Nilsson Model

- Spherical Shell Model
- Deformed Shell Model
  - Anisotropic Harmonic Oscillator
  - Nilsson Model
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    - Choice of Basis
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Spherical shell model

Nuclear properties described in terms of nucleons considered as independent particles moving in an average potential create by all nucleons.

Experimental evidence for shell effects:

Existence of magic numbers: 2,8,20,28,50,82,126

- Large single particle separation energies
- Nuclei are strongly bound at shell closures

Derivation of the average field from microscopic two-body forces (selfconsistent Hartree-Fock method).

Assume the existence of such a potential and construct it phenomenologically

Characteristics of the potential:

$$\left(\frac{\partial V(r)}{\partial r}\right)_{r=0} = 0 \qquad \left(\frac{\partial V}{\partial r}\right)_{r<R_0} > 0 \qquad V(r) \simeq 0, \quad r > R_0$$

Spherical potentials

Infinite square well	Harmonic oscillator	Woods-Saxon potential
$V(r) = -V_0 \quad \text{for } r \le R$ $= +\infty \quad \text{for } r > R$	$V(r) = \frac{1}{2} M \omega_0{}^2 r^2$	$V(r) = -\frac{V_0}{1+\exp\left[(r-R)/a\right]}$

Eigen-functions

Infinite square well	Harmonic oscillator	Woods-Saxon potential
$\psi \sim j_\ell(kr) Y_{\ell m}(\Omega)$	$\psi \sim R_{n\ell}(r) Y_{\ell m}(\Omega)$ $R_{n\ell}(r) = r^\ell e^{-\frac{1}{2}r^2} L_{n-1}^{\ell+1/2}(r^2)$	numerically

Eigen-energies

Infinite square well	Harmonic oscillator	Woods-Saxon potential
$E(n,\ell) = \frac{\hbar^2}{2MR^2} \xi_{n\ell}^2$ $\xi_{n\ell}$: root of $j_\ell(\xi) = 0$	$E(n,\ell) = \hbar\omega_0 (2n + \ell + 3/2)$ $= \hbar\omega_0 (N + 3/2)$	intermediate

Spherical potentials

Woods-Saxon potential

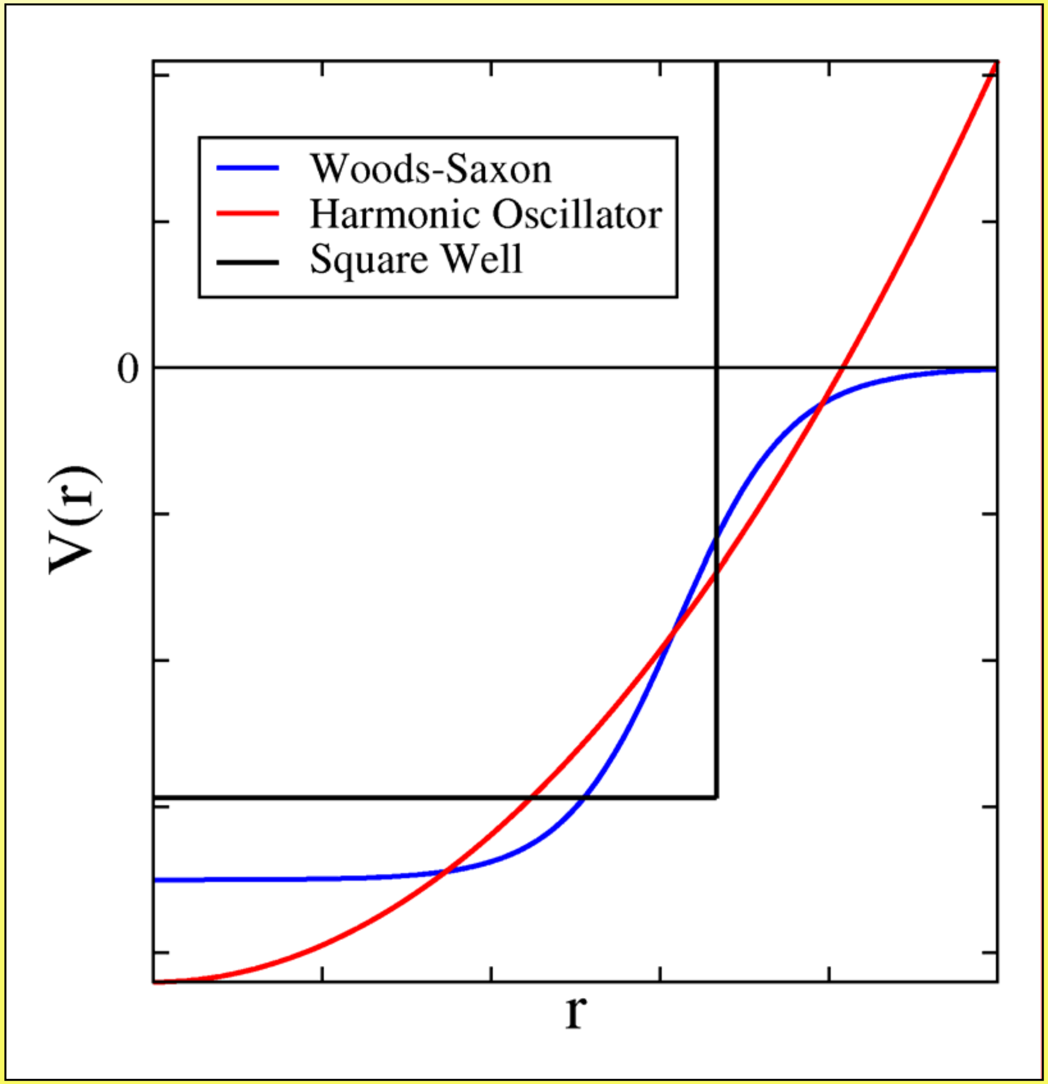
$$V(r) = -\frac{V_0}{1+\exp\left[(r-R)/a\right]}$$

Harmonic oscillator

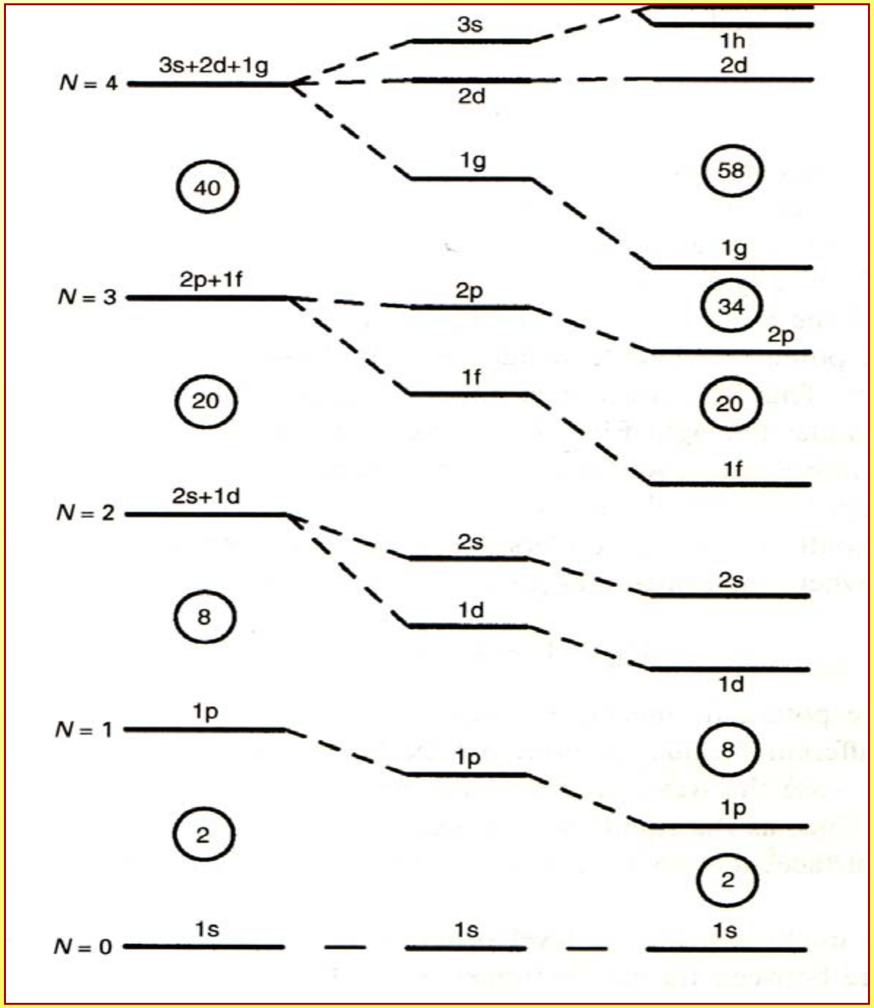
$$V(r) = \frac{1}{2} M \omega_0^2 r^2$$

Infinite square well

$$V(r) = -V_0 \quad \text{for } r \le R$$
$$= +\infty \quad \text{for } r > R$$



Spherical potentials



H.O. W.S. Square

Spherical potentials & spin-orbit

$$V(r) = \frac{1}{2} M \omega_0^2 r^2 + C \vec{\ell} \cdot \vec{s} + D \vec{\ell}^2$$

