



# Survey of neutrino-nucleus interactions

Main goals:

Reviewing of neutrino-nucleus scattering with an emphasis on bringing together the knowledge from different areas

Understanding the language of other communities:

- a) Electron scattering community
- b) Neutrino community
- c) Nuclear structure community
- d) High energy / particle physics community
- e) Monte Carlo and/or experimental community

Identify the common assumptions made which simplify the calculations

Identify the common assumptions made which are not needed because they oversimplify the physics but not (really) the calculations

Identify the places for (easy) improvement



# Survey of neutrino-nucleus interactions

Some material:

<http://nuclear.fis.ucm.es/PDFN/documentos>

yasuo-yo-prc.pdf (appendix)

inclusiv.pdf (first part)

tesis.ps2.pdf (appendix B)

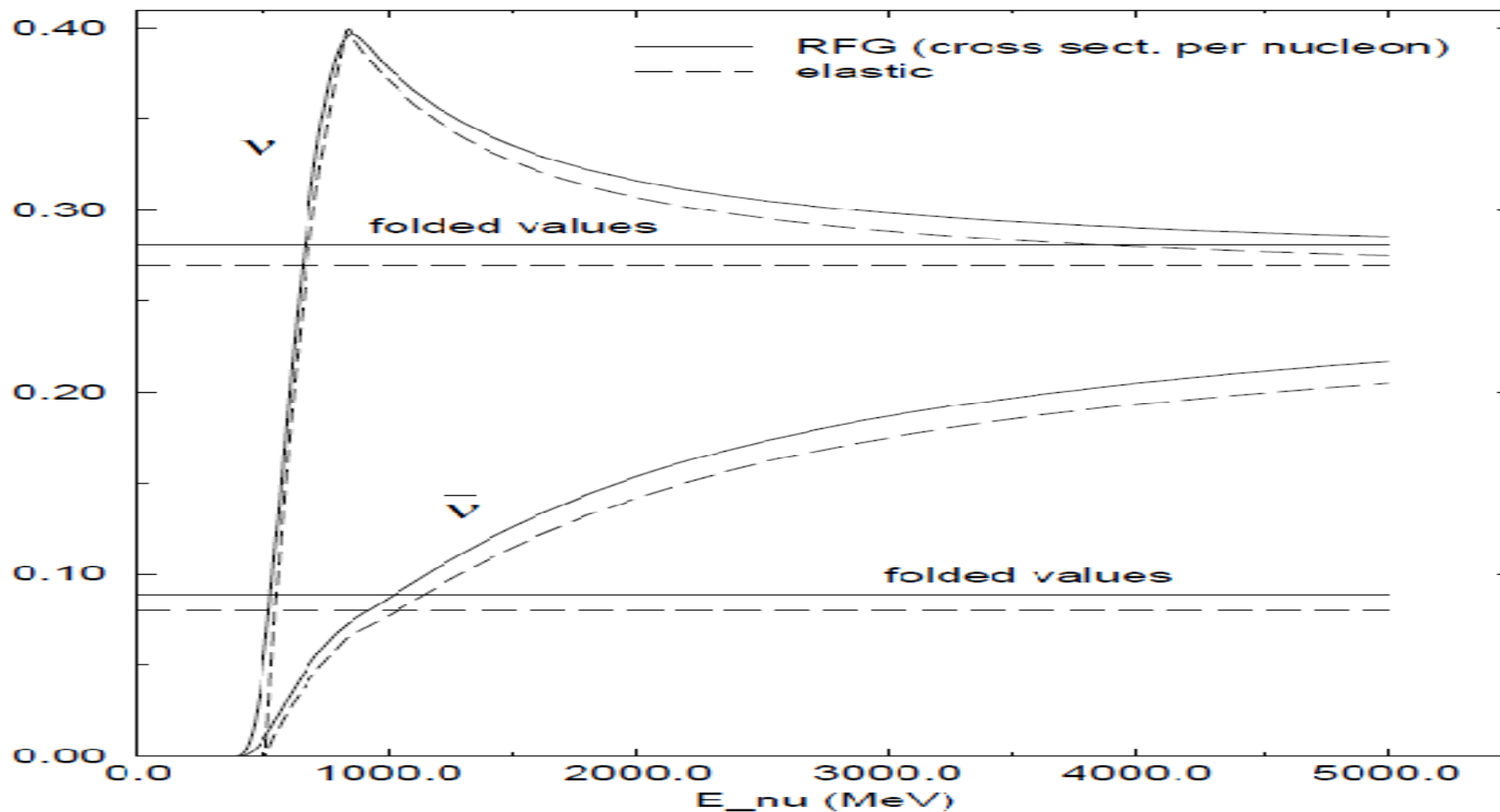
gordon2008.pdf

Folder CODES

We are talking 10% nuclear effects

### NC integrated cross sections

“nuclear model effects”





