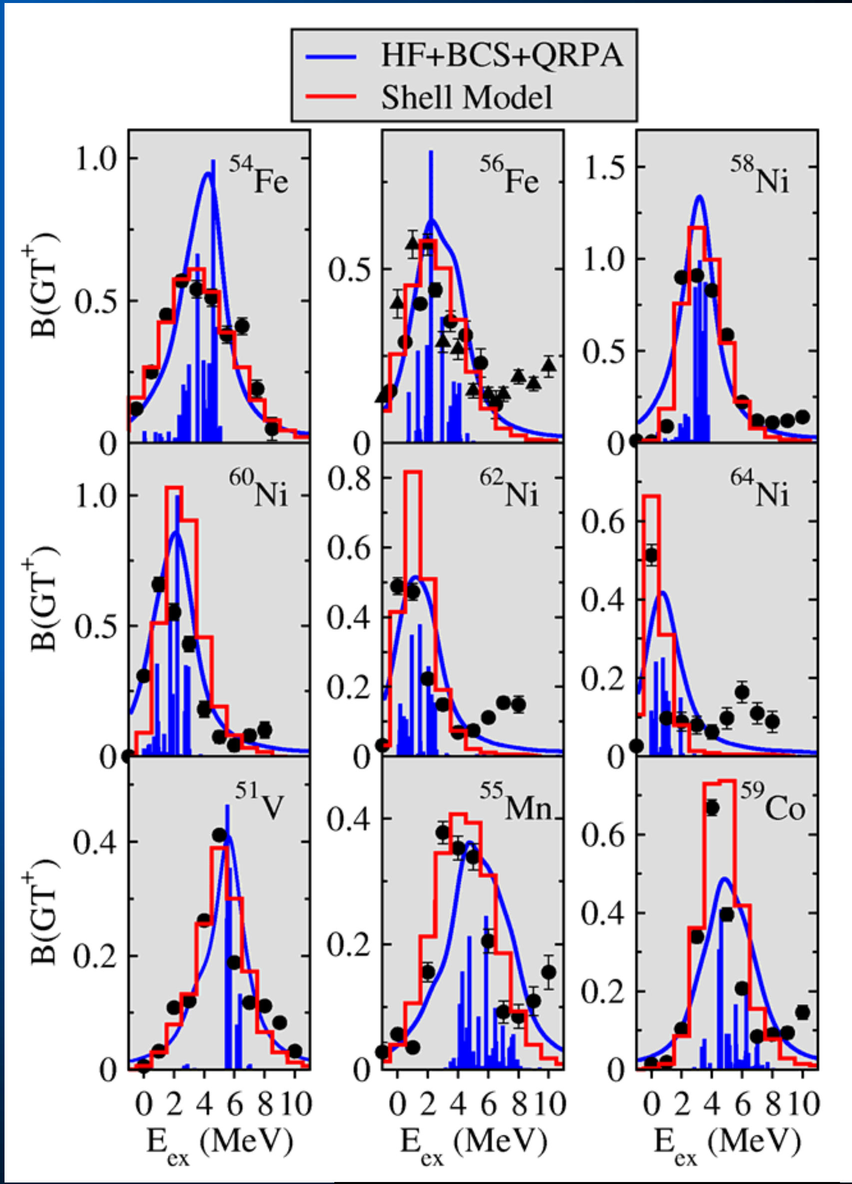
Stable nuclei in Fe-Ni mass region: Theory and experiment



Main constituents of stellar core in presupernovae

Comparison with :

- exp. (n,p), (p,n)
- SM calculations



GT properties: Test of QRPA

SM : Caurier et al. NPA 653(99)439

Stable nuclei in Fe-Ni mass region. GT strength: Theory and experiment

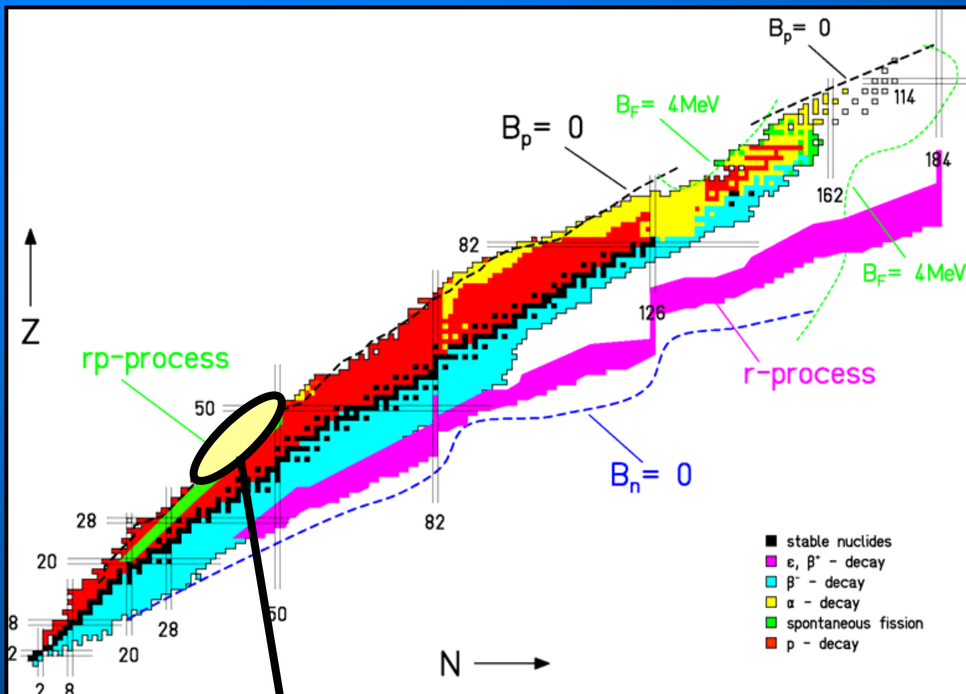
Total B(GT^+) strength

	exp.	QRPA	HF+BCS	SM
^{51}V	1.2 (0.1)	1.61	2.03	1.37
^{54}Fe	3.5 (0.7)	4.24	5.14	3.50
^{55}Mn	1.7 0(.2)	2.18	2.72	2.10
^{56}Fe	2.9 (0.3)	3.24	4.16	2.63
^{58}Ni	3.8 (0.4)	5.00	6.19	4.00
^{59}Co	1.9 (0.1)	2.50	3.26	2.50
^{60}Ni	3.11 (0.08)	3.72	4.97	3.29
^{62}Ni	2.53 (0.07)	2.36	3.40	2.08
^{64}Ni	1.72 (0.09)	1.65	2.65	1.19

Total B(GT^-) strength

	Exp.	QRPA	SM
^{54}Fe	7.8 (1.9)	7.0	6.9
^{56}Fe	9.9 (2.4)	9.0	9.3
^{58}Ni	7.5 (1.8)	7.8	7.7
^{60}Ni	7.2 (1.8)	9.4	10.0