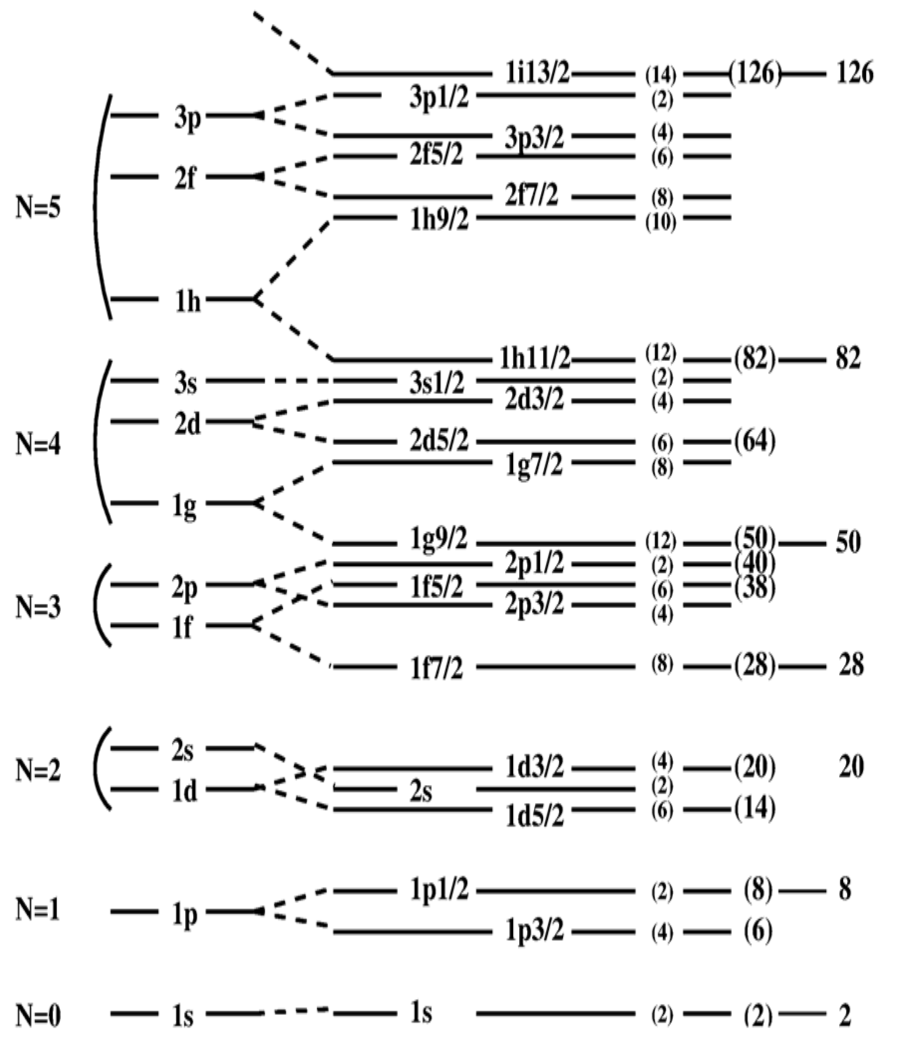
$$\vec{\ell} \cdot \vec{s} = \frac{1}{2}\left(\vec{j}^2 - \vec{\ell}^2 - \vec{s}^2\right)$$

$$E(n,\ell) = \hbar\omega_0 (2n + \ell + 3/2) + D\ell(\ell+1) + C\varepsilon_{SO}$$

$$\begin{cases} \varepsilon_{SO} = \ell & \text{for } j = \ell + \frac{1}{2} \\ \varepsilon_{SO} = -\ell - 1 & \text{for } j = \ell - \frac{1}{2} \end{cases}$$

$$\Delta\varepsilon_{SO} = 2\ell + 1$$

Spherical mean-field



Deformed shell model

Spherical potential well valid for closed shells

Far from closed shells: deformed single particle potential

Experimental evidence:

- Existence of rotational bands: I(I+1) spectra
- Large quadrupole moments and quadrupole transition probabilities
- Single particle structure

Anisotropic Harmonic Oscillator

Generalized Woods-Saxon

$$V(r,\theta,\varphi) = -V_0\left[1+\exp\left(\frac{r-R(\theta,\varphi)}{a(\theta,\varphi)}\right)\right]^{-1}$$

$$V_{LS} = \lambda\left(\vec{\nabla}V(r,\theta,\varphi)\wedge\vec{p}\right)\cdot\vec{s}$$

Anisotropic Harmonic Oscillator

Ellipsoidal distribution: Anisotropic Harmonic Oscillator as average field

$$H_0 = -\frac{\hbar^2}{2m}\Delta + \frac{m}{2}\left(\omega_x{}^2 x^2 + \omega_y{}^2 y^2 + \omega_z{}^2 z^2\right)$$

Frequencies are proportional to the inverse of the ellipsoid axes $\quad \omega_i = \overset{0}{\omega}_0 \frac{R_0}{a_i}$

For axially symmetric shapes,

we introduce the parameter δ

$$\delta = \frac{\omega_\perp - \omega_z}{\omega_0}$$

$$\begin{cases} \omega_\perp{}^2 = \omega_x{}^2 = \omega_y{}^2 = \omega_0{}^2\left(1+\frac{2}{3}\delta\right) \\ \omega_z{}^2 = \omega_0{}^2\left(1-\frac{4}{3}\delta\right) \end{cases}$$

From volume conservation

$$\omega_x\omega_y\omega_z = \left(\overset{0}{\omega}_0\right)^3 \qquad \omega_0\left(\delta\right) = \overset{0}{\omega}_0\left[1-\frac{4}{3}\delta^2 - \frac{16}{27}\delta^3\right]^{-1/6}, \qquad \delta \simeq 0.95\beta$$

Anisotropic Harmonic Oscillator

Introducing dimensionless coordinates through the oscillator length

$$b(\delta) = \sqrt{\frac{\hbar}{m\omega_0(\delta)}} \qquad \vec{r}\,' = \vec{r}/b$$

we get

$$H_0(\delta) = -\frac{\hbar^2}{2m}\frac{m\omega_0(\delta)}{\hbar}\Delta + \frac{m}{2}\left[\omega_0^{\,2}(\delta)\frac{\hbar}{m\omega_0}\left(1+\frac{2}{3}\delta\right)\left(x^2+y^2\right) + \omega_0^{\,2}(\delta)\frac{\hbar}{m\omega_0}\left(1-\frac{4}{3}\delta\right)z^2\right]$$

$$= -\frac{\hbar\omega_0(\delta)}{2}\Delta + \frac{\hbar\omega_0(\delta)}{2}\left[x^2+y^2+z^2+\frac{2}{3}\delta\left(x^2+y^2\right)-\frac{4}{3}\delta z^2\right]$$

$$= \frac{\hbar\omega_0(\delta)}{2}\left[-\Delta + r^2\right] - \delta\hbar\omega_0(\delta)\frac{4}{3}\sqrt{\frac{\pi}{5}}r^2 Y_{20}(\Omega)$$

$$= \overset{0}{H}_0 + H_\delta$$

Axial symmetry: cylindrical basis

$$\left\{N, n_z, n_\rho, m_\ell, m_s\right\}$$

$$N = n_x + n_y + n_z = n_z + 2n_\rho + m_\ell$$

$$\varepsilon\left(n_z, n_\rho, m_\ell\right) = \sum_{i=x,y,z}\hbar\omega_i\left(n_i+\frac{1}{2}\right) = \hbar\omega_z\left(n_z+\frac{1}{2}\right) + \hbar\omega_\perp\left(2n_\rho + m_\ell + 1\right)$$

$$= \hbar\overset{0}{\omega}_0\left[\left(N+\frac{3}{2}\right)+\delta\left(\frac{N}{3}-n_z\right)\right]$$