# About language

Inclusive versus Exclusive reactions

Elastic versus Quasielastic (and inelastic)

Pionic versus non pionic processes

Coherent versus incoherent scattering

(Nucleon/Nuclear) Transparency

Factorization in general and scaling in ***inclusive*** scattering

What is and how good is the RFG?

What means an off-shell effect

## • Introduction

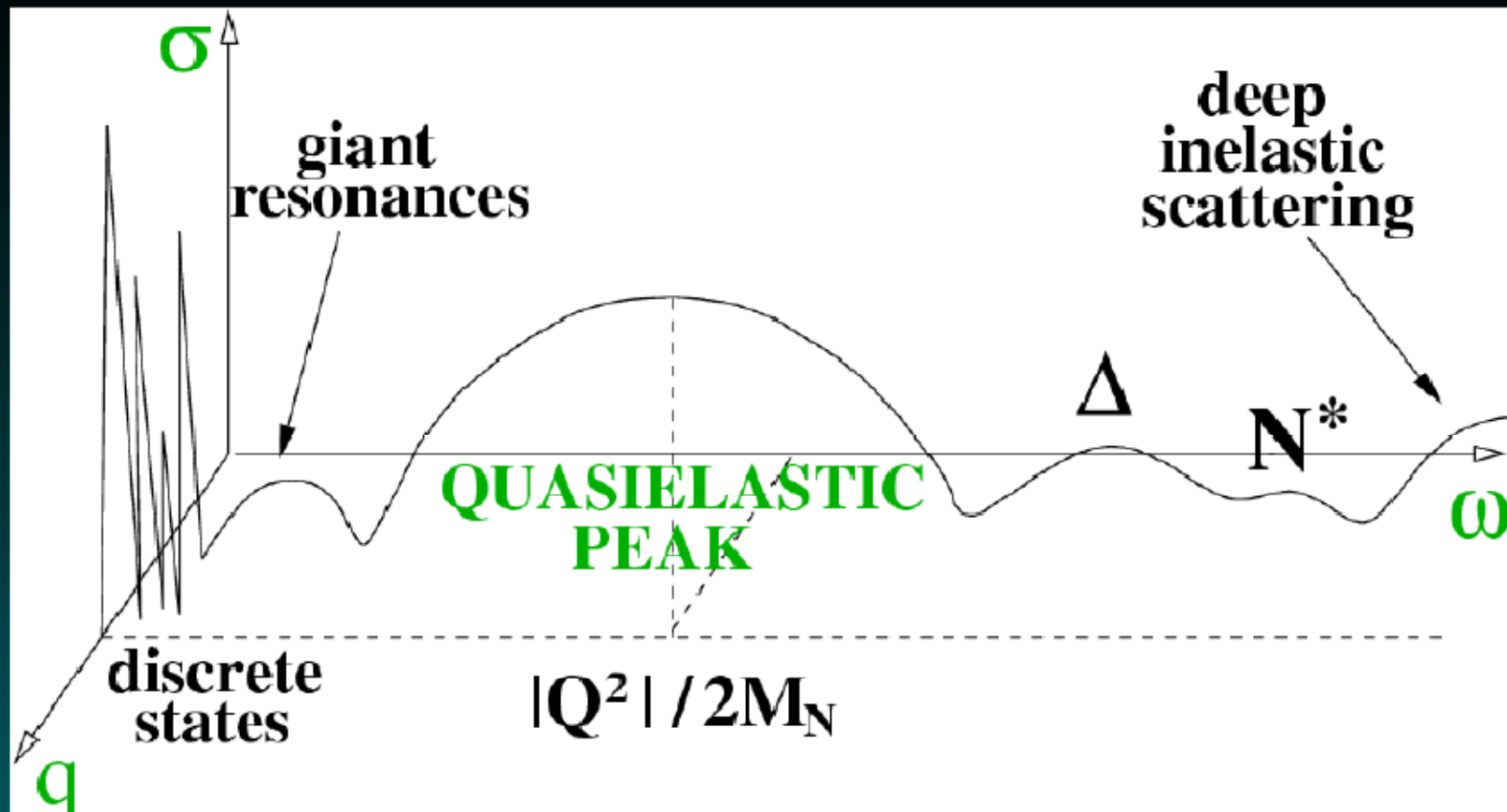
- Toda la materia está formada a partir de combinaciones de tan solo 12 *fermiones* (espín  $\frac{1}{2}$ ) (6 *quarks* y 6 *leptones*)