Medium mass proton rich nuclei: Ge, Se, Kr Sr



Waiting point nuclei in rp-processes

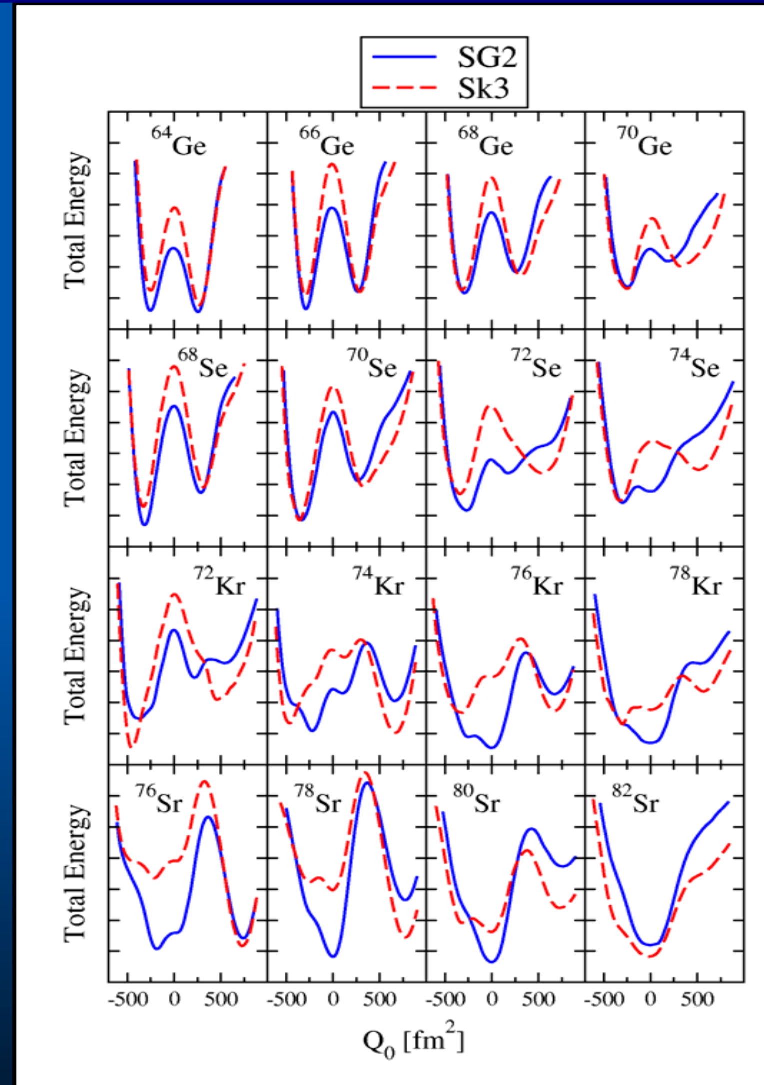
Shape coexistence

Large Q-values

Isotopic chains approaching the drip lines

Beyond full Shell Model

Constrained HF+BCS

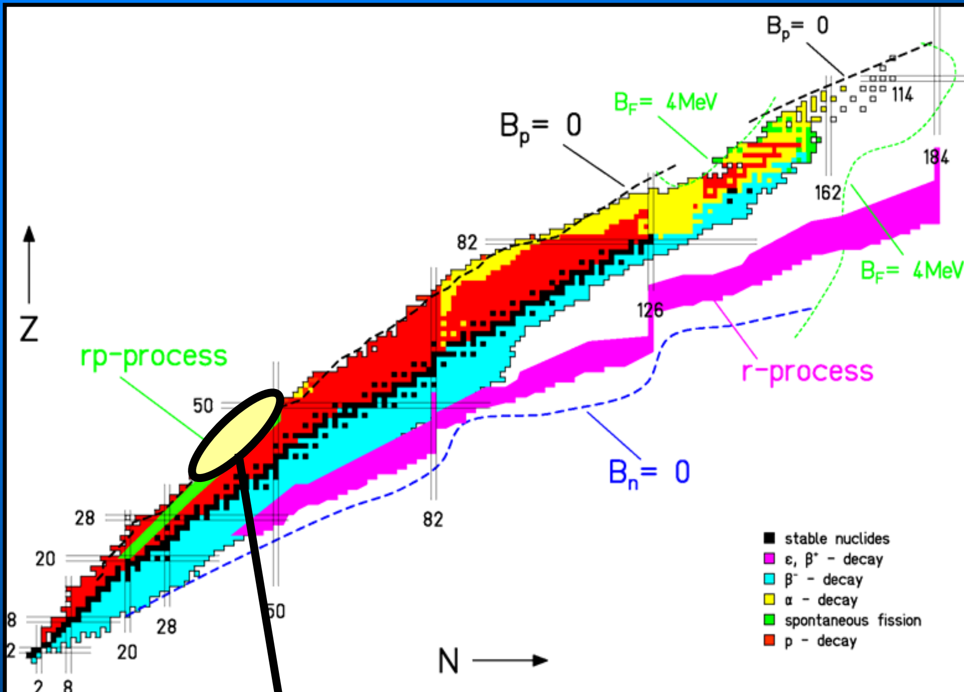


Energy vs. deformation

Shape coexistence

Medium mass proton rich nuclei: Ge, Se, Kr Sr

Constrained HF+BCS



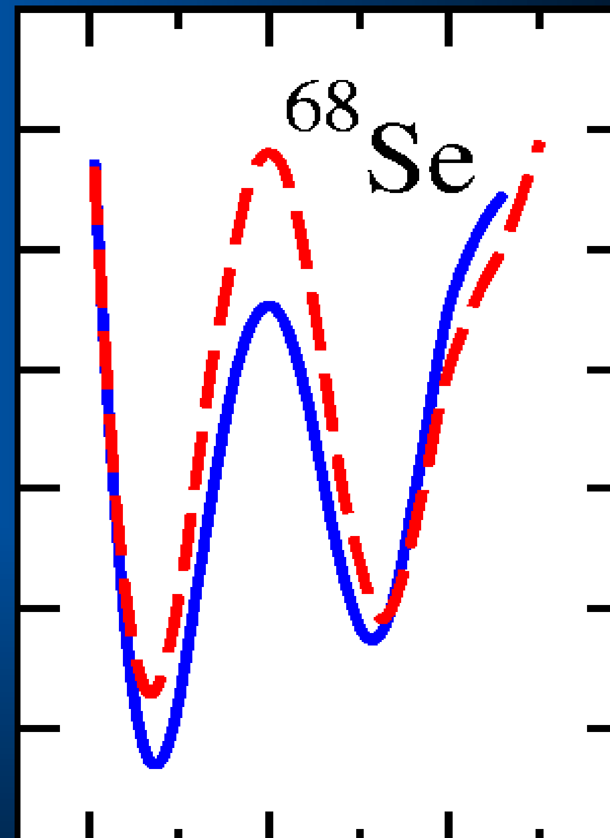
Waiting point nuclei in rp-processes

Shape coexistence

Large Q-values

Isotopic chains approaching the drip lines

Beyond full Shell Model

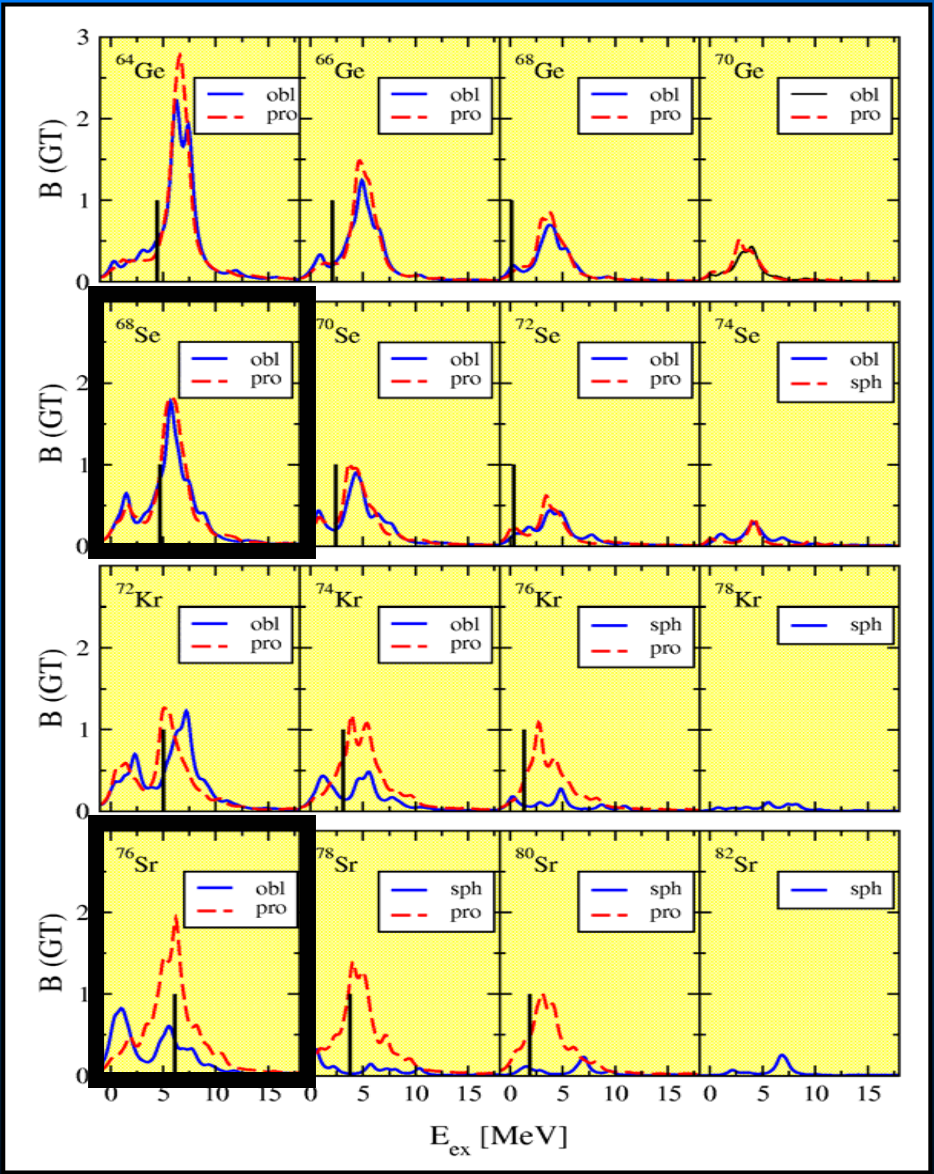


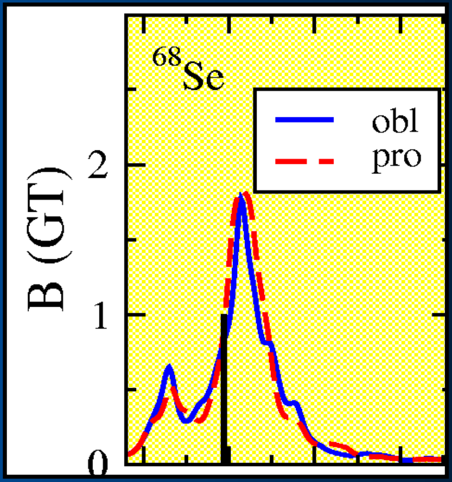
Energy vs. deformation

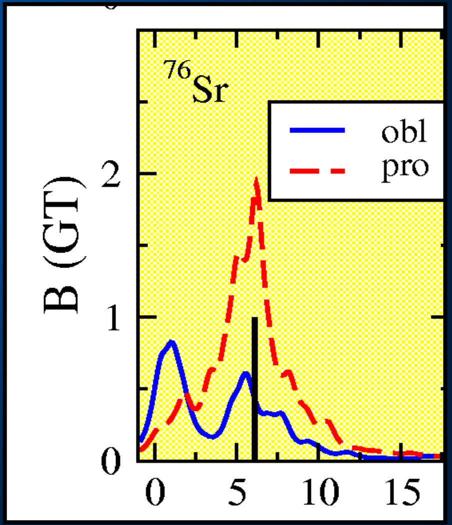
Shape coexistence

Medium mass proton rich nuclei: Ge, Se, Kr Sr

GT strength distributions

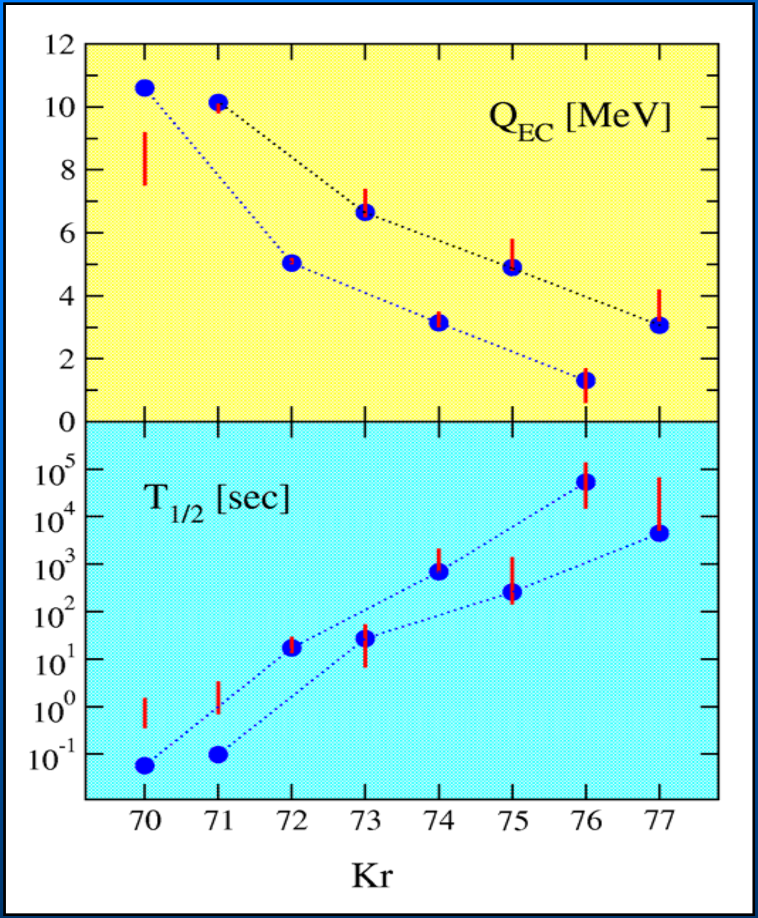


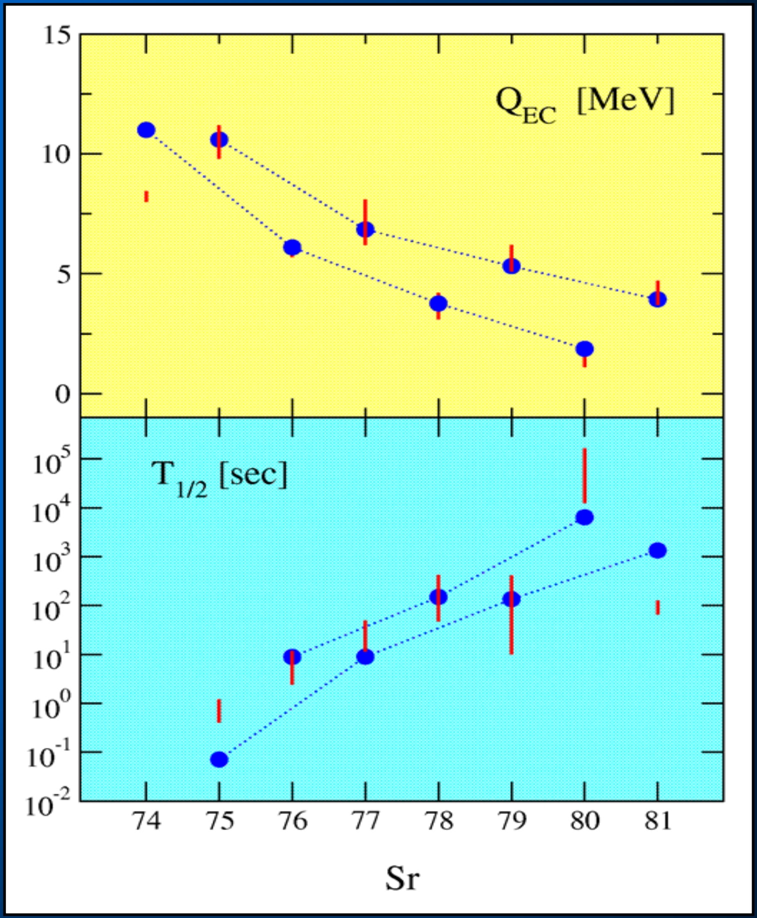




Dependence on deformation

Q_{EC} and $T_{1/2}$: Theory and Experiment



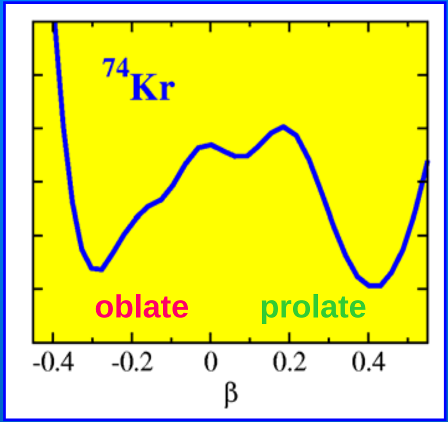


$$Q_{EC} = \left[M_{\text{parent}} - M_{\text{daughter}} + m_e\right] c^2$$

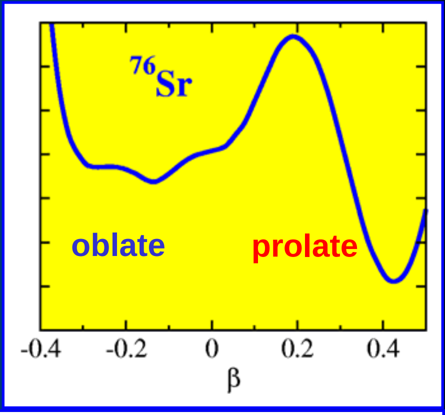
$$T_{1/2}^{-1} = \frac{\kappa^2}{6200} \sum_{\omega} f(Z,\omega) \left|\langle 1_\omega^+ \| \beta^+ \| 0^+ \rangle\right|^2$$

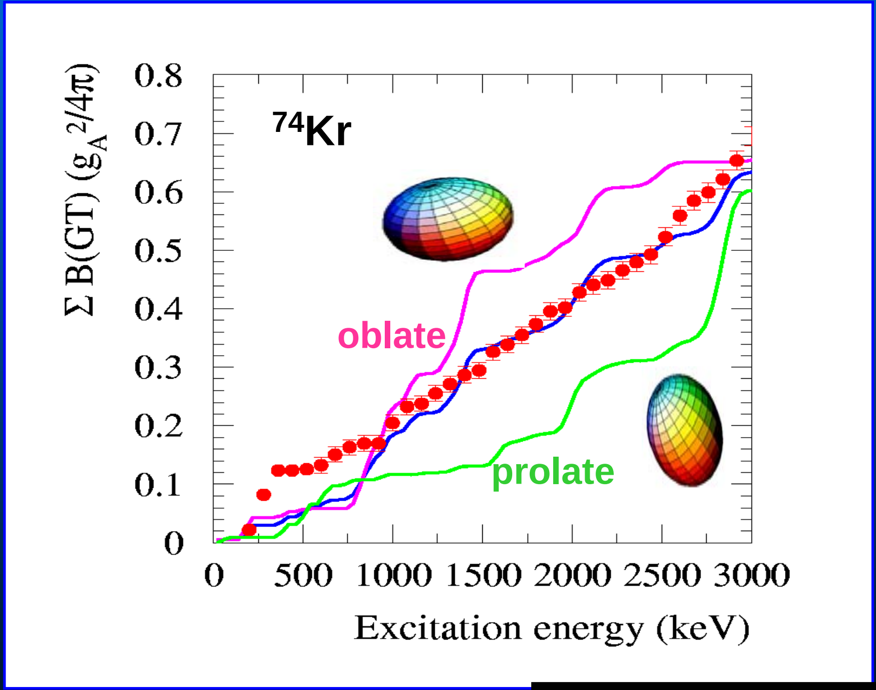
$$\kappa^2 = \left[(g_A/g_V)_{\text{eff}}\right]^2$$