Anisotropic Harmonic Oscillator

Eigen-states characterized by $\Omega\pi[Nn_z m_\ell]$ $\quad \Omega = m_\ell \pm \frac{1}{2}$ $\quad \pi = (-1)^N$

$$\phi_{n_z n_\rho m_\ell m_s}(\vec{R},\sigma) = \psi_{n_\rho}^{m_\ell}(\rho)\psi_{n_z}(z)\frac{e^{im_\ell\varphi}}{\sqrt{2\pi}}\chi_{m_s}(\sigma)$$

$$\psi_{n_\rho}^{m_\ell}(\rho) \sim L_{n_\rho}^{m_\ell}(\rho)$$

$$\psi_{n_z}(z) \sim H_{n_z}(z)$$

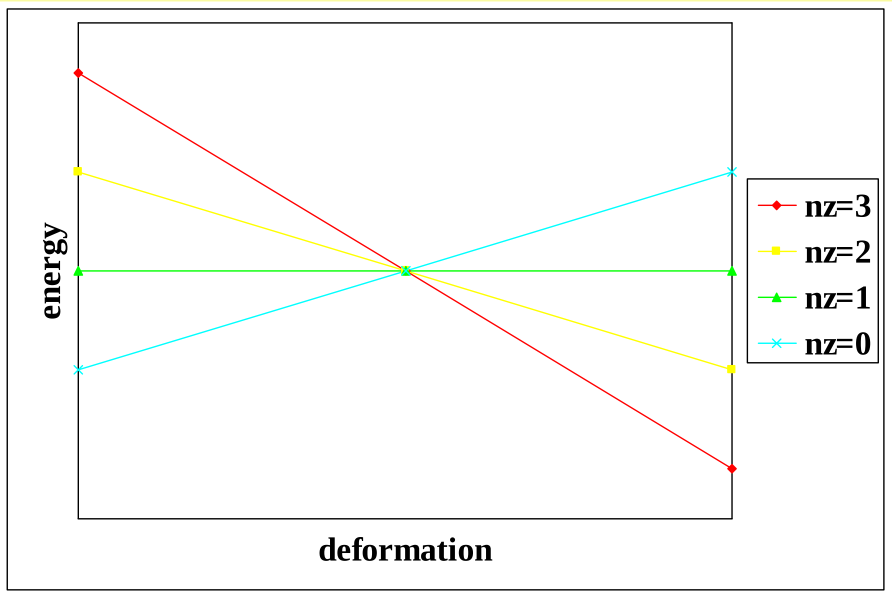
Anisotropic Harmonic Oscillator

Energy level structure: N=3

$$\varepsilon\left(n_z, n_\rho, m_\ell\right) = \sum_{i=x,y,z} \hbar\omega_i \left(n_z + \frac{1}{2}\right) = \hbar\omega_z \left(n_z + \frac{1}{2}\right) + \hbar\omega_\perp \left(2n_\rho + m_\ell + 1\right)$$

$$= \hbar \overset{0}{\omega}_0 \left[\left(N + \frac{3}{2}\right) + \delta\left(\frac{N}{3} - n_z\right)\right]$$

$$\varepsilon^{N=3}\left(n_z n_\rho m_\ell\right) = \frac{9}{2}\hbar\overset{0}{\omega}_0 + \hbar\overset{0}{\omega}_0\,\delta\left(1 - n_z\right)$$



$$N = n_z + 2n_\rho + m_\ell$$

n_z	m_l	n_ρ	Ω	deg
0	3	0	5/2, 7/2	4
	1	1	1/2, 3/2	
1	2	0	3/2, 5/2	3
	0	1	1/2	
2	1	0	1/2, 3/2	2
3	0	0	1/2	1

The Nilsson model: Hamiltonian

Axially symmetric harmonic oscillator potential

+spin-orbit term

+l² term

$$H = H_0 + C\vec{\ell}\cdot\vec{s} + D\left(\vec{\ell}^2 - \left\langle \vec{\ell}^2 \right\rangle_N\right)$$

$$= \hbar\omega_0(\delta)\left[-\frac{1}{2}\Delta + \frac{1}{2}r^2 - \beta r^2 Y_{20}\right] - \kappa\hbar\overset{0}{\omega_0}\left[2\vec{\ell}\cdot\vec{s} + \mu\left(\vec{\ell}^2 - \left\langle \vec{\ell}^2 \right\rangle_N\right)\right]$$

$$C = -2\kappa\hbar\overset{0}{\omega_0}$$

$$D = -\kappa\mu\hbar\overset{0}{\omega_0}$$

$$\left\langle \vec{\ell}^2 \right\rangle_N = \frac{1}{2}N(N+3)$$

The Nilsson model: Hamiltonian

$$H = \overset{0}{H}_0 + \kappa\hbar\overset{0}{\omega}_0\, F$$

$$F = \frac{\delta}{\kappa}\left[1 - \frac{4}{3}\delta^2 - \frac{16}{27}\delta^3\right]^{-1/6}\left\{-\frac{4}{3}\sqrt{\frac{\pi}{5}}r^2 Y_{20}\right\} - 2\vec{\ell}\cdot\vec{s} - \mu\left(\vec{\ell}^{\,2} - \left\langle \vec{\ell}^{\,2}\right\rangle_N\right)$$

$$F = \eta U - 2\vec{\ell}\cdot\vec{s} - \mu\left(\vec{\ell}^{\,2} - \left\langle \vec{\ell}^{\,2}\right\rangle_N\right)$$

$$E = \left(N + \frac{3}{2}\right)\hbar\omega_0\left(\delta\right) + \kappa\hbar\overset{0}{\omega}_0\, f$$

	N,Z<50	50<Z<82	82<N<126	82<Z	126<N
κ	0.08	0.0637	0.0637	0.0577	0.0635
μ	0	0.60	0.42	0.65	0.325

The Nilsson model: Basis

$\vec{\ell} \cdot \vec{s}$ and $\vec{\ell}^{\,2}$ nondiagonal in basis $\{N, n_z, n_\rho, m_\ell, m_s\}$

For large deformations $\vec{\ell} \cdot \vec{s}$, $\vec{\ell}^{\,2}$ can be neglected:

Asymptotic quantum numbers $\{N, n_z, n_\rho, m_\ell, m_s\}$: $[N n_z m_\ell]\Omega\pi$

For small deformations δ-terms can be neglected:

Spherical basis $\{N, \ell, j, \Omega\}$

Nilsson used basis $\{N, \ell, m_\ell, m_s\}$

Diagonal terms

$$\overset{0}{H}_0 \left| N, \ell, m_\ell, m_s \right\rangle = \left(N + \frac{3}{2} \right) \hbar \overset{0}{\omega}_0 \left| N, \ell, m_\ell, m_s \right\rangle$$

$$\vec{\ell}^{\,2} \left| N, \ell, m_\ell, m_s \right\rangle = \ell(\ell+1) \left| N, \ell, m_\ell, m_s \right\rangle$$