Partícula	Sabor			$Q/ e $
Leptones	$e$	$\mu$	$\tau$	-1
	$\nu_e$	$\nu_\mu$	$\nu_\tau$	0
Quarks	$u$	$c$	$t$	$+\frac{2}{3}$
	$d$	$s$	$b$	$-\frac{1}{3}$

• Some traces and cross-sections (see [inclusiv.pdf](#))

# Survey of neutrino-nucleus interactions

## Inclusive electron scattering on nuclei



Many things may happen to the nucleus, depending on the values of  $q$  and  $\omega$

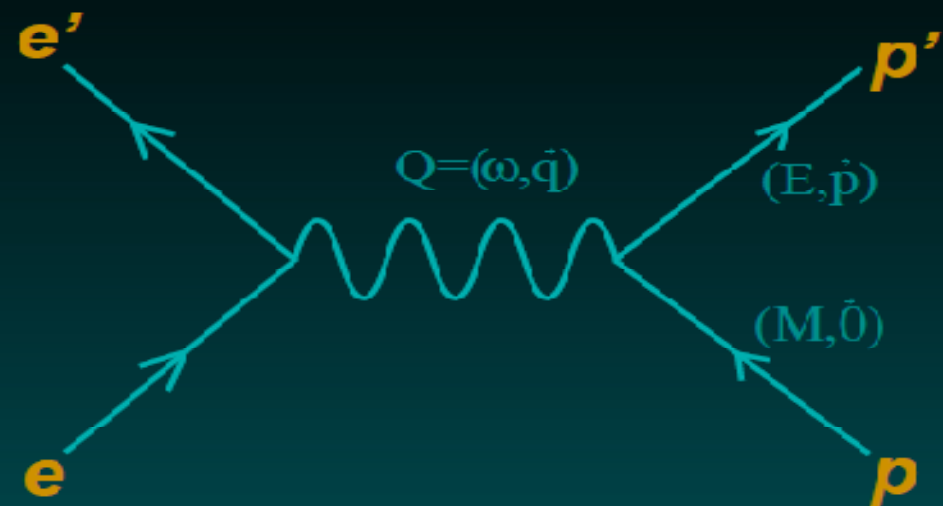
## ELASTIC ELECTRON SCATTERING OFF A FREE NUCLEON AT REST

Conservation of energy and momentum makes that:

$$\omega + M = \sqrt{p^2 + M^2} \Rightarrow$$

$$\omega + M = \sqrt{q^2 + M^2} \Rightarrow$$

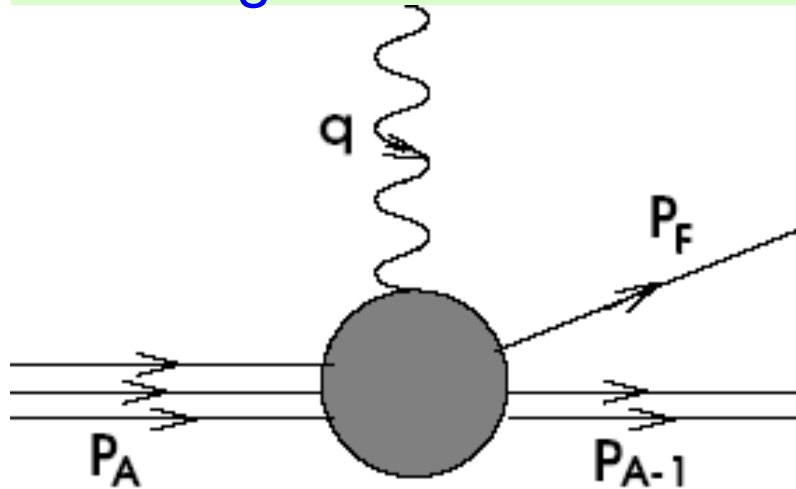
$$\omega = |Q^2|/2M$$



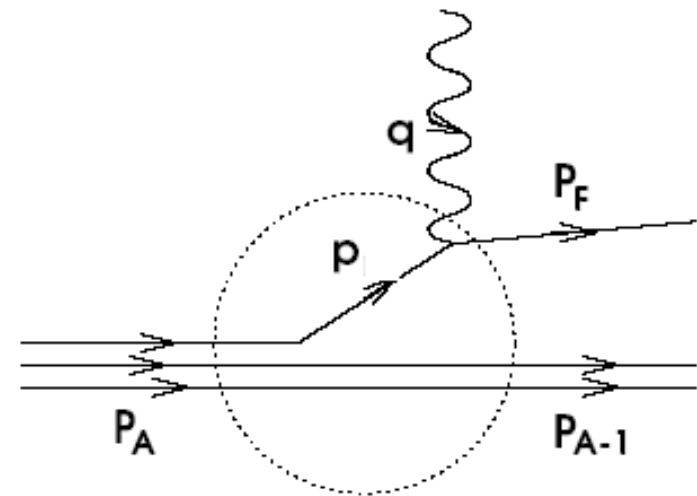
The transferred energy  $\omega$  and momentum  $q$  are related to each other:  
 $\omega$  and  $q$  are not independent degrees of freedom

# OVERVIEW OF THE MODEL (ingredients)

Simple: One photon exchange:



Even simpler: Impulse Approximation



$$J_N^\mu(\omega, \vec{q}) = \int d\vec{p} \bar{\psi}_F(\vec{p} + \vec{q}) \hat{J}_N^\mu(\omega, \vec{q}) \psi_B(\vec{p})$$



## Unpolarized and in plane:

$$\frac{d\sigma}{d\Omega_e d\varepsilon' d\Omega_F} = K \sigma_{Mott} f_{rec} \left[ v_L R^L + v_T R^T + v_{TL} R^{TL} \cos \phi_F + v_{TT} R^{TT} \cos 2\phi_F \right]$$

$$\rho^{exp}(\mathbf{p}_m) = \frac{\left( \frac{d\sigma}{d\varepsilon_f d\Omega_f d\Omega_F} \right)^{exp}}{E_F p_F f_{rec} \sigma_{ep}} .$$

$$A_{TL} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} ,$$

## One-photon exchange approximation yields, for the most general case:

$$\begin{aligned}
 \frac{d\sigma}{d\varepsilon_f d\Omega_f d\Omega_F} &= \frac{E_F p_F}{(2\pi)^3} \sigma_M f_{rec} \frac{1}{2} \left\{ v_L \left( R^L + R_n^L \hat{S}_n \right) + v_T \left( R^T + R_n^T \hat{S}_n \right) \right. \\
 &+ v_{TL} \left[ \left( R^{TL} + R_n^{TL} \hat{S}_n \right) \cos \phi_F + \left( R_l^{TL} \hat{S}_l + R_s^{TL} \hat{S}_s \right) \sin \phi_F \right] \\
 &+ v_{TT} \left[ \left( R^{TT} + R_n^{TT} \hat{S}_n \right) \cos 2\phi_F + \left( R_l^{TT} \hat{S}_l + R_s^{TT} \hat{S}_s \right) \sin 2\phi_F \right] \\
 &+ h \left\{ v_{TL'} \left[ \left( R_l^{TL'} \hat{S}_l + R_s^{TL'} \hat{S}_s \right) \cos \phi_F + \left( R^{TL'} + R_n^{TL'} \hat{S}_n \right) \sin \phi_F \right] \right. \\
 &\left. + v_{T'} \left[ R_l^{T'} \hat{S}_l + R_s^{T'} \hat{S}_s \right] \right\} ,
 \end{aligned}$$