β–decay: Nuclear Structure: Deformation

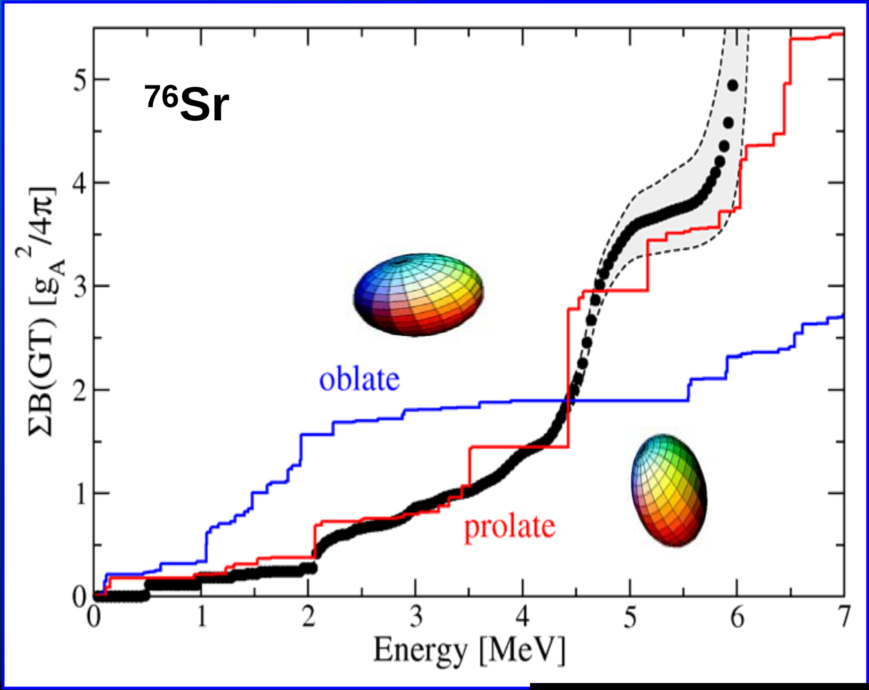


Gamow-Teller strength: Theory and Experiment



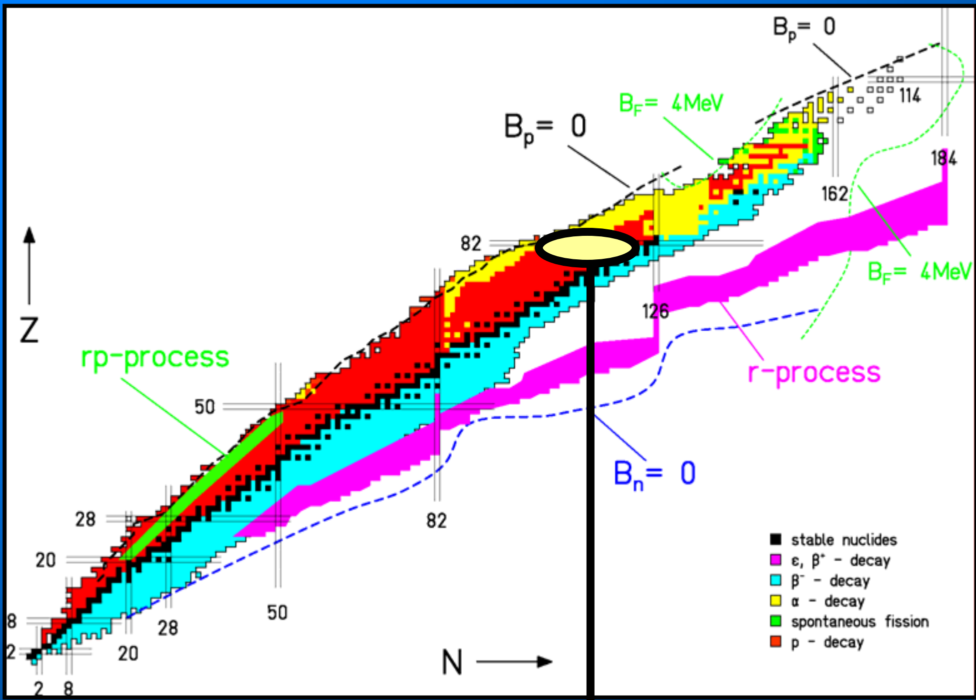


Exp: Poirier et al. 2003

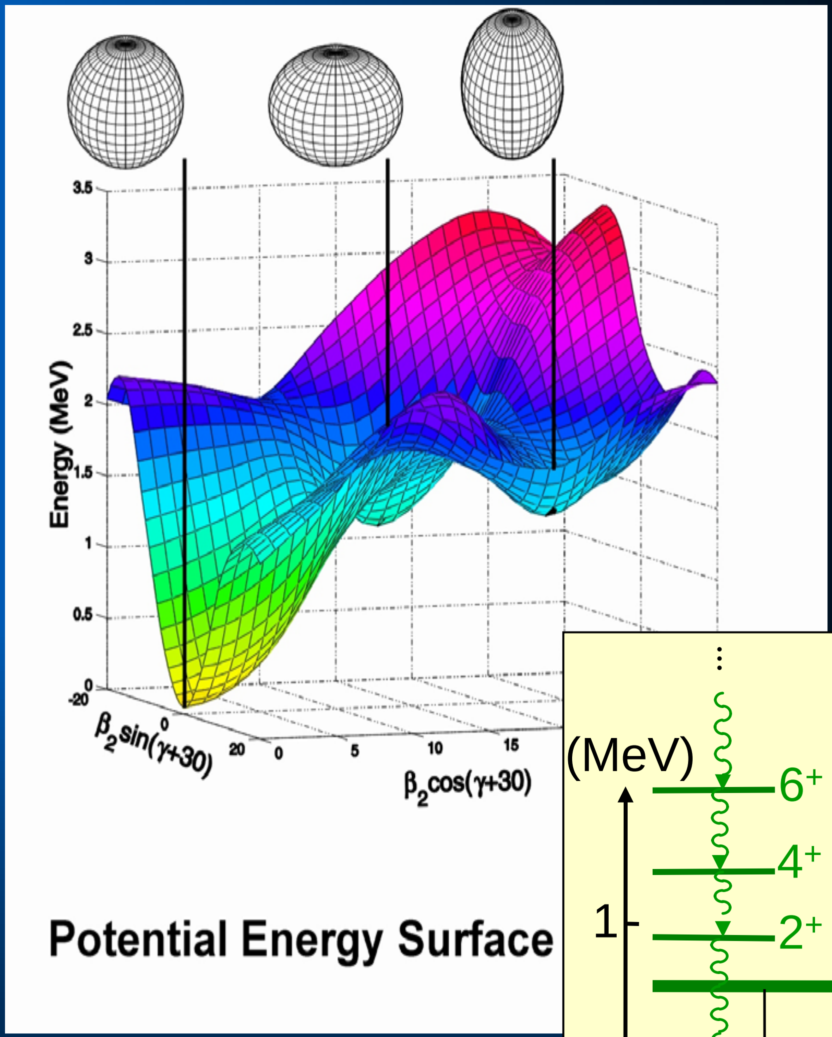


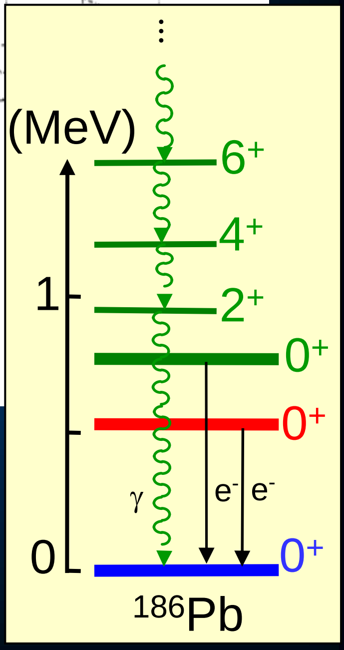
Exp: Nácher et al. 2004

Shape dependence of GTdistributions in neutron-deficient Hg, Pb, Po isotopes

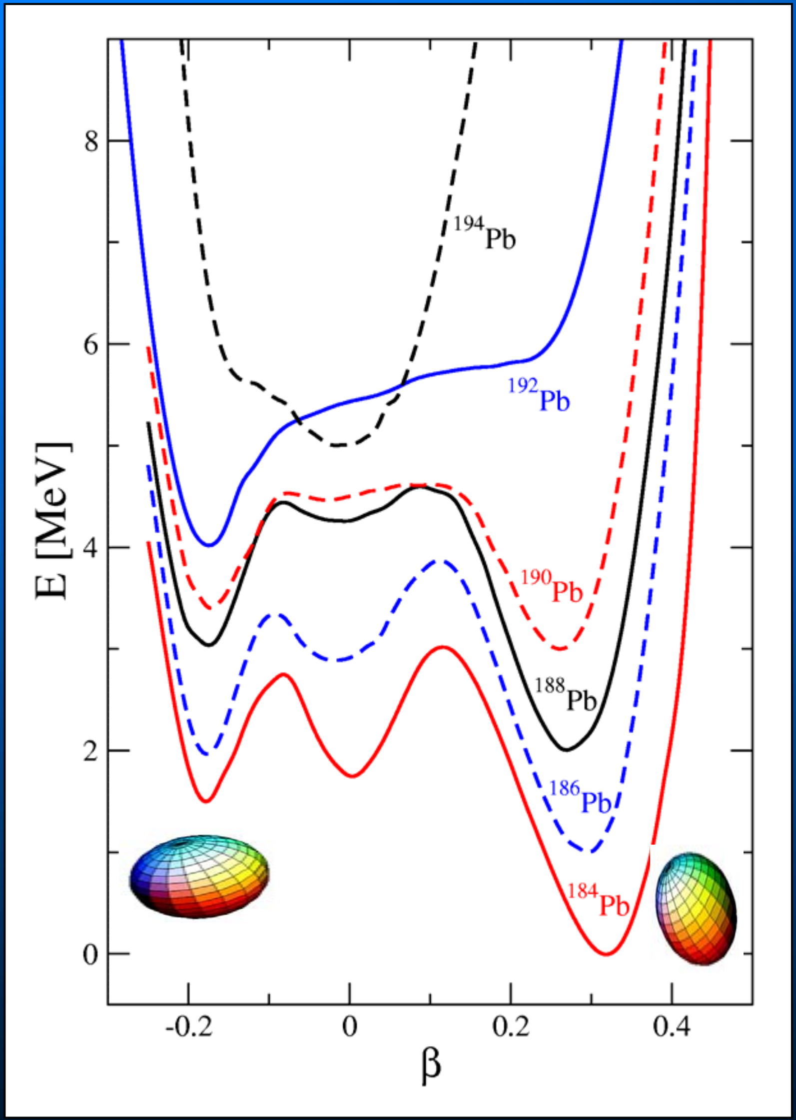


- Triple shape coexistence at low excitation energy
- Search for signatures of deformation on their beta-decay patterns

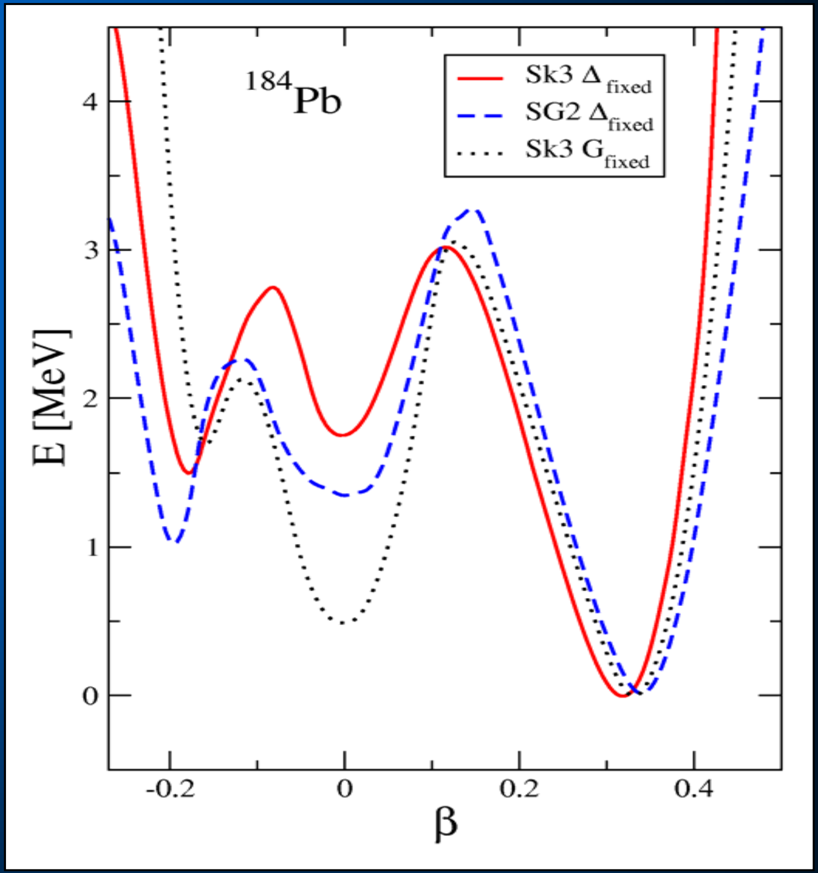




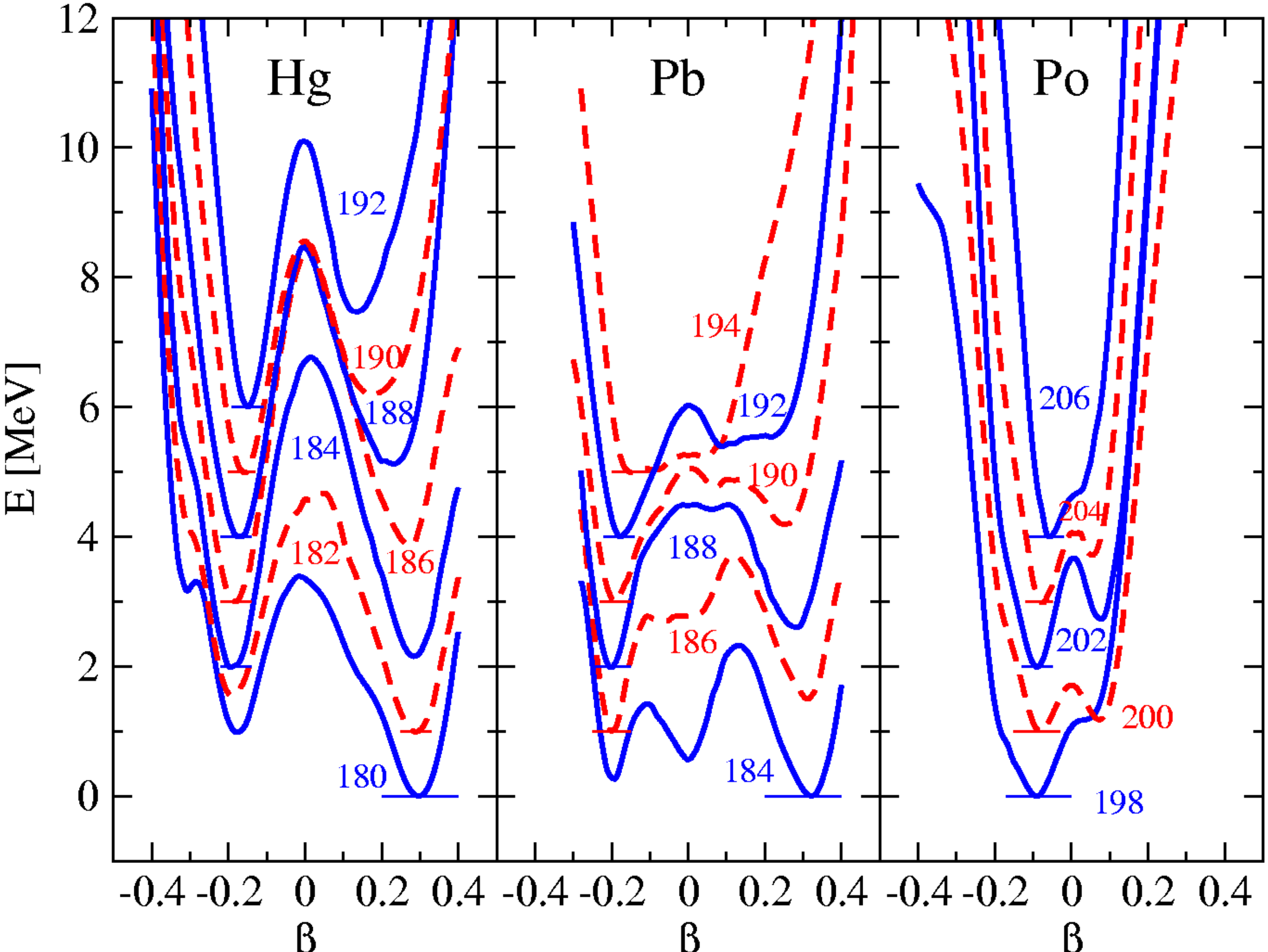
Energy-deformation curves : Pb isotopes



Skyrme force and pairing treatment



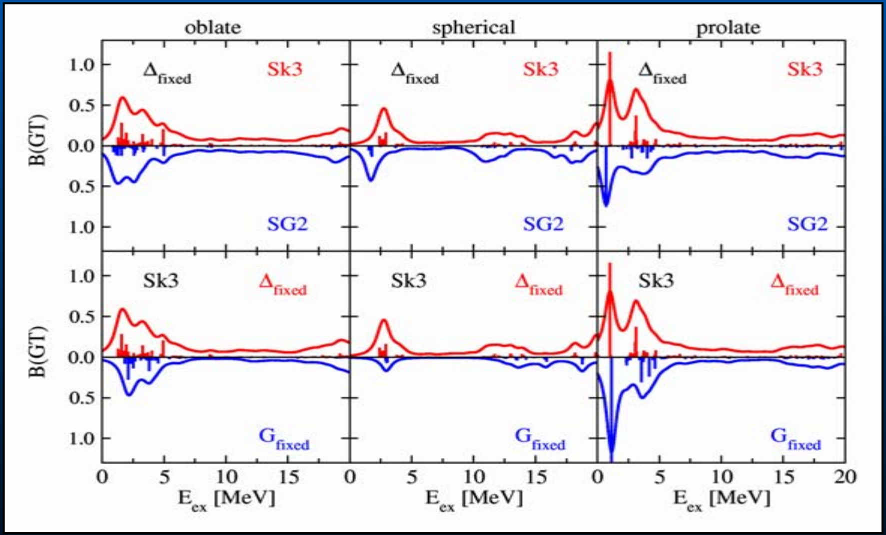
- Influence: Relative energy of minima
- Little influence : location of minima

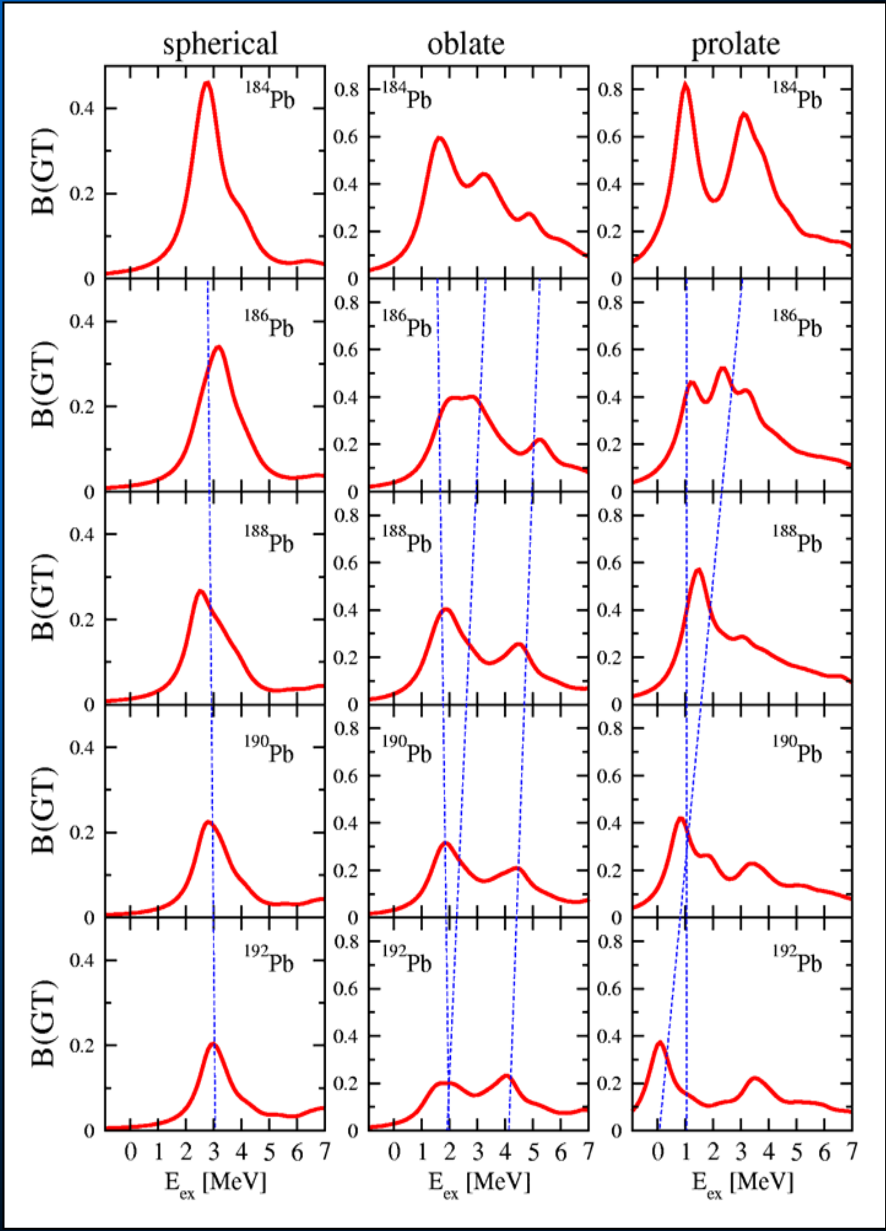

Hg
Pb
Po
E [MeV]
β
192
190
188
184
182
186
180
194
192
190
188
186
184
206
204
202
200
198
12
10
8
6
4
2
0
-0.4
-0.2
0
0.2
0.4

GT strength distributions : Pb isotopes

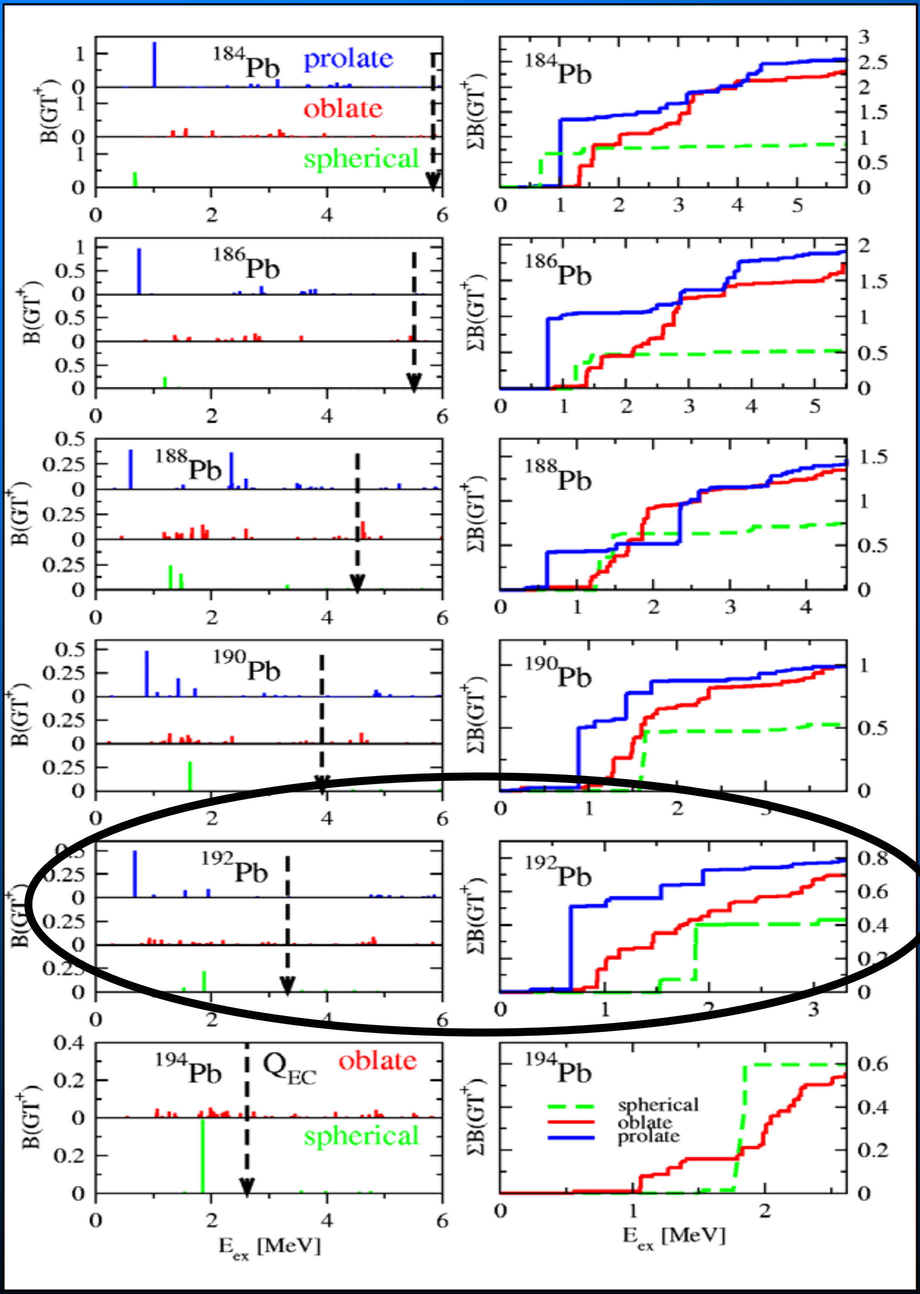
B(GT) strength distributions

- **Not very sensitive to** : Skyrme force and pairing treatment
- **Sensitive to** : Nuclear shape