The Nilsson model:Matrix elements

Matrix elements

$$\left\langle \ell' m'_\ell m'_s \left| \vec{\ell}\cdot\vec{s} \right| \ell m_\ell m_s \right\rangle \quad \begin{cases} \ell = \ell' \\ m_\ell = m'_\ell, m'_\ell \pm 1 \\ m_s = m'_s \pm 1, m'_s \\ m_\ell + m_s = m'_\ell + m'_s \end{cases}$$

$$\left\langle \ell, m_\ell \pm 1, \mp \left| \vec{\ell}\cdot\vec{s} \right| \ell, m_\ell, \pm \right\rangle = \frac{1}{2}\sqrt{(\ell \mp m_\ell)(\ell \pm m_\ell + 1)}$$

$$\left\langle \ell, m_\ell, \pm \left| \vec{\ell}\cdot\vec{s} \right| \ell, m_\ell, \pm \right\rangle = \pm\frac{1}{2} m_\ell$$

$$\left\langle \ell' m'_\ell \left| Y_{20} \right| \ell m_\ell \right\rangle = i^{\ell-\ell'} \sqrt{\frac{5}{4\pi}} \sqrt{\frac{2\ell+1}{2\ell'+1}} \left\langle \ell 2 m_\ell 0 \middle| \ell 2 \ell' m'_\ell \right\rangle \left\langle \ell 2 0 0 \middle| \ell 2 \ell' 0 \right\rangle$$

$$\begin{cases} m_\ell = m'_\ell \quad m_s = m'_s \\ \ell = \ell', \ell' \pm 2 \quad N = N' \pm 2 \end{cases}$$

The Nilsson model: Matrix elements

Radial matrix elements

$$\left|N\ell\right\rangle = \sqrt{\frac{2(n-1)!}{b^3\left[\Gamma(n+\ell+1/2)\right]^3}}\left(\frac{r}{b}\right)^{\ell} e^{-\frac{1}{2}\left(\frac{r}{b}\right)^2} L_{n-1}^{\ell+1/2}\left(r^2/b^2\right)$$

$$\left\langle N'\ell'\right|r^2\left|N\ell\right\rangle = \left[\frac{(n'-1)!(n-1)!}{\left[\Gamma(n'+\ell'+1/2)\right]\left[\Gamma(n+\ell+1/2)\right]}\right]^{1/2} b^2 (-1)^{n'+n} \mu!\nu!$$

$$\times\sum_{\sigma}\frac{\Gamma(p+\sigma+1)}{\sigma!(n'-1-\sigma)!(n-1-\sigma)!(\sigma+\mu-n'+1)!(\sigma+\nu-n+1)!}$$

$$p=\frac{1}{2}(\ell+\ell'+3) \qquad \mu = p-\ell'-1/2 \qquad \nu = p-\ell-1/2$$

$$N = 2(n-1)+\ell$$

$N, N\pm 2$ admixtures

$$\left\langle N\ell\right|r^2\left|N\ell\right\rangle = (2n+\ell-1/2) = (N+3/2)$$

$$\left\langle N\ell-2\right|r^2\left|N\ell\right\rangle = 2\sqrt{n(n+\ell-1/2)}$$

$$\left\langle N-2\ell\right|r^2\left|N\ell\right\rangle = -\sqrt{(n-1)(n+\ell-1/2)}$$

$$\left\langle N-2\ell-2\right|r^2\left|N\ell\right\rangle = \sqrt{(n+\ell-1/2)(n+\ell-3/2)}$$

$$\left\langle N-2\ell+2\right|r^2\left|N\ell\right\rangle = \sqrt{(n-1)(n-2)}$$

The Nilsson model

Nilsson states:

$$\left|i\right\rangle_{\{\Omega\pi\}} = \sum_{\alpha} C_i^{\alpha} \left|\alpha\right\rangle; \quad \alpha\left\{N\ell m_\ell m_s\right\}$$

$N \qquad \Omega \qquad \left|N\ell m_\ell m_s\right\rangle_{\Omega\pi}$

$N=0 \quad \Omega=1/2 \quad \left|000+\right\rangle_{1/2^+}$

$N=1 \quad \Omega=3/2 \quad \left|111+\right\rangle_{3/2^-}$

$\Omega=1/2 \quad \left|110+\right\rangle_{1/2^-} \quad \left|111-\right\rangle_{1/2^-}$

$N=2 \quad \Omega=5/2 \quad \left|222+\right\rangle_{5/2^+}$

$\Omega=3/2 \quad \left|221+\right\rangle_{3/2^+} \quad \left|222-\right\rangle_{3/2^+}$

$\Omega=1/2 \quad \left|220+\right\rangle_{5/2^+} \quad \left|200+\right\rangle_{5/2^+} \quad \left|221-\right\rangle_{5/2^+}$

$N=3 \quad \Omega=7/2 \quad \left|333+\right\rangle_{7/2^-}$

$\Omega=5/2 \quad \left|332+\right\rangle_{5/2^-} \quad \left|333-\right\rangle_{5/2^-}$

$\Omega=3/2 \quad \left|331+\right\rangle_{3/2^-} \quad \left|311+\right\rangle_{3/2^-} \quad \left|332-\right\rangle_{3/2^-}$

$\Omega=1/2 \quad \left|330+\right\rangle_{1/2^-} \quad \left|331+\right\rangle_{3/2^-} \quad \left|331-\right\rangle_{1/2^-} \quad \left|311-\right\rangle_{1/2^-}$

The Nilsson model: N=0

Diagonalization in blocks Ω,π (with or without ΔN=2 admixtures)

$$H = \overset{0}{H}_0 + \kappa\hbar\overset{0}{\omega}_0 F = \overset{0}{H}_0 + \kappa\hbar\overset{0}{\omega}_0 \left\{\eta U - 2\vec{\ell}\cdot\vec{s} - \mu\vec{\ell}^{\,2}\right\}$$

$$\eta = \frac{\delta}{\kappa}\left[1 - \frac{4}{3}\delta^2 - \frac{16}{27}\delta^3\right]^{-1/6} \qquad U = \left\{-\frac{4}{3}\sqrt{\frac{\pi}{5}}\, r^2 Y_{20}\right\}$$

$$N = 0 \quad \Omega = 1/2 \quad \rightarrow \quad n = 1, \ell = 0$$

$$\left|N\ell m_\ell m_s\right\rangle \quad \rightarrow \quad \left|000+\right\rangle$$

$$N = 0 \ \rightarrow \ \mu = 0$$

$$\left\langle 000+\right|\vec{\ell}\cdot\vec{s}\left|000+\right\rangle = 0$$

$$\left\langle 000+\right|Y_{20}\left|000+\right\rangle \sim \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 0$$

$$\left\langle 000+\right|\eta U - 2\vec{\ell}\cdot\vec{s} - \mu\vec{\ell}^{\,2}\left|000+\right\rangle = 0$$

The Nilsson model: N=1

$$N=1 \quad \Omega=3/2 \quad \rightarrow \quad n=1, \ell=1$$
$$|N\ell m_\ell m_s\rangle \quad \rightarrow \quad |111+\rangle$$

$$N=1 \quad \Omega=1/2 \quad \rightarrow \quad n=1, \ell=1$$
$$|N\ell m_\ell m_s\rangle \quad \rightarrow \quad |110+\rangle, |111\text{-}\rangle$$

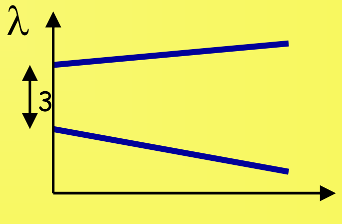
$$N=1 \ \rightarrow \ \mu=0$$
$$\langle 111+|\vec{\ell}\cdot\vec{s}|111+\rangle = \frac{1}{2}$$
$$\langle 111+|-\eta\frac{4}{3}\sqrt{\frac{\pi}{5}}r^2Y_{20}|111+\rangle = -\eta\frac{4}{3}\sqrt{\frac{\pi}{5}}\langle 11|r^2|11\rangle\langle 11|Y_{20}|11\rangle = \frac{1}{3}\eta$$
$$\langle 11|r^2|11\rangle = 1+\frac{3}{2}$$
$$\langle 11|Y_{20}|11\rangle = \sqrt{\frac{5}{4\pi}}\langle 1210|1211\rangle\langle 1200|1210\rangle = -\sqrt{\frac{5}{4\pi}}\frac{1}{5}$$

$$\langle 111+|\eta U - 2\vec{\ell}\cdot\vec{s} - \mu\vec{\ell}^2|111+\rangle = \frac{1}{3}\eta - 1$$

$$N=1 \ \rightarrow \ \mu=0$$
$$\langle 110+|\vec{\ell}\cdot\vec{s}|110+\rangle = 0$$
$$\langle 110+|-\eta\frac{4}{3}\sqrt{\frac{\pi}{5}}r^2Y_{20}|110+\rangle = -\eta\frac{4}{3}\sqrt{\frac{\pi}{5}}\langle 11|r^2|11\rangle\langle 10|Y_{20}|10\rangle = -\frac{2}{3}\eta$$
$$\langle 11|r^2|11\rangle = 1+\frac{3}{2}$$
$$\langle 10|Y_{20}|10\rangle = \sqrt{\frac{5}{4\pi}}\langle 1200|1210\rangle\langle 1200|1210\rangle = \sqrt{\frac{5}{4\pi}}\frac{2}{5}$$

$$\langle 110+|\eta U - 2\vec{\ell}\cdot\vec{s} - \mu\vec{\ell}^2|110+\rangle = -\frac{2}{3}\eta$$
$$\langle 111-|\eta U - 2\vec{\ell}\cdot\vec{s} - \mu\vec{\ell}^2|111\text{-}\rangle = 1+\frac{1}{3}\eta$$
$$\langle 110+|\eta U - 2\vec{\ell}\cdot\vec{s} - \mu\vec{\ell}^2|111\text{-}\rangle = -\sqrt{2}$$

$$\begin{array}{cc} |110+\rangle & |111\text{-}\rangle \end{array}$$
$$\begin{pmatrix} -\frac{2}{3}\eta & -\sqrt{2} \\ -\sqrt{2} & 1+\frac{1}{3}\eta \end{pmatrix} \qquad \lambda = \frac{1-\frac{1}{3}\eta \pm \sqrt{\eta^2+2\eta+9}}{2}$$



The Nilsson model: N=2

$N=2 \quad \Omega=5/2 \quad \rightarrow \quad \{n=1,\ell=2\}\{n=2,\ell=0\}$

$|N\ell m_\ell m_s\rangle \quad \rightarrow \quad |222+\rangle$

$\langle 222+|\eta U-2\vec{\ell}\cdot\vec{s}-\mu\vec{\ell}^{\,2}|222+\rangle=-2+\frac{2}{3}\eta$

$N=2 \quad \Omega=3/2$

$|N\ell m_\ell m_s\rangle \quad \rightarrow \quad |221+\rangle, |222\text{-}\rangle$

$\langle 221+|\eta U-2\vec{\ell}\cdot\vec{s}-\mu\vec{\ell}^{\,2}|221+\rangle=-1-\frac{1}{3}\eta$

$\langle 222-|\eta U-2\vec{\ell}\cdot\vec{s}-\mu\vec{\ell}^{\,2}|222\text{-}\rangle=2+\frac{2}{3}\eta$

$\langle 222-|\eta U-2\vec{\ell}\cdot\vec{s}-\mu\vec{\ell}^{\,2}|221+\rangle=-2$

$$\begin{array}{cc} |221+\rangle & |222\text{-}\rangle \end{array}$$
$$\begin{pmatrix} -\left(1+\frac{1}{3}\eta\right) & -2 \\ -2 & 2+\frac{2}{3}\eta \end{pmatrix}$$

$N=2 \quad \Omega=1/2$

$|N\ell m_\ell m_s\rangle \quad \rightarrow \quad |220+\rangle, |200+\rangle, |221\text{-}\rangle$

$\langle 220+|\eta U-2\vec{\ell}\cdot\vec{s}-\mu\vec{\ell}^{\,2}|220+\rangle=-\frac{2}{3}\eta$

$\langle 200+|\eta U-2\vec{\ell}\cdot\vec{s}-\mu\vec{\ell}^{\,2}|200+\rangle=0$

$\langle 221-|\eta U-2\vec{\ell}\cdot\vec{s}-\mu\vec{\ell}^{\,2}|221+\rangle=1-\frac{1}{3}\eta$

$\langle 220+|\eta U-2\vec{\ell}\cdot\vec{s}-\mu\vec{\ell}^{\,2}|200+\rangle=\frac{2\sqrt{2}}{3}\eta$

$\langle 220+|\eta U-2\vec{\ell}\cdot\vec{s}-\mu\vec{\ell}^{\,2}|221\text{-}\rangle=-\sqrt{6}$

$\langle 200+|\eta U-2\vec{\ell}\cdot\vec{s}-\mu\vec{\ell}^{\,2}|221\text{-}\rangle=0$

$$\begin{array}{ccc} |220+\rangle & |200+\rangle & |221\text{-}\rangle \end{array}$$
$$\begin{pmatrix} -\frac{2}{3}\eta & \frac{2\sqrt{2}}{3}\eta & -\sqrt{6} \\ & 0 & 0 \\ & & 1-\frac{1}{3}\eta \end{pmatrix}$$

The Nilsson model: N=3

$$N=3 \quad \Omega=7/2$$

$$|333+\rangle$$

$$\left(\eta-3-12\mu\right)$$

$$N=3 \quad \Omega=5/2$$

$$|332+\rangle \qquad |333-\rangle$$

$$\begin{pmatrix} -1-12\mu & -\sqrt{6} \\ & \eta+3-12\mu \end{pmatrix}$$

$$N=3 \quad \Omega=3/2$$

$$|331+\rangle \qquad |311+\rangle \qquad |332-\rangle$$

$$\begin{pmatrix} -\frac{3}{5}\eta-1-12\mu & \frac{4}{5}\eta & -\sqrt{10} \\ & \frac{3}{5}\eta-1-2\mu & 0 \\ & & 2-12\mu \end{pmatrix}$$

$$N=3 \quad \Omega=1/2$$

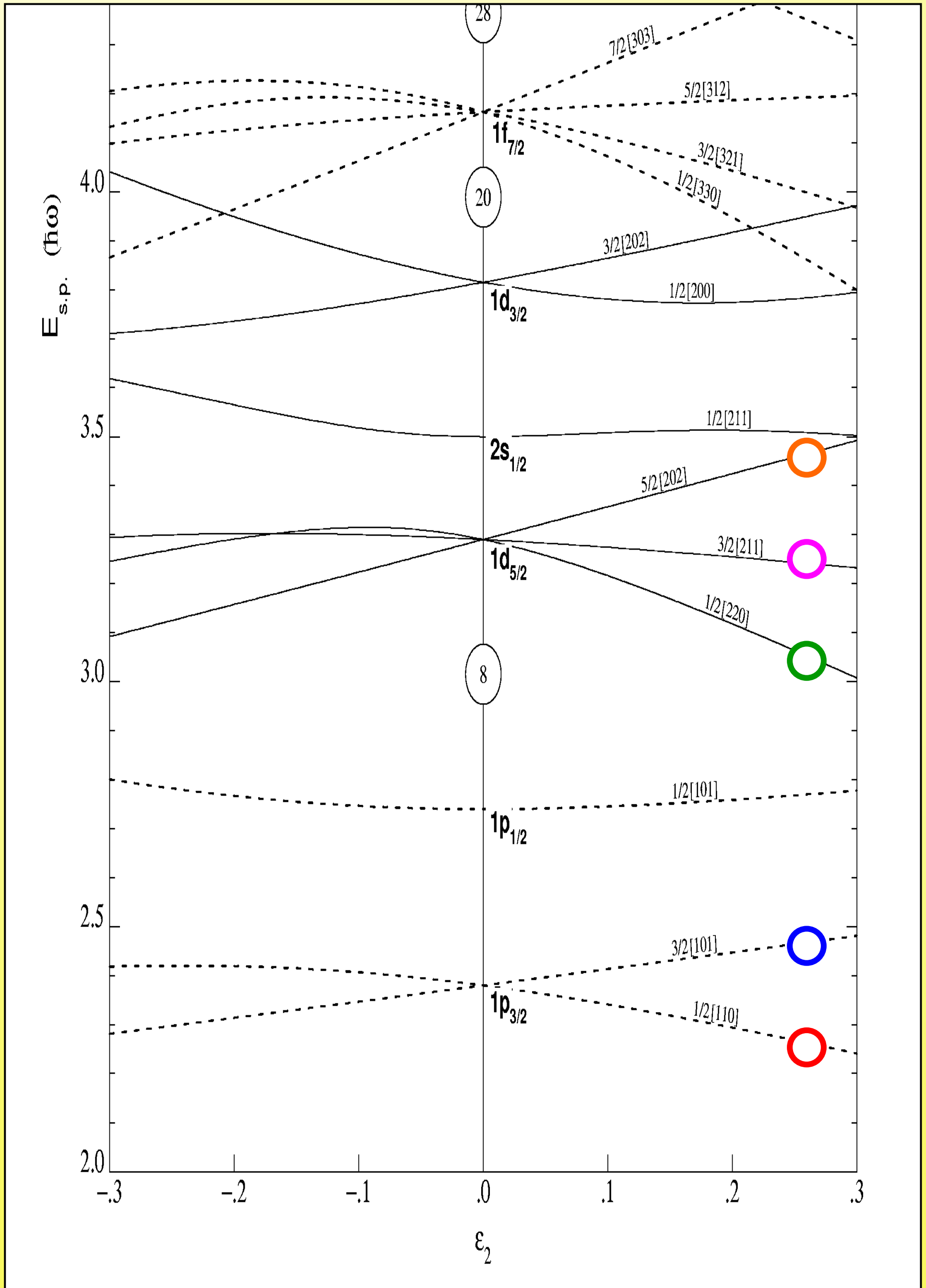
$$|330+\rangle \qquad |310+\rangle \qquad |331-\rangle \qquad |311-\rangle$$