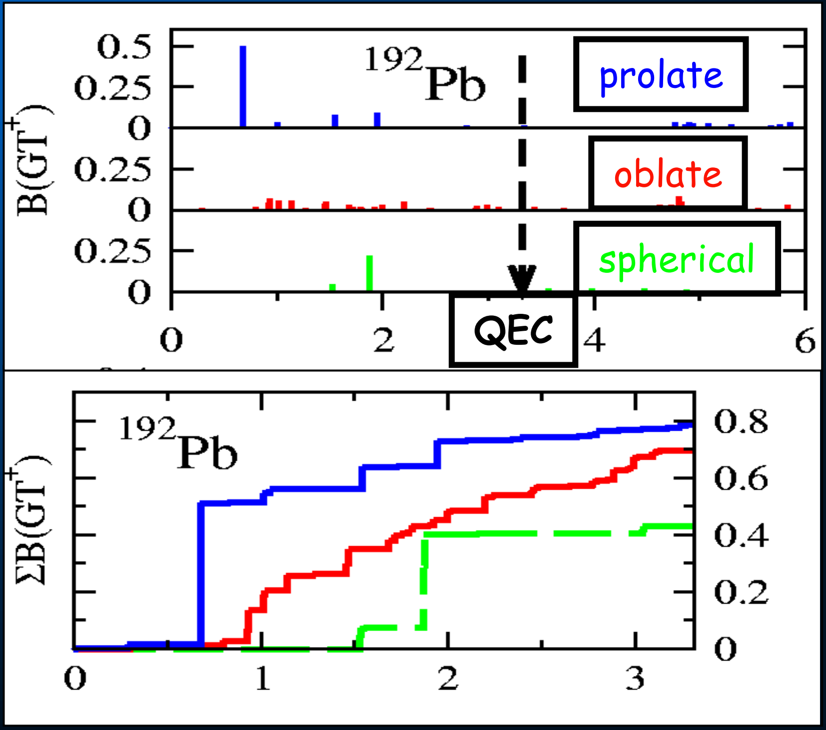




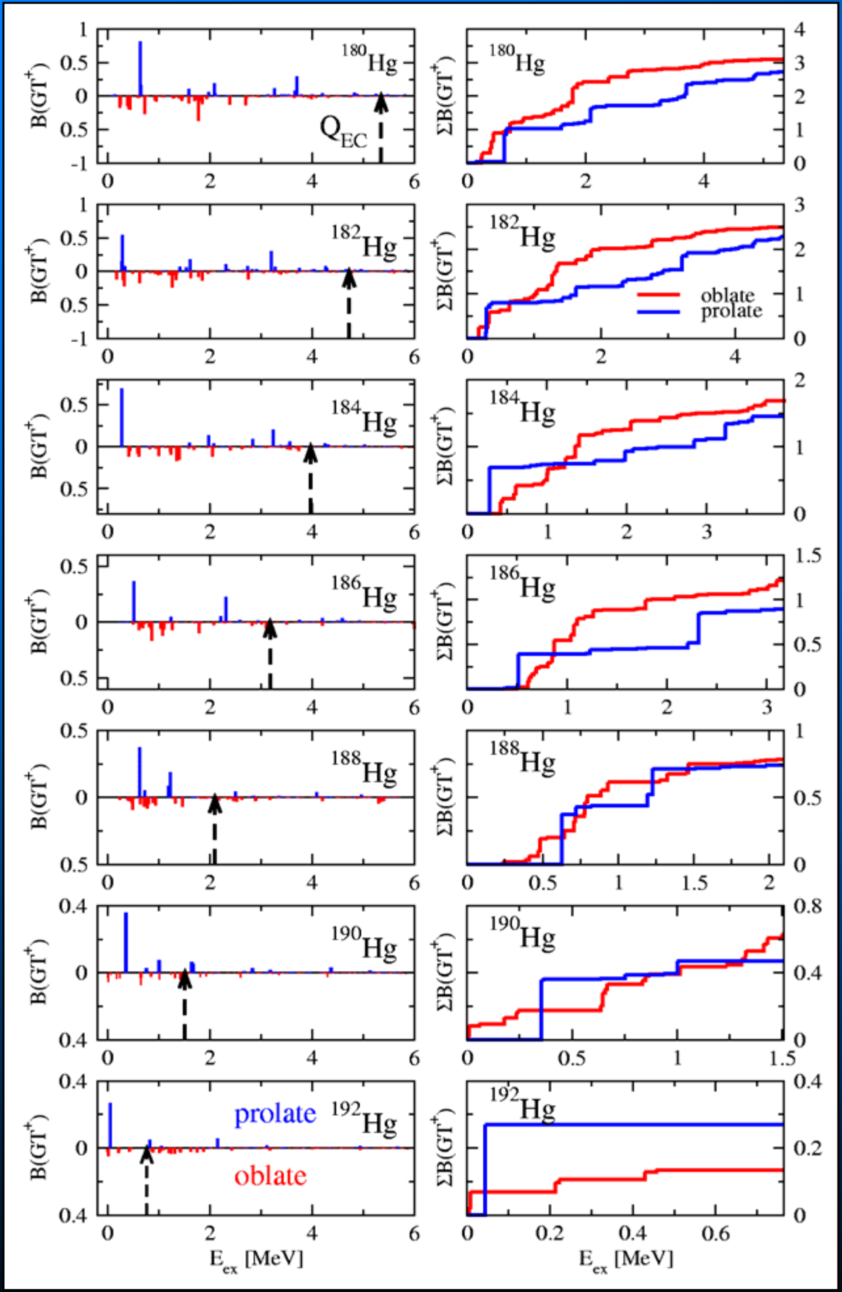
Shape dependence of GTdistributions in neutron-deficient: Pb isotopes

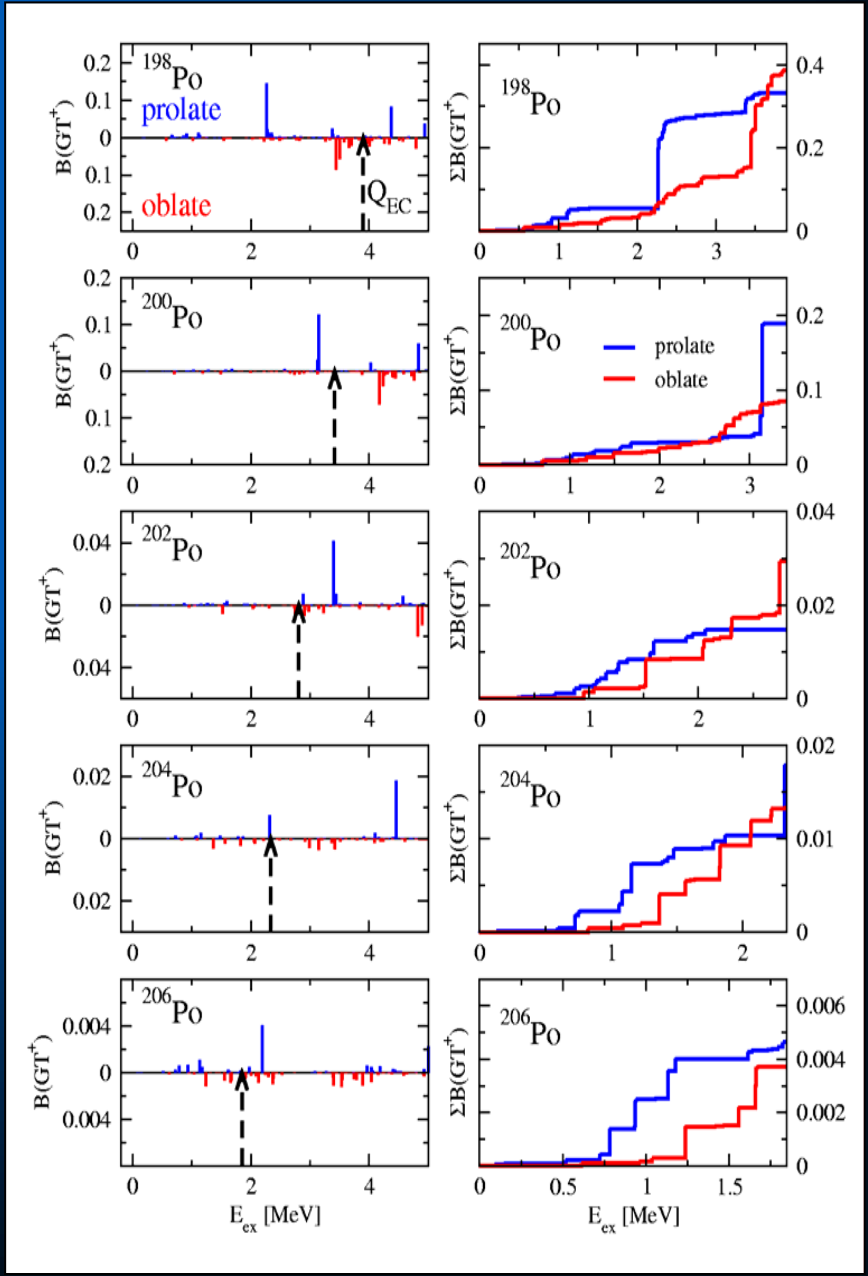


Signatures of deformation

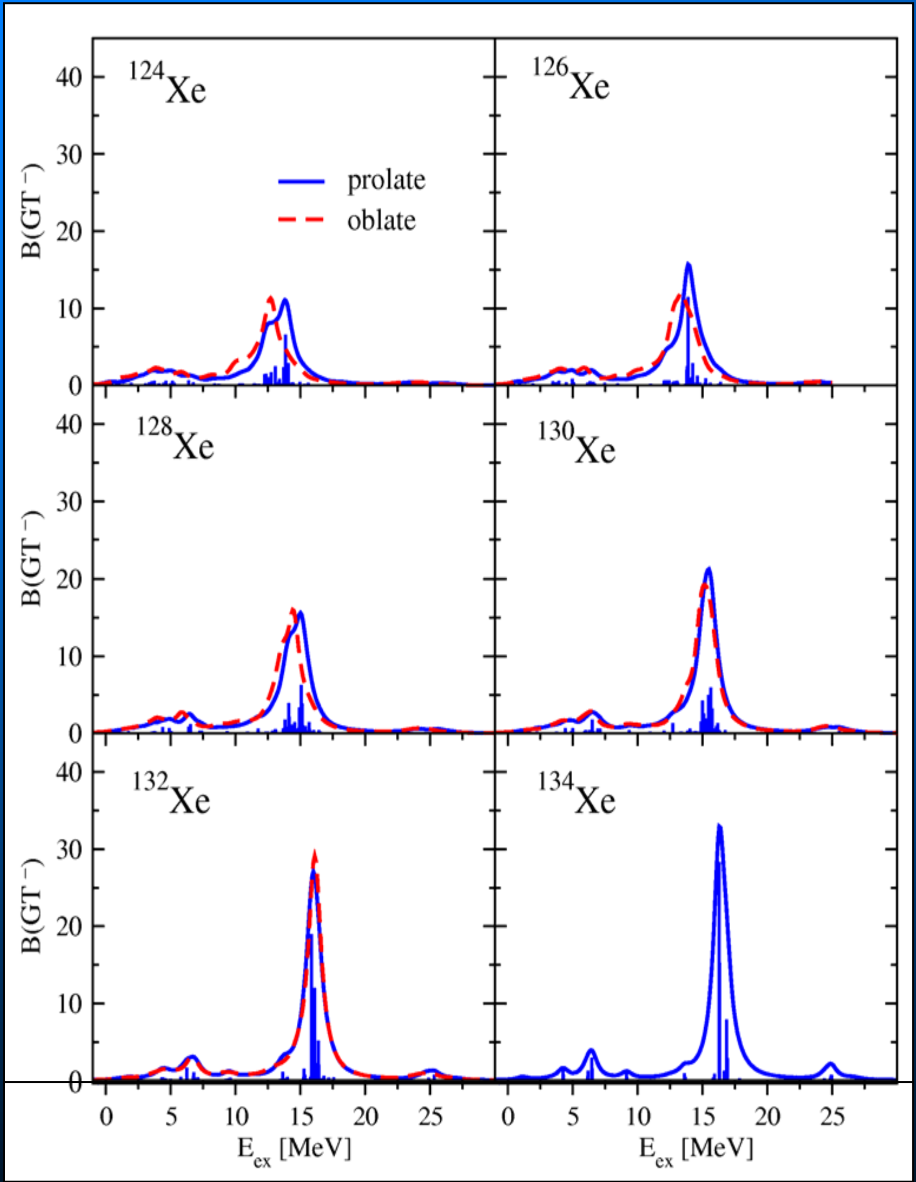


Shape dependence of GTdistributions in neutron-deficient Hg, Po isotopes

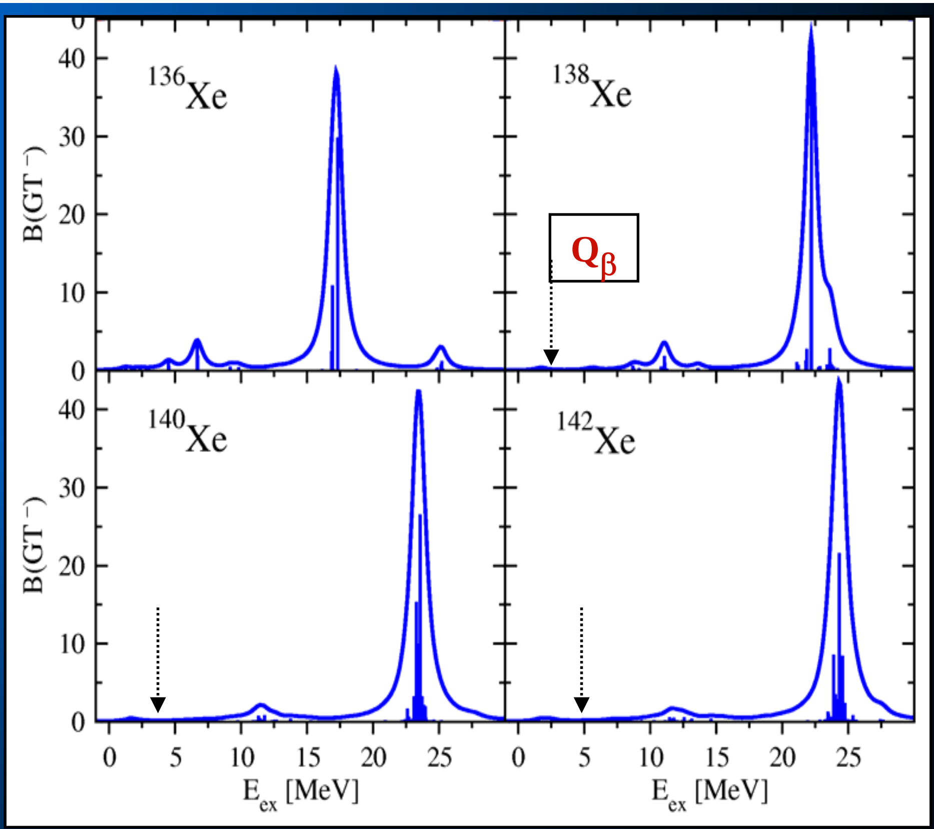




GT⁻ strength distributions : Xe isotopes

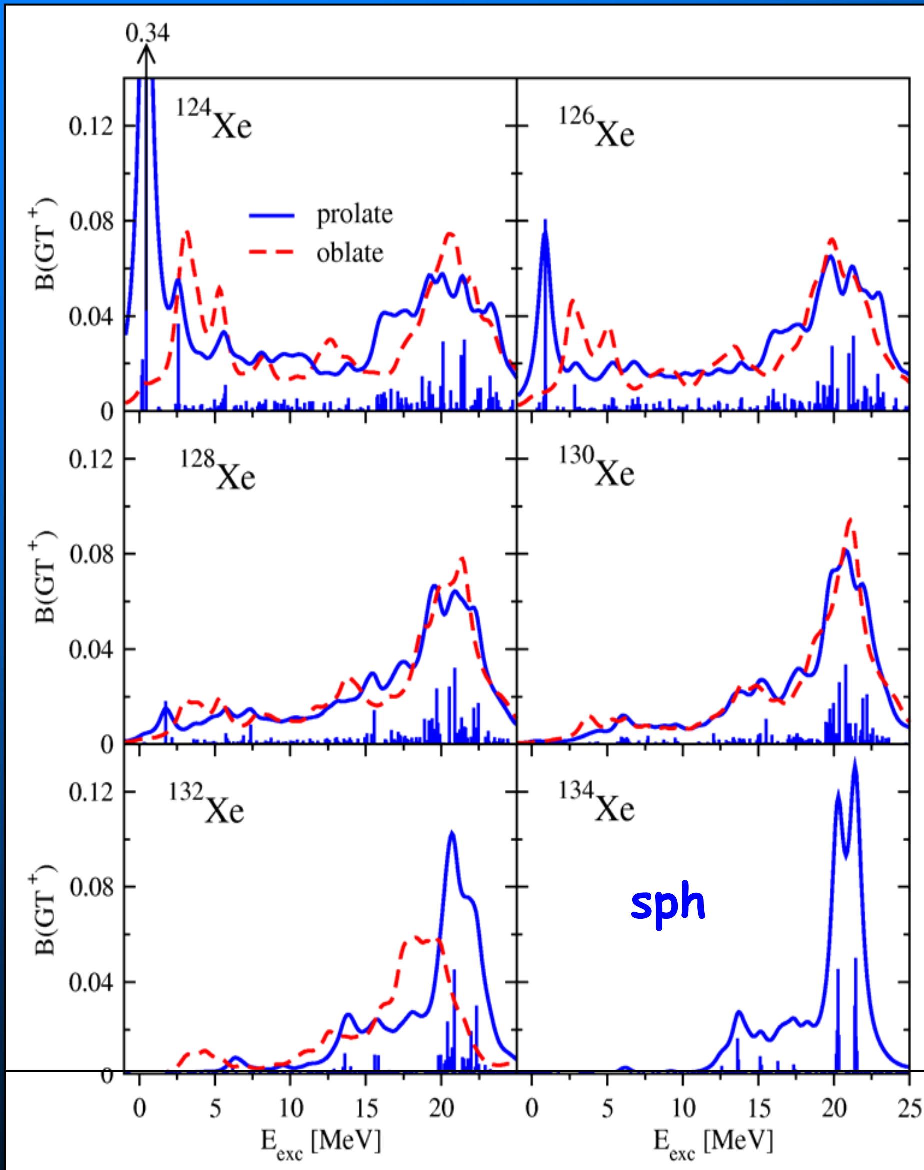


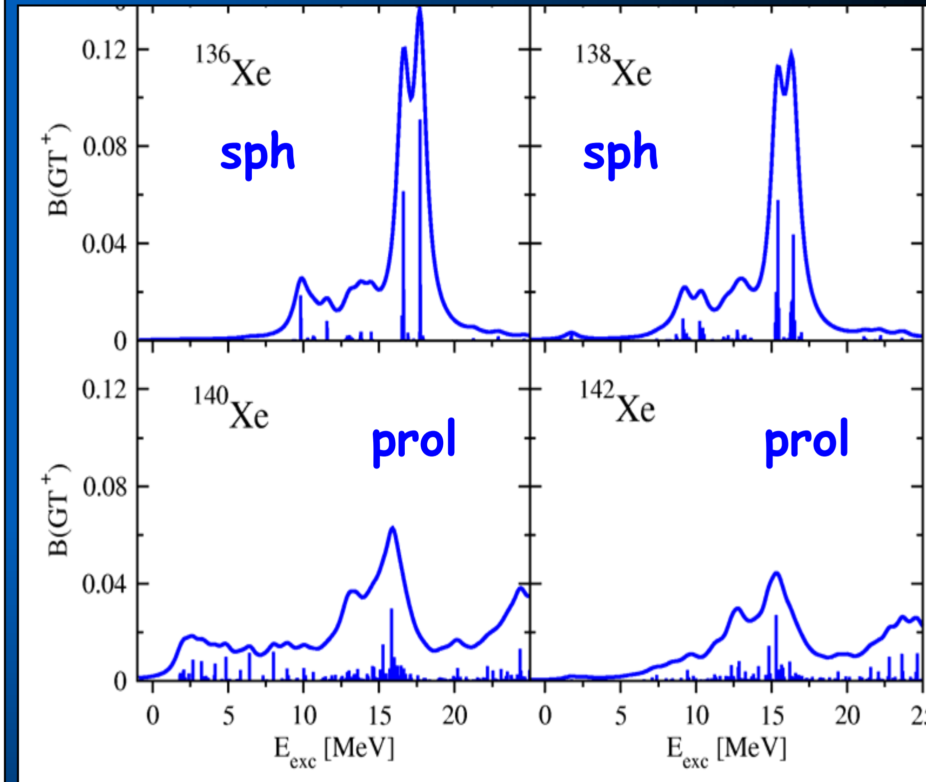
^{136}Xe: moving target in collisions with a hydrogen gas jet to explore the feasibility of EXL at FAIR-GSI



$$^{A}Xe \left\{ \begin{array}{c} (p,n) \\ (^{3}He,^{3}H) \\ (d,2n) \end{array} \right\} {}^{A}Cs$$

GT⁺ strength distributions : Xe isotopes





$$^{A}\mathrm{Xe} \left\{ \begin{array}{c} (n,p) \\ (^{3}H,{}^{3}He) \\ (d,2p) \end{array} \right\} {}^{A}\mathrm{I}$$

Exotic Nuclei : Nuclear Astrophysics

Beta-decay half-lives of waiting point nuclei in rp-processes

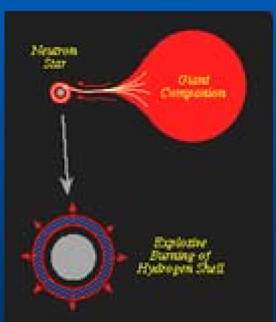
Quality of astrophysical models depends critically on the quality of input (nuclear)

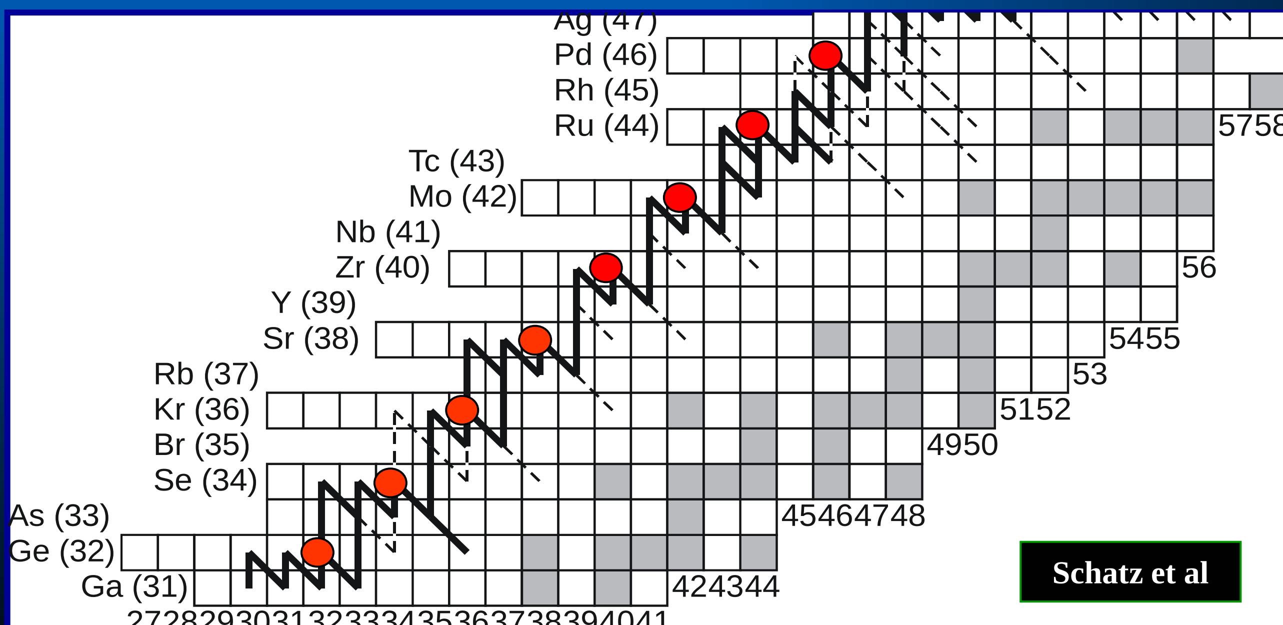
rp-process:

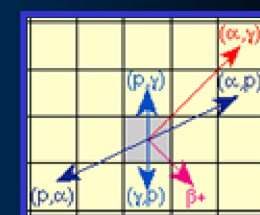
Proton capture reaction rates orders of magnitude faster than the competing β^+-decays

Waiting point nuclei:

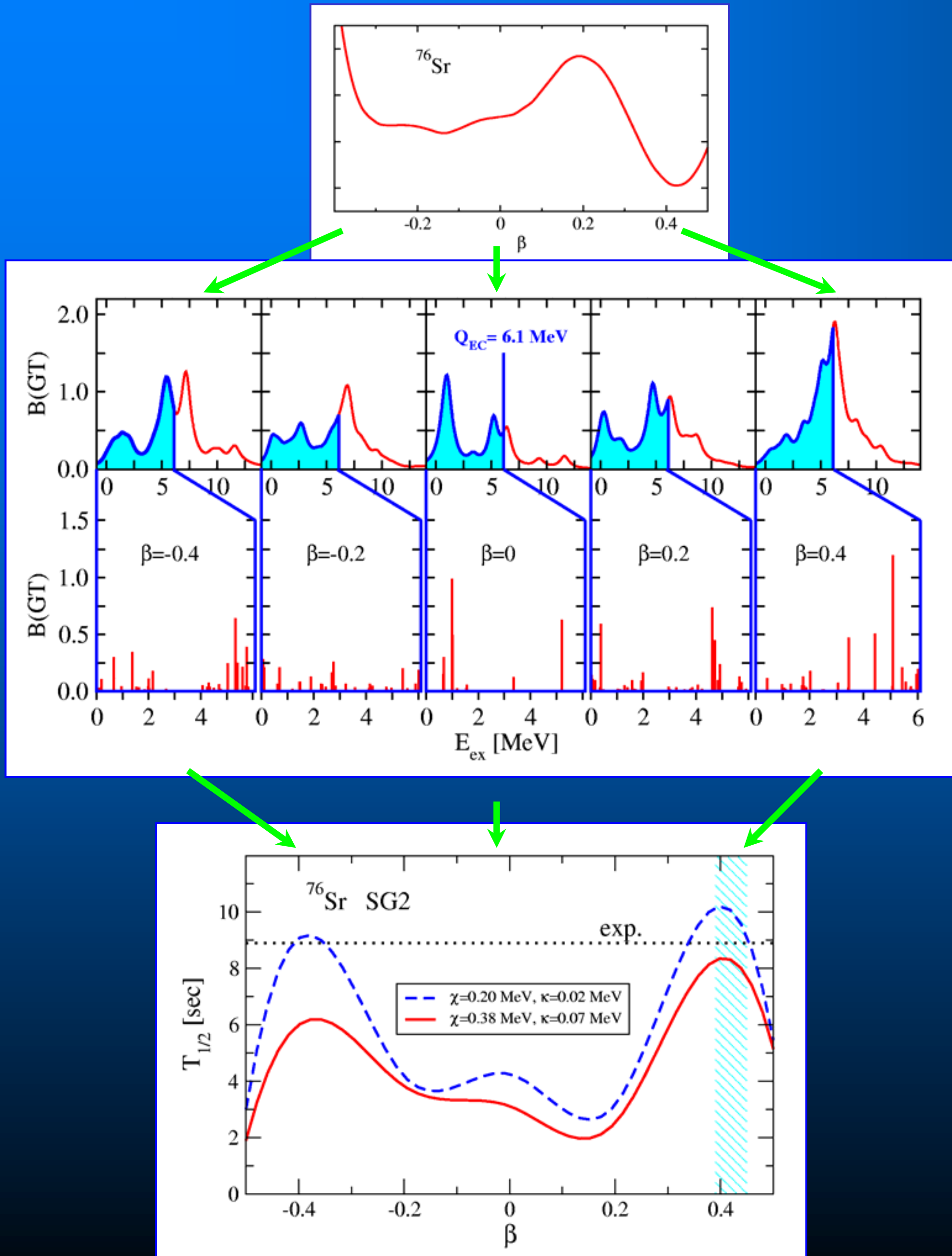
p-capture is inhibited: the reaction flow waits for a slow beta-decay to proceed