$$\begin{pmatrix} -\frac{4}{5}\eta-12\mu & \frac{2\sqrt{6}}{5}\eta & -2\sqrt{3} & 0 \\ & -\frac{6}{5}\eta-2\mu & 0 & -\sqrt{2} \\ & & -\frac{3}{5}\eta+1-12\mu & \frac{4}{5}\eta \\ & & & \frac{3}{5}\eta+1-2\mu \end{pmatrix}$$

The Nilsson model: ΔN=2

Ω=1/2⁺ with ΔN=2 admixtures

	$\|000+\rangle$	$\|220+\rangle$	$\|200+\rangle$	$\|221-\rangle$	$\|440+\rangle$	$\|420+\rangle$	$\|400+\rangle$	$\|441-\rangle$	$\|421-\rangle$
$\|000+\rangle$	0	$\frac{1}{\sqrt{3}}\eta$	0	0	0	0	0	0	0
$\|220+\rangle$		$-\frac{2}{3}\eta$	$-\frac{2\sqrt{2}}{3}\eta$	$-\sqrt{6}$	$\frac{6}{\sqrt{35}}\eta$	$\frac{2}{3}\sqrt{\frac{2}{7}}\eta$	$\frac{2}{3}\sqrt{\frac{2}{5}}\eta$	0	0
$\|200+\rangle$			0	0	0	$\frac{1}{3}\sqrt{7}\eta$	0	0	0
$\|221-\rangle$				$-\frac{1}{3}\eta+1$	0	0	0	$\sqrt{\frac{6}{7}}\eta$	$\frac{1}{3}\sqrt{\frac{2}{7}}\eta$
$\|440+\rangle$					$-\frac{20}{21}\eta-20\mu$	$\frac{12\sqrt{10}}{35}\eta$	0	$-\sqrt{20}$	0
$\|420+\rangle$						$-\frac{22}{21}\eta-6\mu$	$\frac{4}{15}\sqrt{35}\eta$	0	$-\sqrt{6}$
$\|400+\rangle$							0	0	0
$\|441-\rangle$								$-\frac{17}{21}\eta+1-20\mu$	$\frac{4}{7}\sqrt{3}\eta$
$\|421-\rangle$									$-\frac{11}{21}\eta+1-6\mu$



$${}^{A}_{Z}Nucleus_{N} \rightarrow K^{\pi}$$

$${}^{27}_{14}\mathrm{Si}_{13} \rightarrow 5/2^{+}$$

$${}^{27}_{13}\mathrm{Al}_{14} \rightarrow 5/2^{+} \qquad {}^{25}_{12}\mathrm{Mg}_{13} \rightarrow 5/2^{+}$$

$${}^{23}_{11}\mathrm{Na}_{12} \rightarrow 3/2^{+} \qquad {}^{19}_{8}\mathrm{O}_{11} \rightarrow 3/2^{+}$$

$${}^{19}_{9}\mathrm{F}_{10} \rightarrow 1/2^{+} \qquad {}^{19}_{10}\mathrm{Ne}_{9} \rightarrow 1/2^{+}$$

$${}^{9}_{4}\mathrm{Be}_{5} \rightarrow 3/2^{-}$$

$${}^{7}_{3}\mathrm{Li}_{4} \rightarrow 1/2^{-}$$

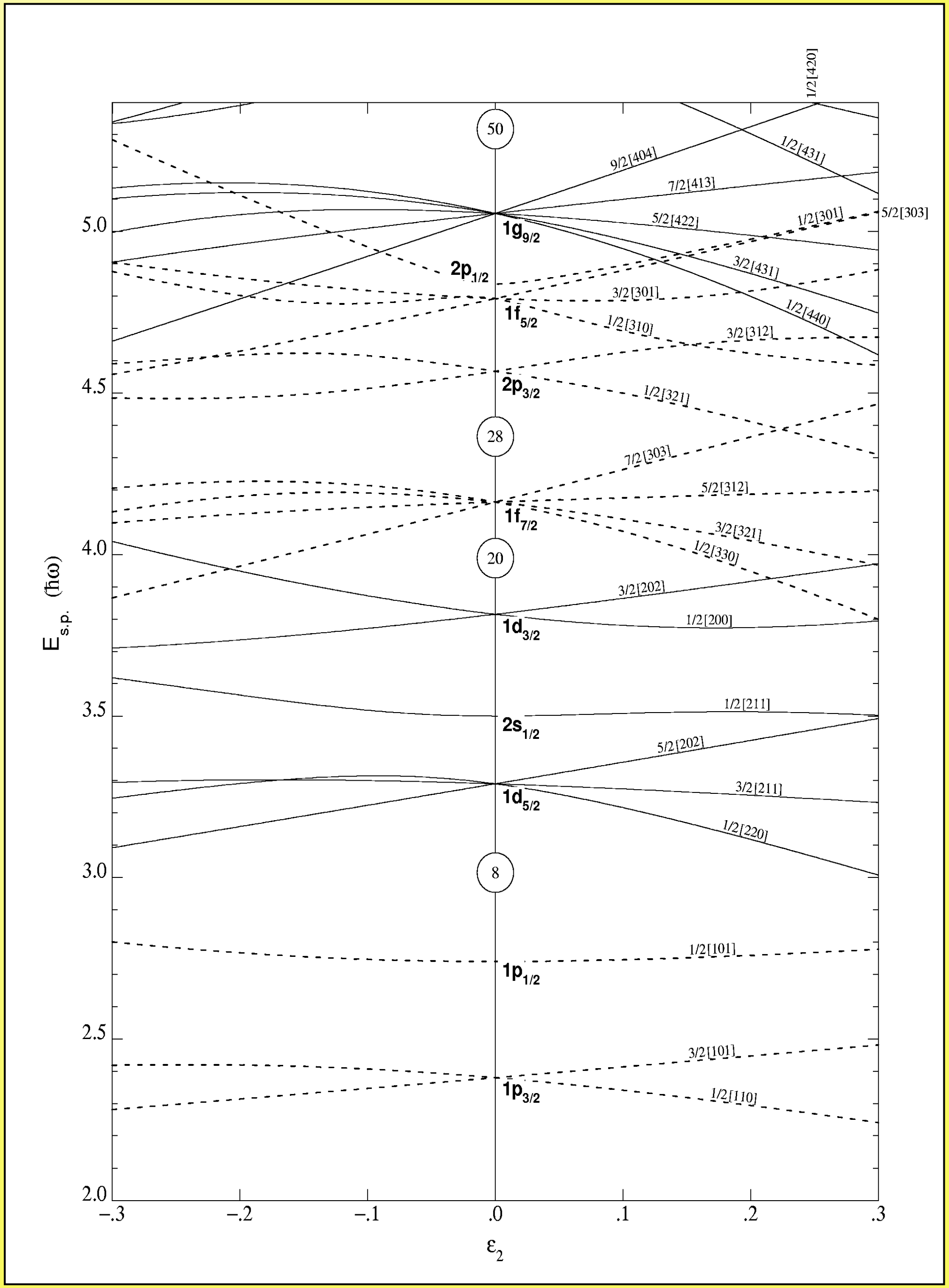
• Spherical levels split into (2j+1)/2 levels