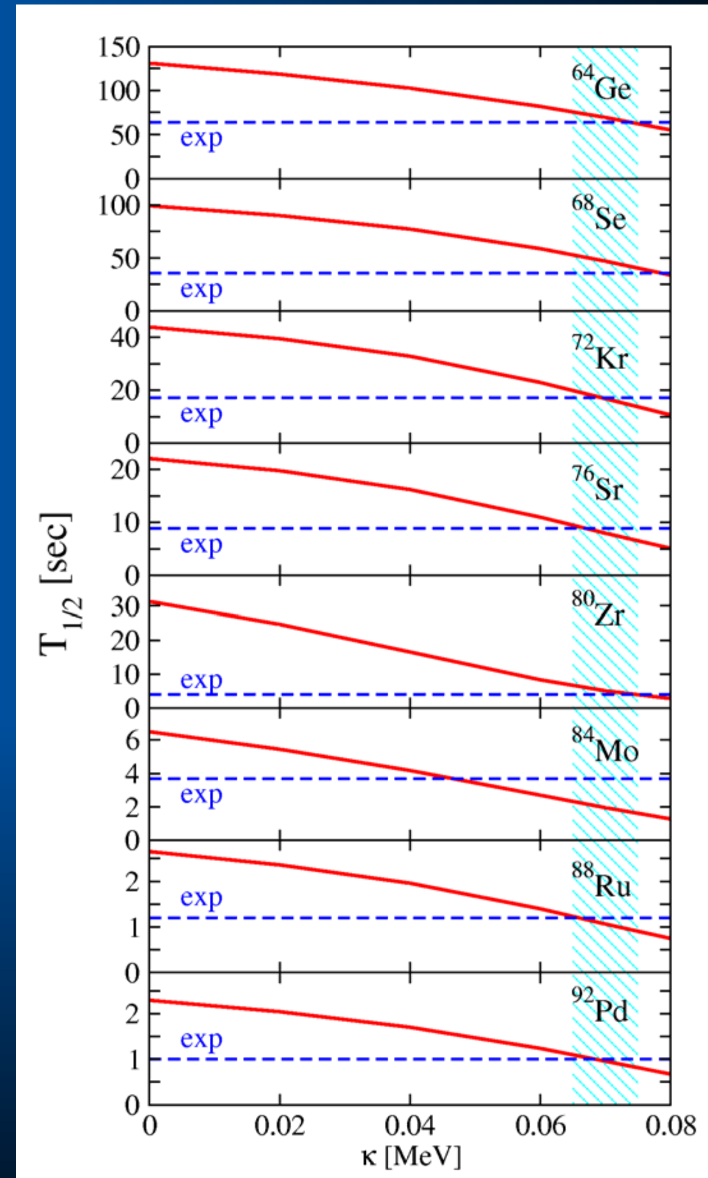




Dependence of $T_{1/2}$ on deformation

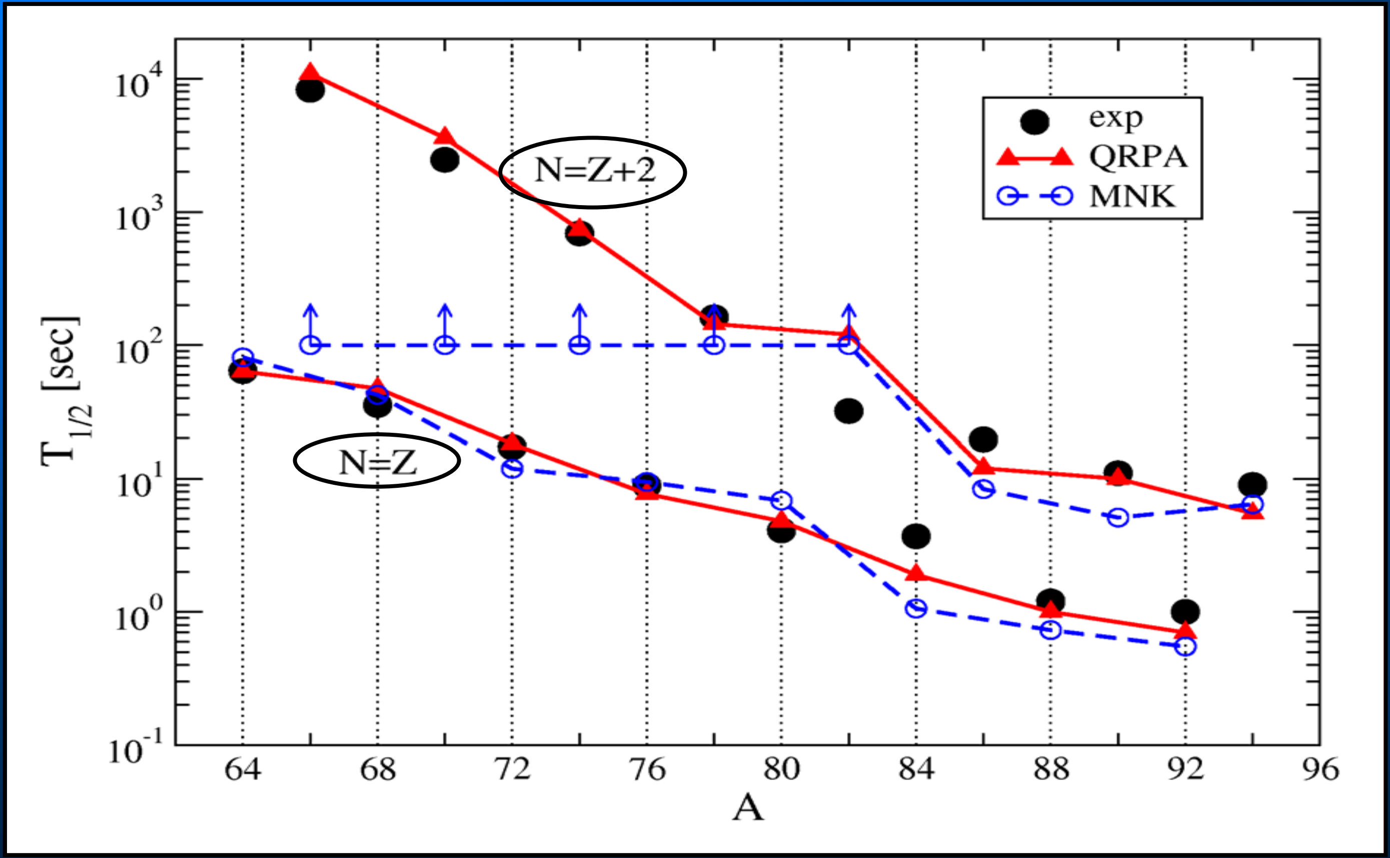


Dependence of $T_{1/2}$ on κ_{pp}



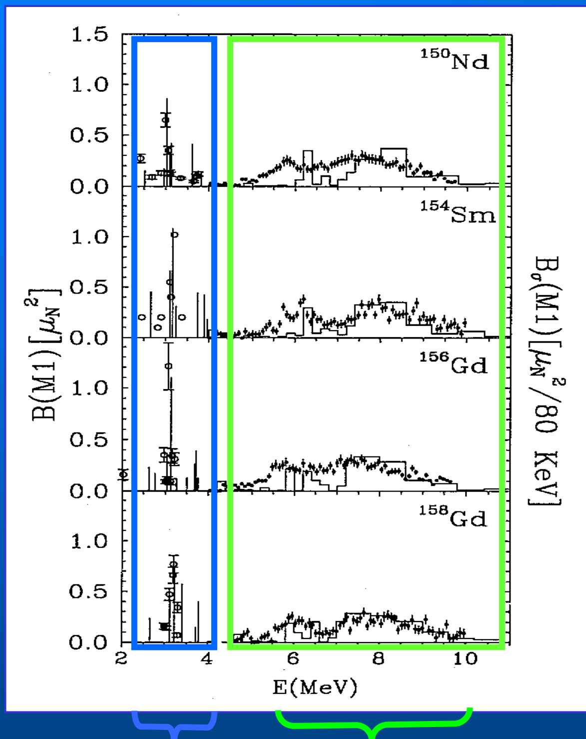
Half-lives of waiting poinr nuclei

$$T_{1/2}^{-1} = \frac{\kappa^2}{6200} \sum_{\omega} f(Z,\omega) \left| \langle 1_\omega^+ \| \beta^+ \| 0^+ \rangle \right|^2$$



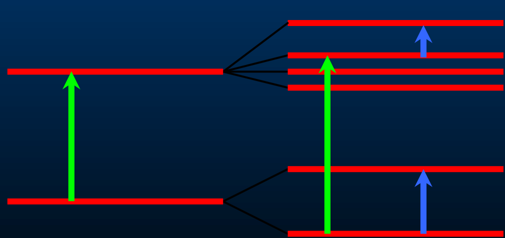
Good agreement with experiment: Reliable extrapolations

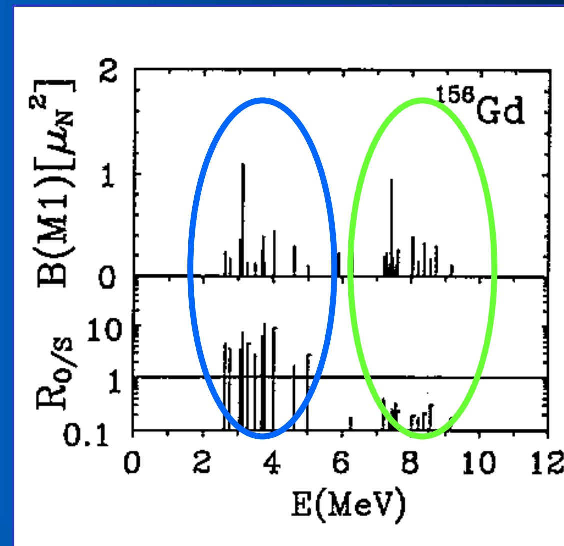
M1 excitations in deformed nuclei



orbital
(e,e') (γ,γ')

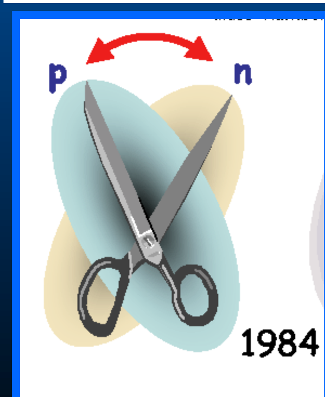
spin
(p,p')





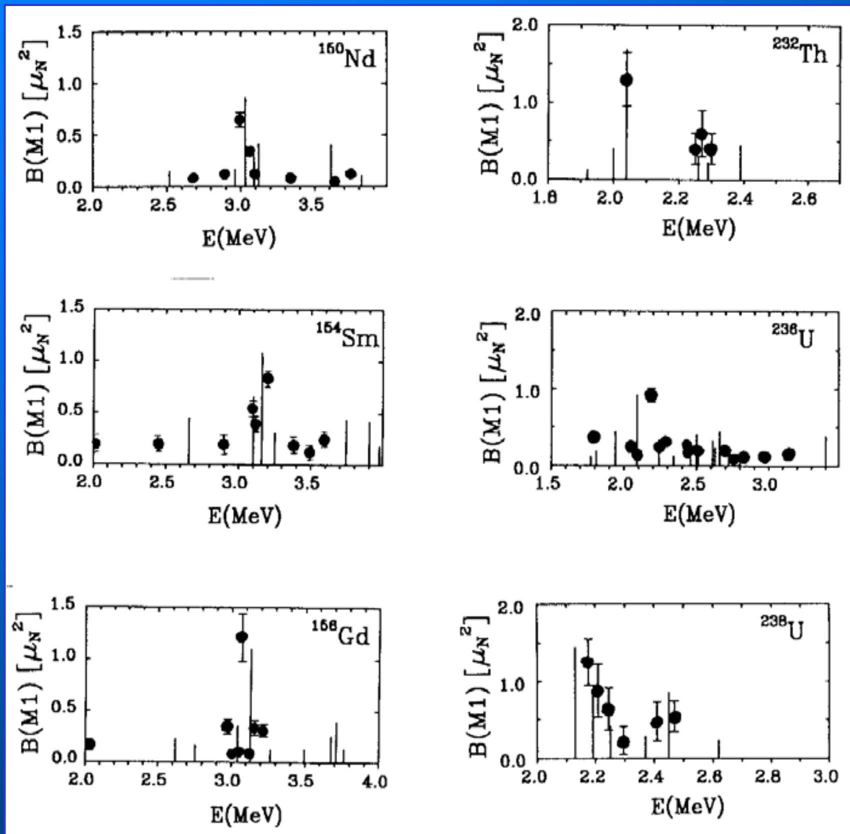
(p,p') Triumf 90's

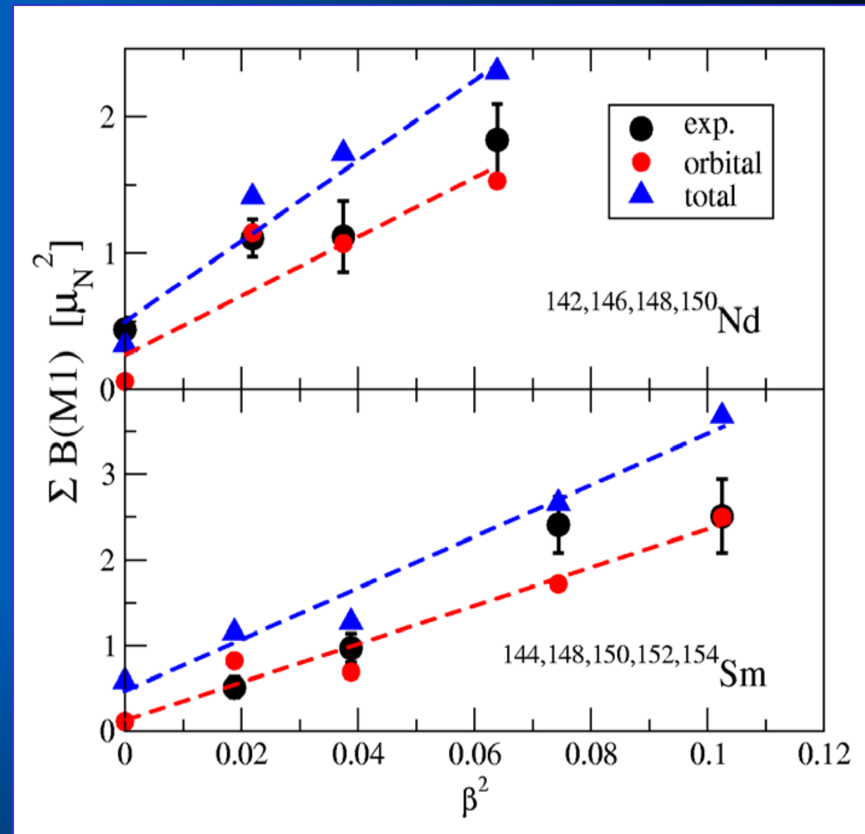
(e,e') Darmstadt 1984
(γ,γ') Stuttgart 90's



Scissors mode
~ β^2

ORBITAL M1 excitations in deformed nuclei: Theory and experiment





Orbital M1 excitations

Excitation spectra and β^2 dependence well reproduced

in rare-earths and actinides

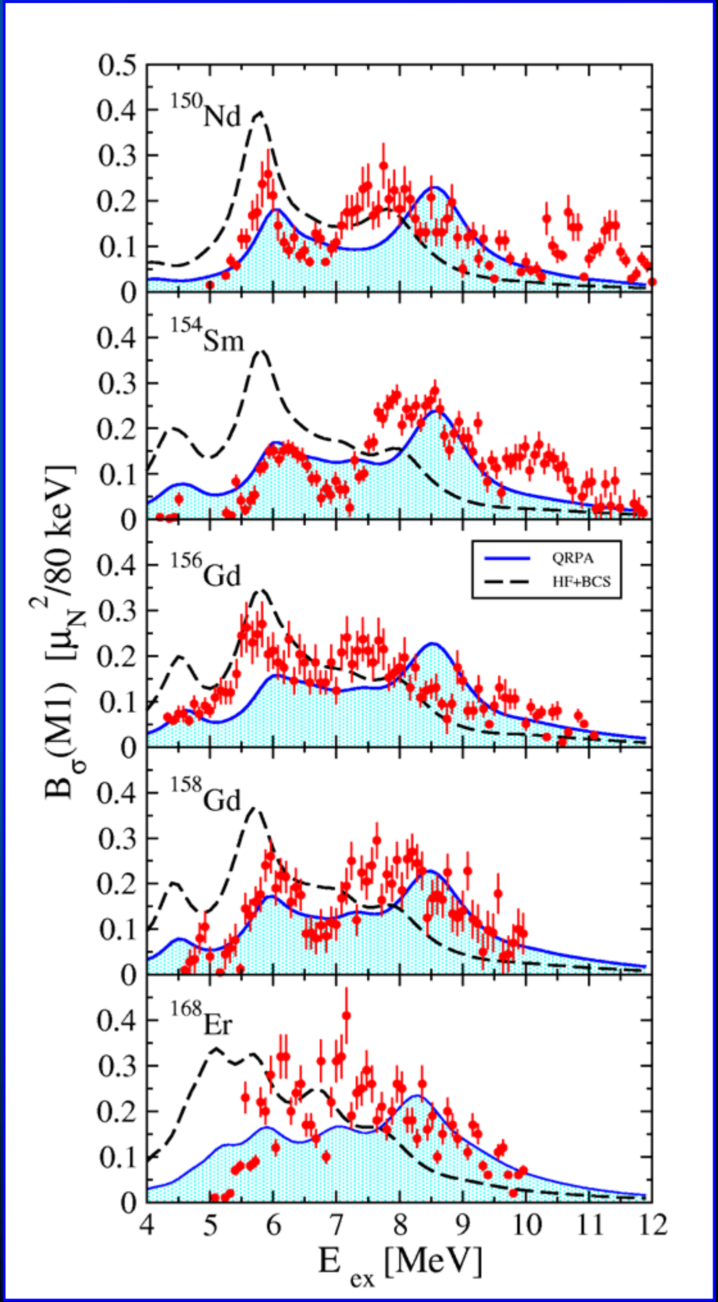
SPIN M1 excitations in deformed nuclei: Theory and experiment

Spin M1 strength

$\Delta T_z=0$ isospin counterparts of $\Delta T_z=1$

GT transitions

$$\mathbf{M1} = \sqrt{\frac{3}{4\pi}}\mu_N \left[\mathbf{J_p} + \left(g_p^s - 1\right)\mathbf{S_p} + g_n^s \mathbf{S_n}\right]$$



Spin M1

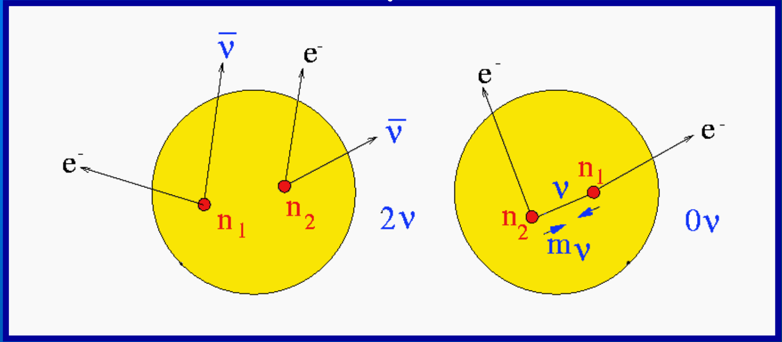
total strength and peak structure

Well reproduced

Double beta-decay: Test of the Standard Model

One of the most rare events in nature $T \sim 10^{20}$ years

2 decay modes



$(A,Z) \rightarrow (A,Z+2)+2e^-+2\nu$

$(A,Z) \rightarrow (A,Z+2)+2e^-$

2ν : 2 successive β decays through intermediate virtual states.

Observed in ^{48}Ca ^{76}Ge ^{82}Se ^{96}Zr ^{100}Mo ^{116}Cd ^{128}Te ^{130}Te ^{150}Nd **($T \sim 10^{19}$-10^{21} years)**

0ν : Lepton number not conserved. Forbidden in the SM.