• Levels ($\Omega\pi$) are twofold degenerate

• Asymptotic q-numbers not conserved for small deformations but useful to classify levels

• For positive deformations (PROLATE SHAPES), levels with lower Ω are shifted downwards

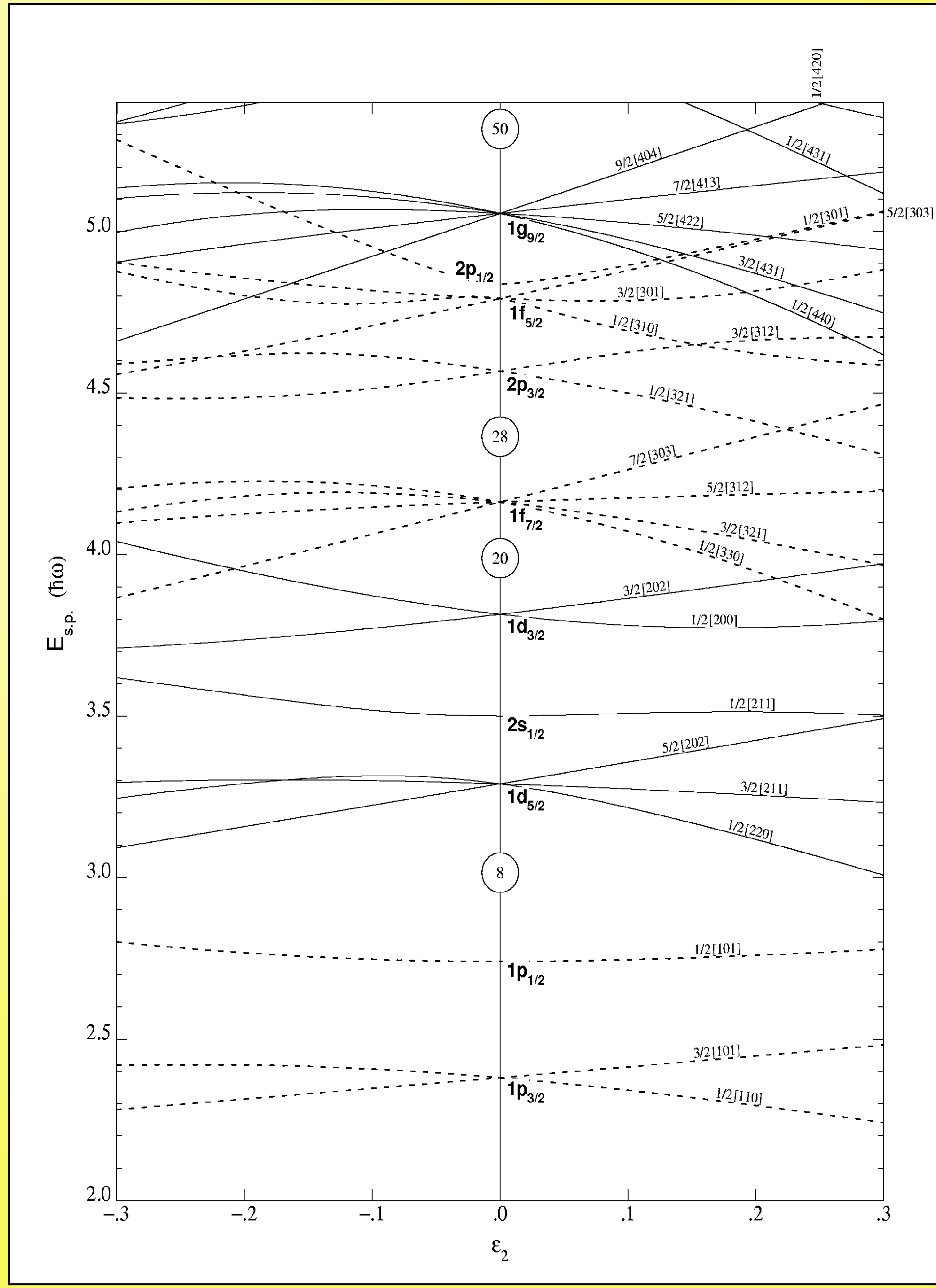
• For negative deformations (OBLATE SHAPES), levels with lower Ω are shifted upwards



$N = n_z + 2n_\rho + m_\ell$

$[N n_z m_\ell]\Omega^\pi$

$N=0$	$\Omega=1/2^+$	$n_z=0$ $m_\ell=0$	$[000]1/2^+$
$N=1$	$\Omega=1/2^-$	$n_z=1$ $m_\ell=0$	$[110]1/2^-$
		$n_z=0$ $m_\ell=1$	$[101]1/2^-$
$N=1$	$\Omega=3/2^-$	$n_z=0$ $m_\ell=1$	$[101]3/2^-$
$N=2$	$\Omega=1/2^+$	$n_z=2$ $m_\ell=0$	$[220]1/2^+$
		$n_z=1$ $m_\ell=1$	$[211]1/2^+$
		$n_z=0$ $m_\ell=0$	$[200]1/2^+$
$N=2$	$\Omega=3/2^+$	$n_z=1$ $m_\ell=1$	$[211]3/2^+$
		$n_z=0$ $m_\ell=2$	$[202]3/2^+$
$N=2$	$\Omega=5/2^+$	$n_z=0$ $m_\ell=2$	$[202]5/2^+$
$N=3$	$\Omega=1/2^-$	$n_z=3$ $m_\ell=0$	$[330]1/2^-$
		$n_z=2$ $m_\ell=1$	$[321]1/2^-$
		$n_z=1$ $m_\ell=0$	$[310]1/2^-$
		$n_z=0$ $m_\ell=1$	$[301]1/2^-$
$N=3$	$\Omega=3/2^-$	$n_z=2$ $m_\ell=1$	$[321]3/2^-$
		$n_z=1$ $m_\ell=2$	$[312]3/2^-$
		$n_z=0$ $m_\ell=1$	$[301]3/2^-$
$N=3$	$\Omega=5/2^-$	$n_z=1$ $m_\ell=2$	$[312]5/2^-$
		$n_z=0$ $m_\ell=3$	$[303]5/2^-$
$N=3$	$\Omega=7/2^-$	$n_z=0$ $m_\ell=3$	$[303]7/2^-$