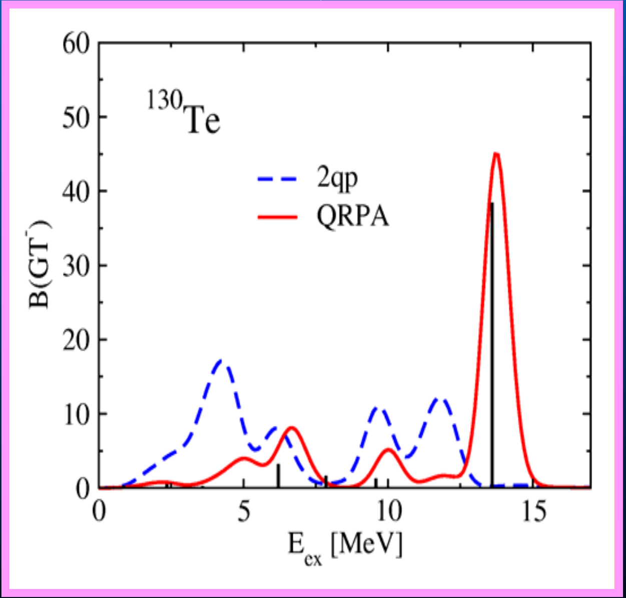
ν emitted is absorbed : Massive Majorana particle. **(T (^{76}Ge) > 10^{25} years)**

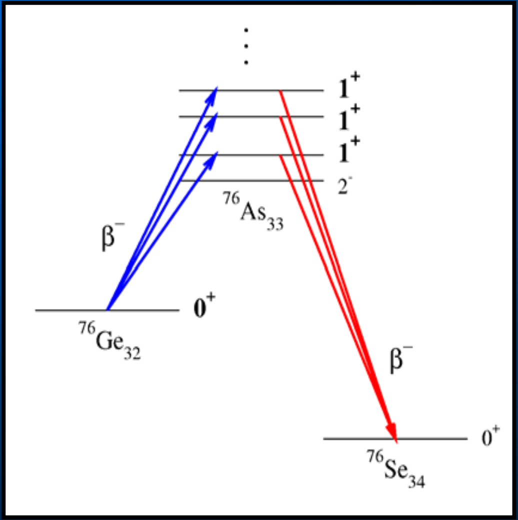
Double beta-decay: Deformation

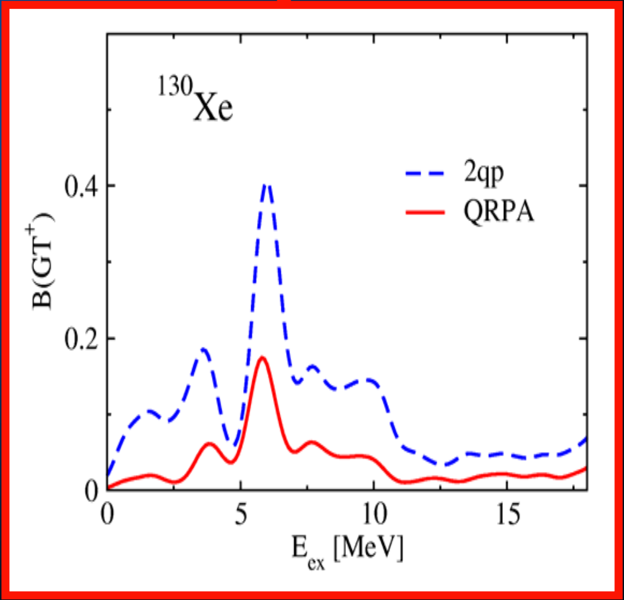
$$[T^{2\nu}_{1/2}(0^+_{g.s.} \rightarrow 0^+_{g.s.})]^{-1} = G^{2\nu}\ (g_A)^4\ |M^{2\nu}_{GT}|^2.$$

Description of the nuclear structure involved in the process

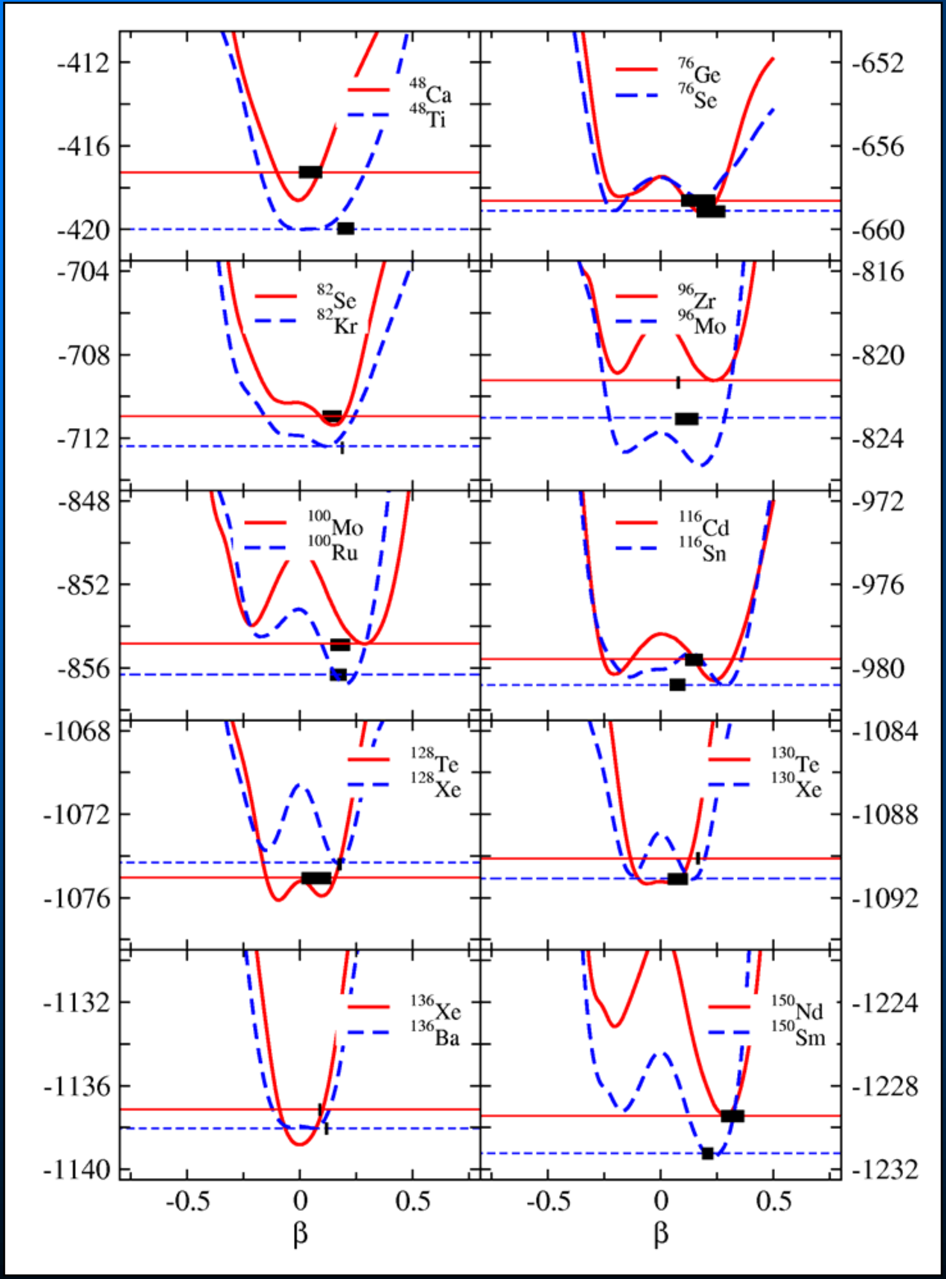
$$M^{2\nu}_{GT} = \sum_{m_i m_f} \sum_{K=0,\pm 1} \frac{< 0^+_f \parallel \beta^- \parallel 1(K), m_f >< 1(K), m_f | 1(K), m_i >< 1(K), m_i \parallel \beta^- \parallel 0^+_i >}{(\omega^{m_f}_K + \omega^{m_i}_K)/2}.$$



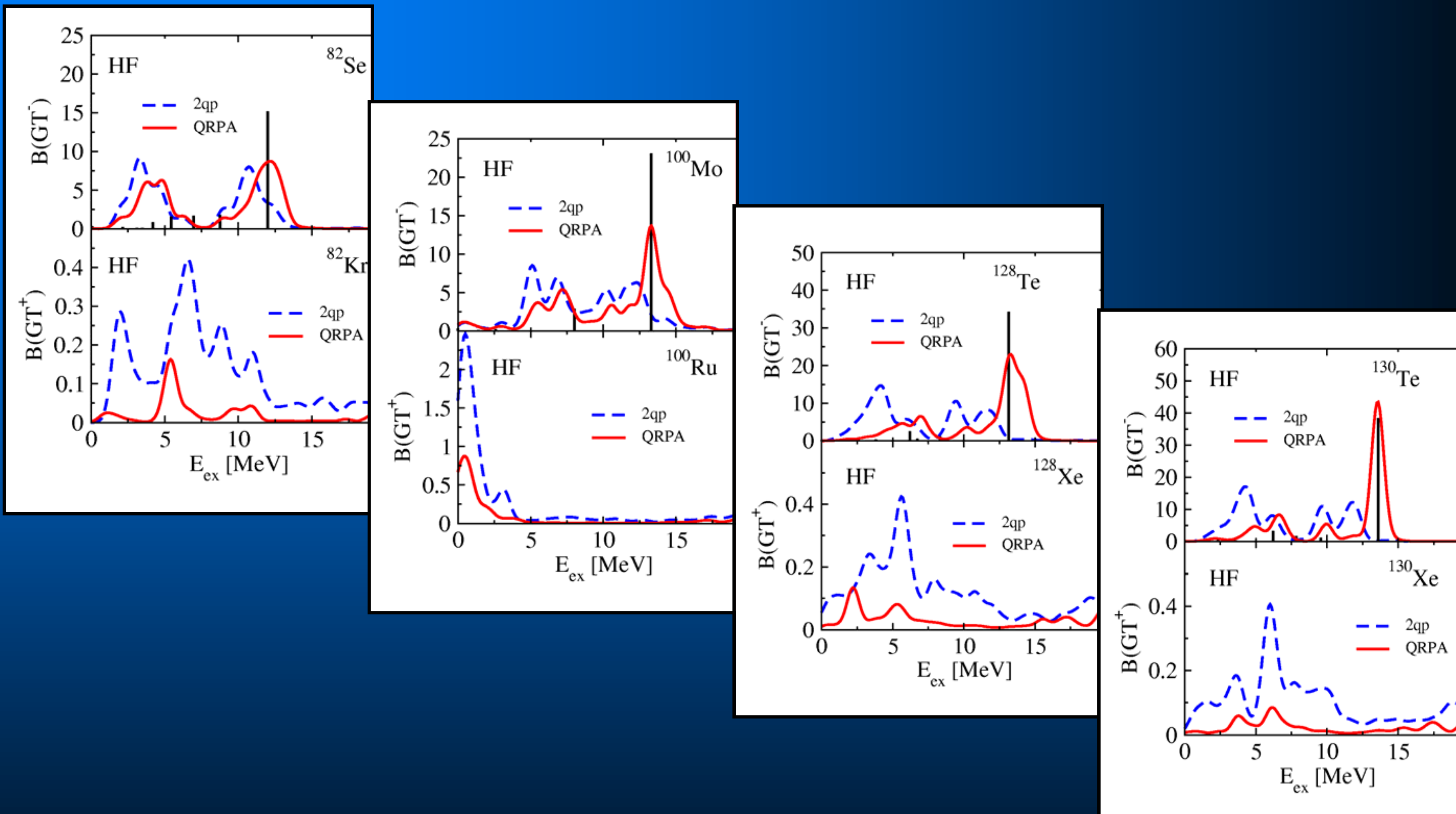




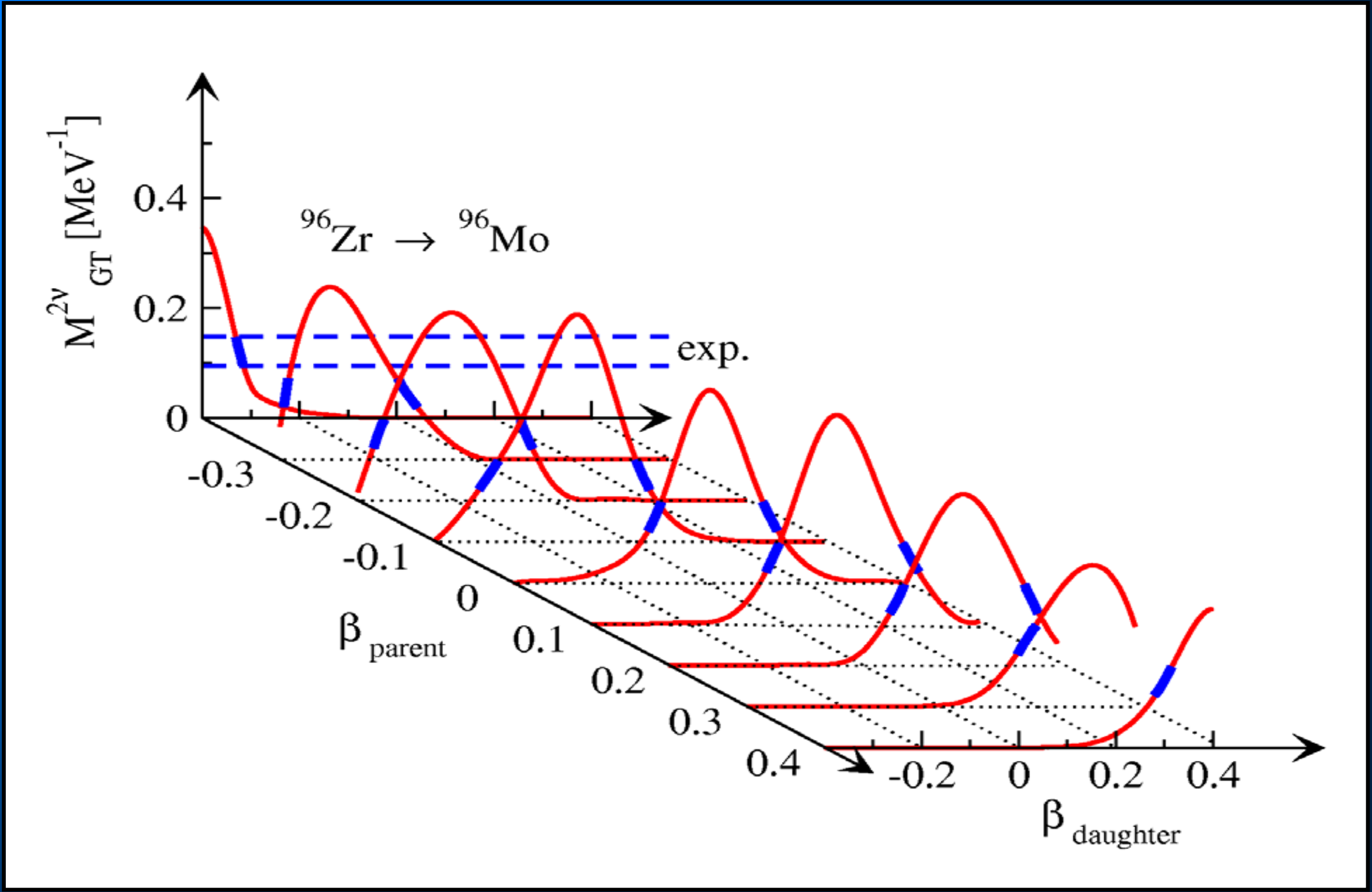
Constrained HF calculations in double beta partners