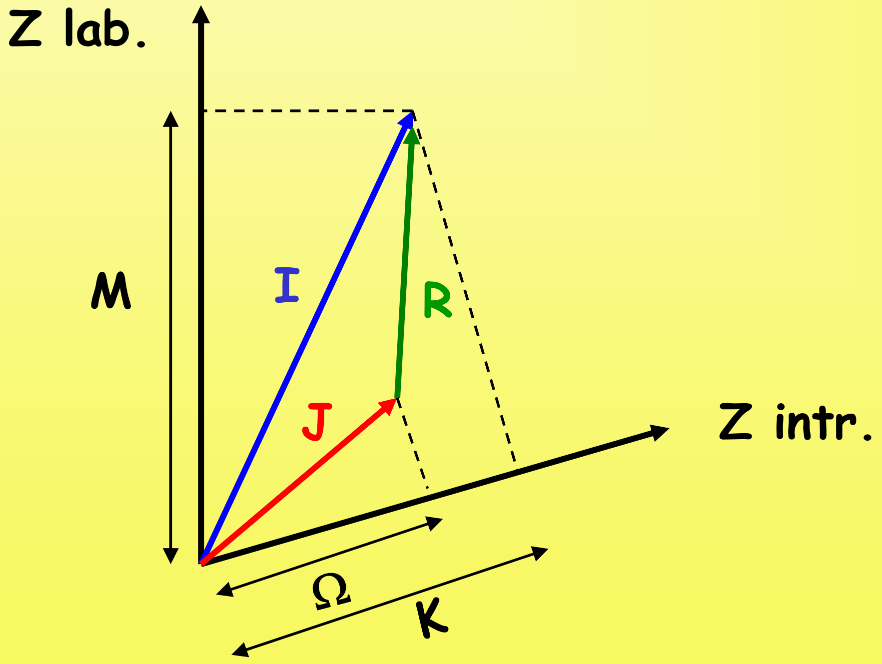


Laboratory frame

Wave functions in the unified model $\Psi \sim \phi\left(\vec{r}\right)\Phi\left(\theta_k\right)$

$$\Psi\left(IKM\right) = \sqrt{\frac{2I+1}{16\pi^2}}\left\{D_{MK}^{I*}\left(\theta_k\right)\phi_K\left(\vec{r}\right) + \left(-1\right)^{I-J} D_{M-K}^{I*}\left(\theta_k\right)\phi_{-K}\left(\vec{r}\right)\right\}$$

Intrinsic wave functions (Nilsson) Rotation matrices



3-j

$$\begin{pmatrix} a & a+\frac{1}{2} & \frac{1}{2} \\ \alpha & -\alpha-\frac{1}{2} & \frac{1}{2} \end{pmatrix} = (-)^{a-\alpha-1} \left[\frac{a+\alpha+1}{(2a+2)(2a+1)}\right]^{\frac{1}{2}}$$

$$\begin{pmatrix} a & a & 1 \\ \alpha & -\alpha-1 & 1 \end{pmatrix} = (-)^{a-\alpha} \left[\frac{(a-\alpha)(a+\alpha+1)}{2a(a+1)(2a+1)}\right]^{\frac{1}{2}}$$

$$\begin{pmatrix} a & a & 1 \\ \alpha & -\alpha & 0 \end{pmatrix} = (-)^{a-\alpha} \frac{\alpha}{[a(a+1)(2a+1)]^{\frac{1}{2}}}$$

$$\begin{pmatrix} a & a+1 & 1 \\ \alpha & -\alpha-1 & 1 \end{pmatrix} = (-)^{a-\alpha} \left[\frac{(a+\alpha+1)(a+\alpha+2)}{(2a+1)(2a+2)(2a+3)}\right]^{\frac{1}{2}}$$

$$\begin{pmatrix} a & a+1 & 1 \\ \alpha & -\alpha & 0 \end{pmatrix} = (-)^{a-\alpha-1} \left[\frac{(a-\alpha+1)(a+\alpha+1)}{(a+1)(2a+1)(2a+3)}\right]^{\frac{1}{2}}$$

$$\begin{pmatrix} a & a+\frac{1}{2} & \frac{3}{2} \\ \alpha & -\alpha-\frac{3}{2} & \frac{3}{2} \end{pmatrix} = (-)^{a-\alpha-1} \left[\frac{3(a+\alpha+1)(a+\alpha+2)(a-\alpha)}{2a(2a+1)(2a+2)(2a+3)}\right]^{\frac{1}{2}}$$

$$\begin{pmatrix} a & a+\frac{1}{2} & \frac{3}{2} \\ \alpha & -\alpha-\frac{1}{2} & \frac{1}{2} \end{pmatrix} = (-)^{a-\alpha} \left[\frac{a+\alpha+1}{2a(2a+1)(2a+2)(2a+3)}\right]^{\frac{1}{2}}(a-3\alpha)$$

$$\begin{pmatrix} a & a+\frac{3}{2} & \frac{3}{2} \\ \alpha & -\alpha-\frac{3}{2} & \frac{3}{2} \end{pmatrix} = (-)^{a-\alpha-1} \left[\frac{(a+\alpha+1)(a+\alpha+2)(a+\alpha+3)}{(2a+1)(2a+2)(2a+3)(2a+4)}\right]^{\frac{1}{2}}$$

$$\begin{pmatrix} a & a+\frac{3}{2} & \frac{3}{2} \\ \alpha & -\alpha-\frac{1}{2} & \frac{1}{2} \end{pmatrix} = (-)^{a-\alpha} \left[\frac{3(a-\alpha+1)(a+\alpha+1)(a+\alpha+2)}{(2a+1)(2a+2)(2a+3)(2a+4)}\right]^{\frac{1}{2}}$$

$$\begin{pmatrix} a & a & 2 \\ \alpha & -\alpha-2 & 2 \end{pmatrix} = (-)^{a-\alpha} \left[\frac{3(a+\alpha+1)(a+\alpha+2)(a-\alpha-1)(a-\alpha)}{a(2a+3)(2a+2)(2a+1)(2a-1)}\right]^{\frac{1}{2}}$$

$$\begin{pmatrix} a & a & 2 \\ \alpha & -\alpha-1 & 1 \end{pmatrix} = (-)^{a-\alpha} \left[\frac{3(a-\alpha)(a+\alpha+1)}{a(2a+3)(2a+2)(2a+1)(2a-1)}\right]^{\frac{1}{2}}(2\alpha+1)$$

$$\begin{pmatrix} a & a & 2 \\ \alpha & -\alpha & 0 \end{pmatrix} = (-)^{a-\alpha} \frac{3\alpha^2-a(a+1)}{[a(a+1)(2a+3)(2a+1)(2a-1)]^{\frac{1}{2}}}$$

$$\begin{pmatrix} a & a+1 & 2 \\ \alpha & -\alpha-2 & 2 \end{pmatrix} = (-)^{a-\alpha} \left[\frac{(a+\alpha+1)(a+\alpha+2)(a+\alpha+3)(a-\alpha)}{a(a+1)(2a+4)(2a+3)(2a+1)}\right]^{\frac{1}{2}}$$

$$\begin{pmatrix} a & a+1 & 2 \\ \alpha & -\alpha-1 & 1 \end{pmatrix} = (-)^{a-\alpha-1} \left[\frac{(a+\alpha+2)(a+\alpha+1)}{a(a+1)(2a+4)(2a+3)(2a+1)}\right]^{\frac{1}{2}}(a-2\alpha)$$

$$\begin{pmatrix} a & a+1 & 2 \\ \alpha & -\alpha & 0 \end{pmatrix} = (-)^{a-\alpha-1}\, \alpha \left[\frac{3(a+\alpha+1)(a-\alpha+1)}{a(a+1)(a+2)(2a+3)(2a+1)}\right]^{\frac{1}{2}}$$

$$\begin{pmatrix} a & a+2 & 2 \\ \alpha & -\alpha-2 & 2 \end{pmatrix} = (-)^{a-\alpha} \left[\frac{(a+\alpha+1)(a+\alpha+2)(a+\alpha+3)(a+\alpha+4)}{(2a+5)(2a+4)(2a+3)(2a+2)(2a+1)}\right]^{\frac{1}{2}}$$

$$\begin{pmatrix} a & a+2 & 2 \\ \alpha & -\alpha-1 & 1 \end{pmatrix} = (-)^{a-\alpha-1} \left[\frac{(a+\alpha+1)(a+\alpha+2)(a+\alpha+3)(a-\alpha+1)}{(a+1)(a+2)(2a+1)(2a+3)(2a+5)}\right]^{\frac{1}{2}}$$

$$\begin{pmatrix} a & a+2 & 2 \\ \alpha & -\alpha & 0 \end{pmatrix} = (-)^{a-\alpha} \left[\frac{3(a+\alpha+1)(a+\alpha+2)(a-\alpha+1)(a-\alpha+2)}{(a+1)(2a+5)(2a+4)(2a+3)(2a+1)}\right]^{\frac{1}{2}}$$

$$\langle ab\alpha\beta \mid c-\gamma\rangle = (-1)^{a-b-\gamma}\sqrt{2c+1}\begin{pmatrix} a & b & c \\ \alpha & \beta & \gamma \end{pmatrix}$$