GT strength in the single-beta branches of double beta partners



Double beta matrix elements

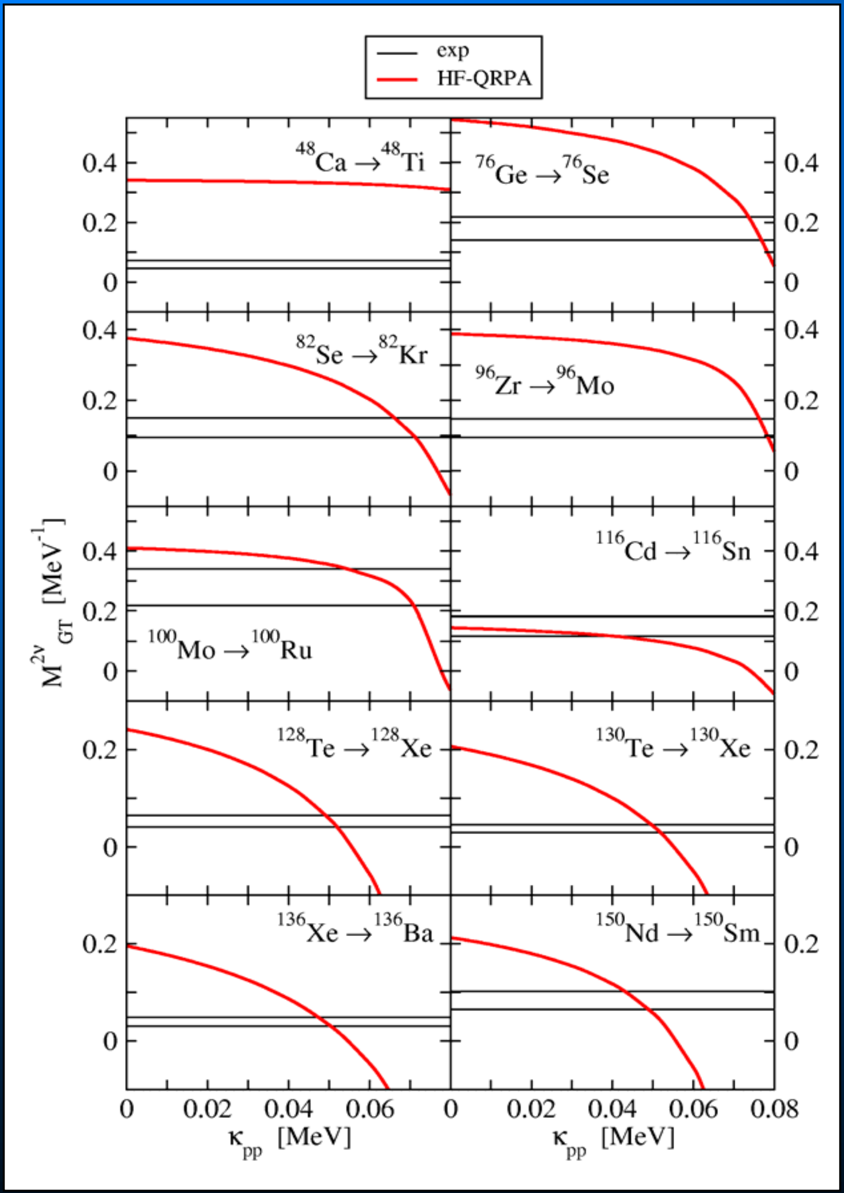


2νββ matrix elements vs. deformation

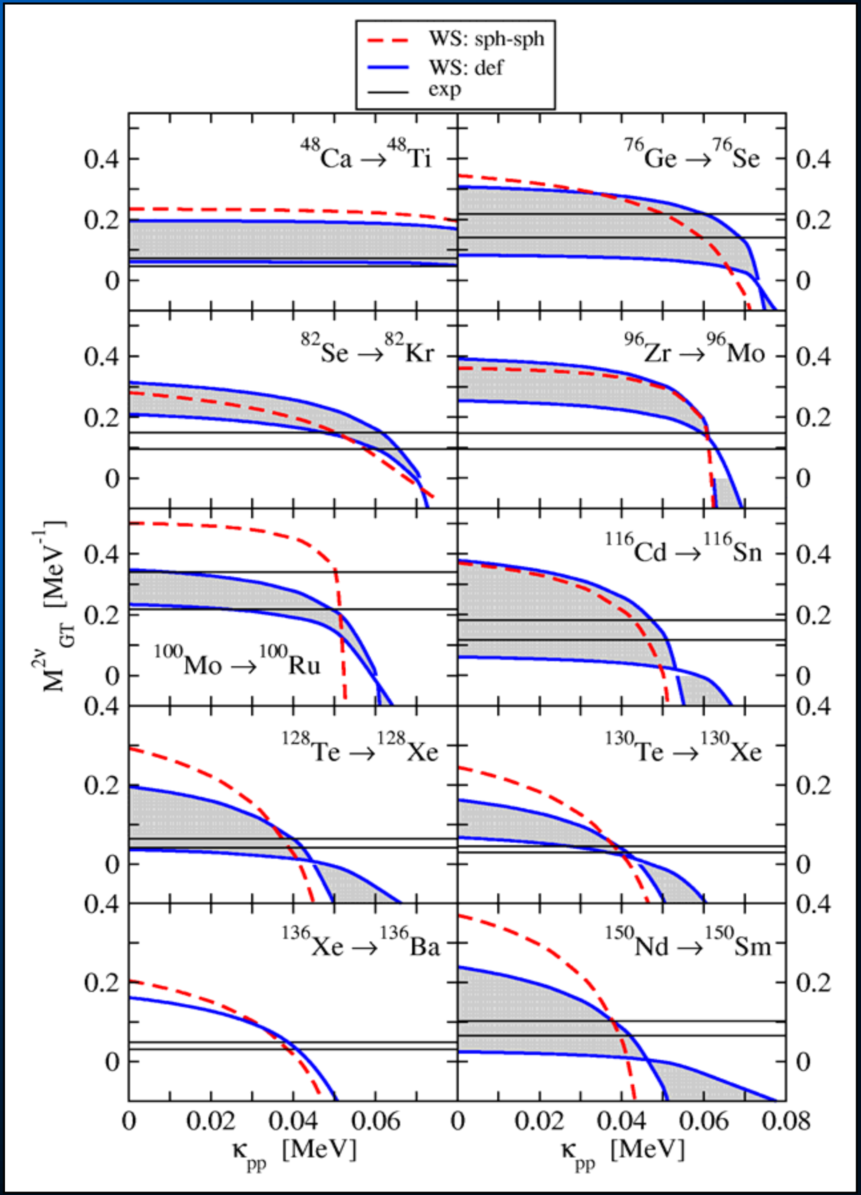
Suppression mechanism due to deformation

Double beta matrix elements vs. kpp

HF-QRPA



WS-QRPA



Conclusions

Theoretical approach based on a deformed Skyrme HF+BCS+QRPA

- Used to describe spin-isospin nuclear properties (GT & M1) in stable and exotic nuclei
- Tested along the nuclear chart

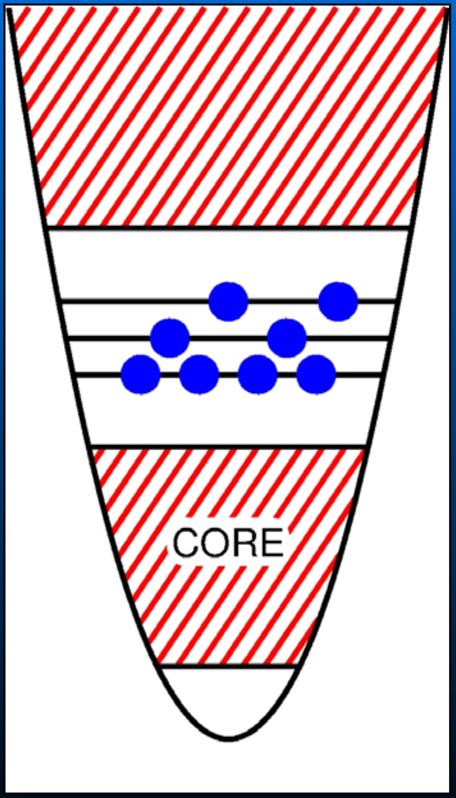
Find

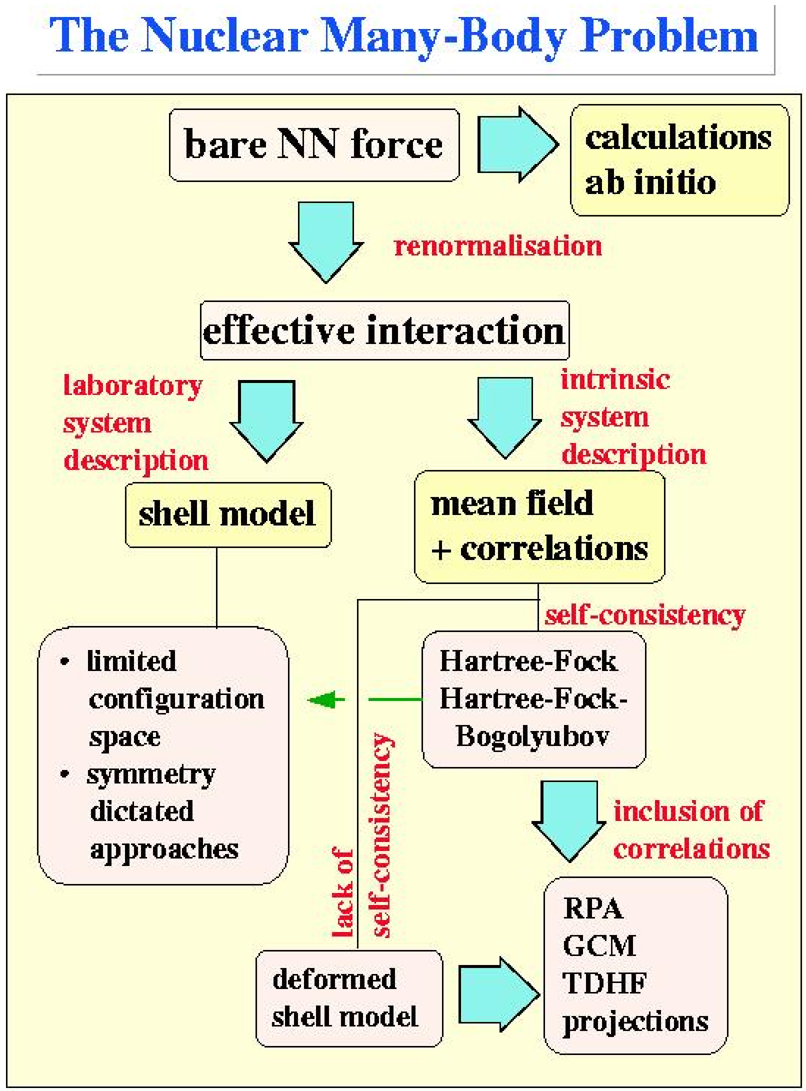
- Good agreement with experiment
  - GT strength distributions (Fe-Ni, p-rich A=70, $2\nu\beta\beta$ emitters)
  - Half-lives (waiting point nuclei)
  - Spin M1 strength distributions (rare earths & actinides)
  - $2\nu\beta\beta$ matrix elements
- Dependence on nuclear shape (shape coexistence)
  - Signatures of deformation in GT strength distributions in A=70 and neutron-deficient Pb isotopes
  - Suppresion mechanism in $2\nu\beta\beta$ matrix elements

Nuclear Structure models

Shell model

- Valence space
- Eff. interaction
- Diagonalization





QRPA

- Simplicity
- No core
- Highly excited states

Not all many-body correlations are taken into account

pnQRPA with separable forces

Transition amplitudes $\left\langle \omega_K \middle| \beta_K^{\pm} \middle| 0 \right\rangle = \mp M_{\pm}^{\omega_K}$

$$\Gamma_{\omega_K}^{+} = \sum_{\pi\nu} \left[X_{\pi\nu}^{\omega_K} \alpha_{\nu}^{+} \alpha_{\bar{\pi}}^{+} - Y_{\pi\nu}^{\omega_K} \alpha_{\bar{\nu}} \alpha_{\pi} \right]$$

$$X_{\pi\nu}^{\omega_K} = \frac{1}{\omega_K - \epsilon_{\pi\nu}} \left[2\chi_{GT}^{ph} \left(q_{\pi\nu} M_{-}^{\omega_K} + \tilde{q}_{\pi\nu} M_{+}^{\omega_K} \right) - 2\kappa_{GT}^{pp} \left(q_{\pi\nu}^{U} M_{--}^{\omega_K} + q_{\pi\nu}^{V} M_{++}^{\omega_K} \right) \right]$$

$$Y_{\pi\nu}^{\omega_K} = \frac{-1}{\omega_K + \epsilon_{\pi\nu}} \left[2\chi_{GT}^{ph} \left(q_{\pi\nu} M_{+}^{\omega_K} + \tilde{q}_{\pi\nu} M_{-}^{\omega_K} \right) + 2\kappa_{GT}^{pp} \left(q_{\pi\nu}^{U} M_{++}^{\omega_K} + q_{\pi\nu}^{V} M_{--}^{\omega_K} \right) \right]$$

$$M_{-}^{\omega_K} = \sum_{\pi\nu} \left(q_{\pi\nu} X_{\pi\nu}^{\omega_K} + \tilde{q}_{\pi\nu} Y_{\pi\nu}^{\omega_K} \right)$$

$$M_{+}^{\omega_K} = \sum_{\pi\nu} \left(\tilde{q}_{\pi\nu} X_{\pi\nu}^{\omega_K} + q_{\pi\nu} Y_{\pi\nu}^{\omega_K} \right)$$

$$M_{--}^{\omega_K} = \sum_{\pi\nu} \left(q_{\pi\nu}^{U} X_{\pi\nu}^{\omega_K} - q_{\pi\nu}^{V} Y_{\pi\nu}^{\omega_K} \right)$$

$$M_{++}^{\omega_K} = \sum_{\pi\nu} \left(q_{\pi\nu}^{V} X_{\pi\nu}^{\omega_K} - q_{\pi\nu}^{U} Y_{\pi\nu}^{\omega_K} \right)$$

$$\tilde{q}_{\pi\nu} = u_{\nu} v_{\pi} \Sigma_K^{\nu\pi}; \quad q_{\pi\nu} = v_{\nu} u_{\pi} \Sigma_K^{\nu\pi}; \quad q_{\pi\nu}^{V} = v_{\nu} v_{\pi} \Sigma_K^{\nu\pi}; \quad q_{\pi\nu}^{U} = u_{\nu} u_{\pi} \Sigma_K^{\nu\pi}$$

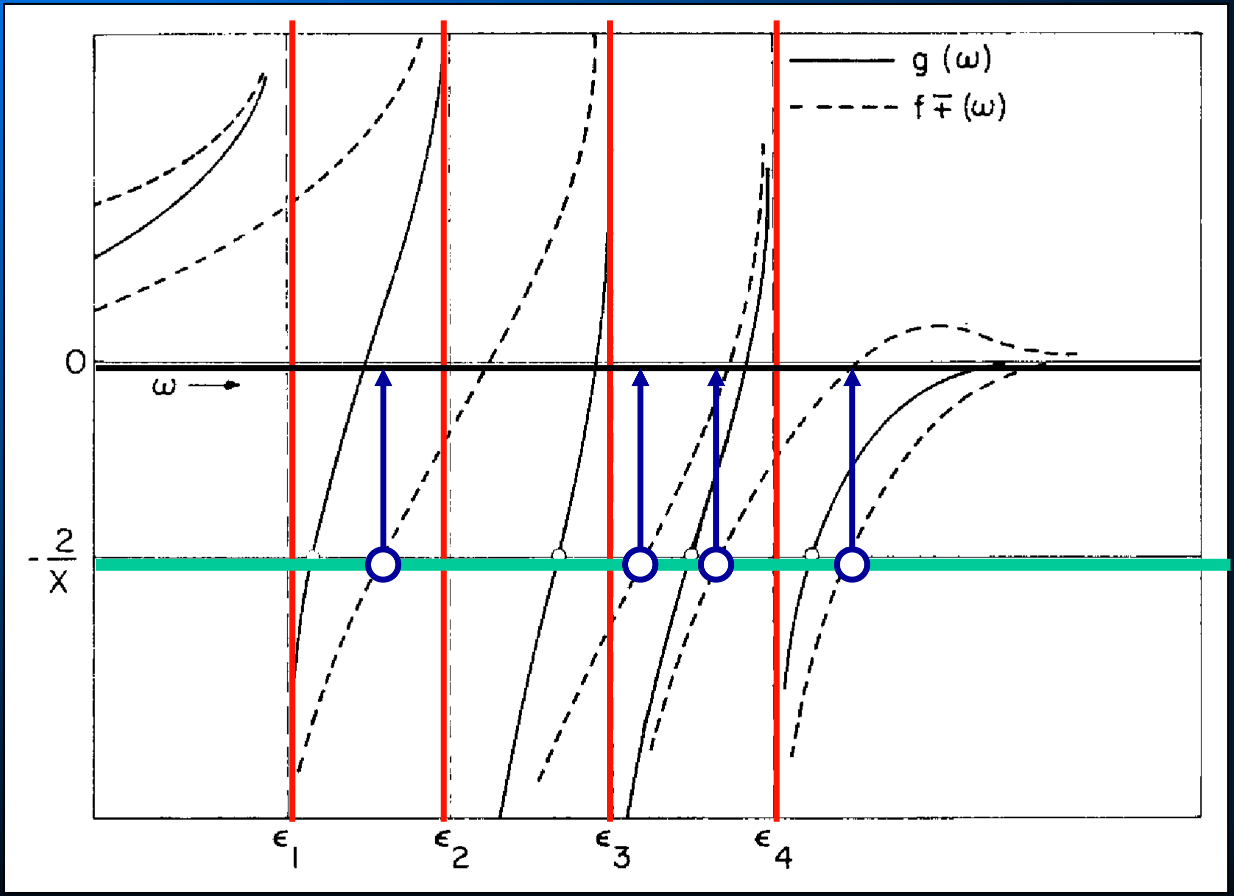
Secular equation

$$\left(\frac{1}{4\chi_{GT}}\right)^2 = \frac{1}{2\chi_{GT}}\sum_{i_0}\frac{(a_{i_0}^2+b_{i_0}^2)}{\omega_0^2-\mathcal{E}_{i_0}^2}\mathcal{E}_{i_0} + \left(\sum_{i_0} a_{i_0}b_{i_0}\frac{2\mathcal{E}_{i_0}}{\omega_0^2-\mathcal{E}_{i_0}^2}\right)^2 - \sum_{i_0}\left(\frac{a_{i_0}^2}{\omega_0+\mathcal{E}_{i_0}} - \frac{b_{i_0}^2}{\omega_0-\mathcal{E}_{i_0}}\right)\sum_{i_0}\left(\frac{b_{i_0}^2}{\omega_0+\mathcal{E}_{i_0}} - \frac{a_{i_0}^2}{\omega_0-\mathcal{E}_{i_0}}\right)$$

$$-\frac{2}{\chi} = (A+D)\mp\{(A-D)^2+4B^2\}^{\frac{1}{2}} \equiv f^{\mp}(\omega).$$

$$-\frac{2}{\chi} = (A+D) = -4\sum_i \frac{\varepsilon_i(q_i^2+\tilde{q}_i^2)}{\omega^2-\varepsilon_i^2} \equiv g(\omega).$$

Determine RPA energies ω



Odd-A nucleus

$$K_f = K_i + 1 \rightarrow I_f = K_f$$

$$K_f = K_i \rightarrow I_f = K_f,\quad I_f = K_f + 1$$

$$K_f = K_i - 1 \rightarrow I_f = K_f,\quad I_f = K_f + 1,\quad I_f = K_f + 2$$

$$E_{rot} = \frac{1}{2\mathcal{I}}\left[I(I+1) - K^2 + \delta_{K,1/2}(-1)^{I+1/2}(I+1/2)a\right]$$