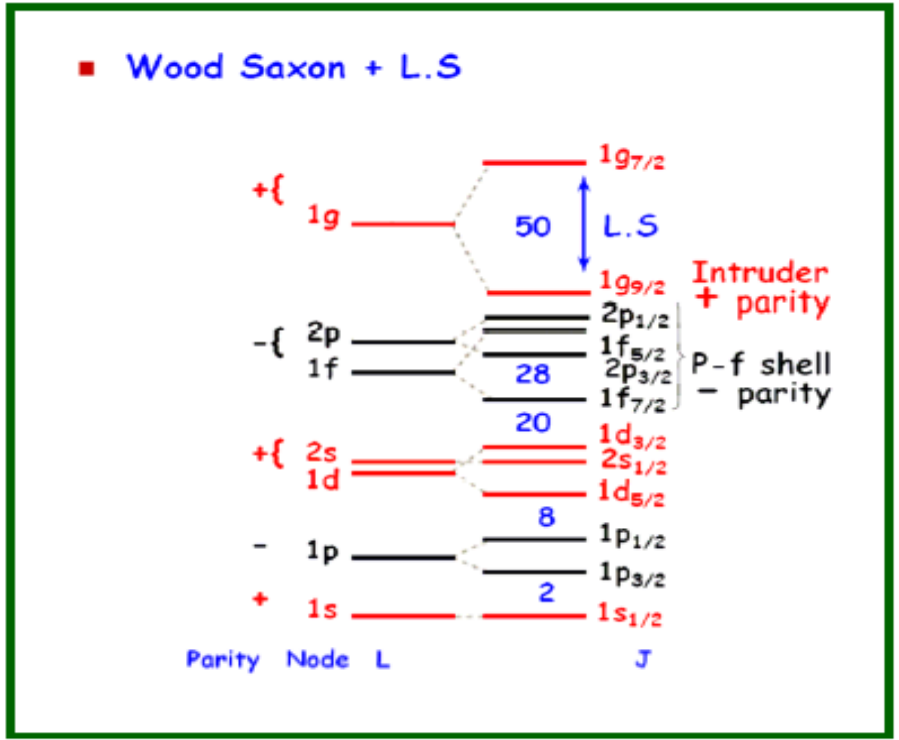
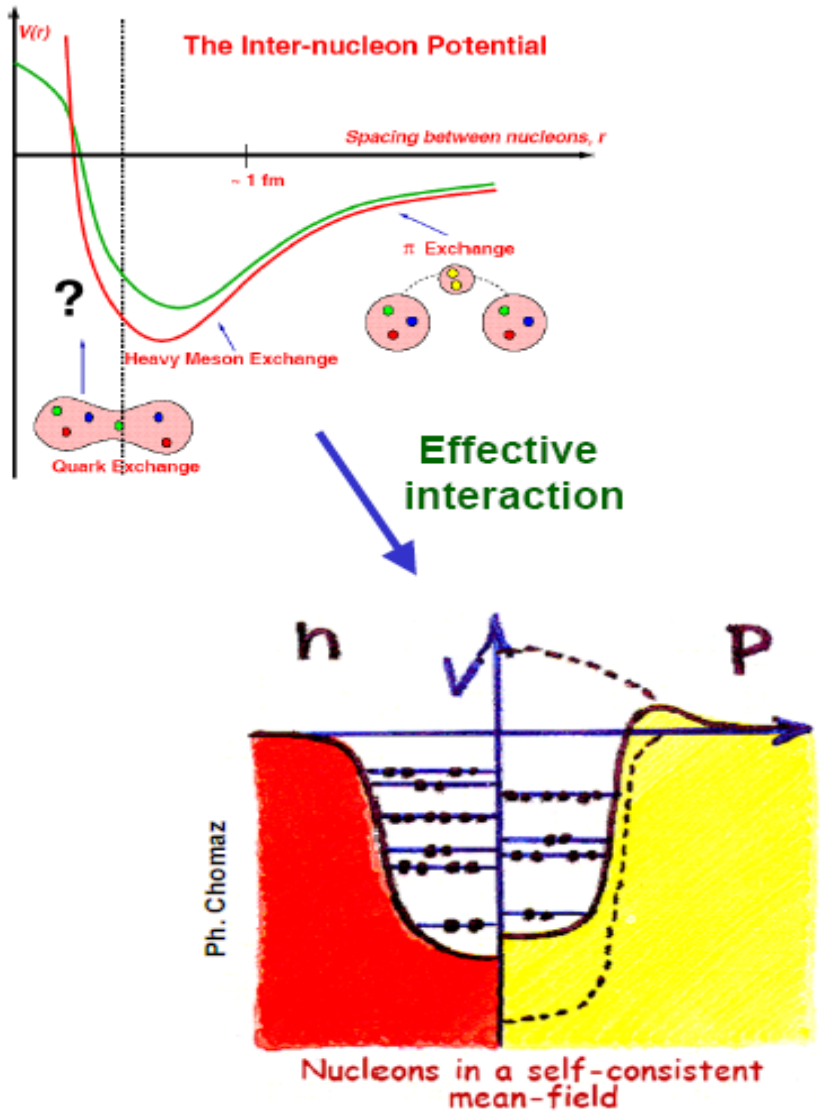
R's proportional to  $W^{\mu\nu}$ :

$$W^{\mu\nu} = \frac{1}{2j_b + 1} \sum_{\mu_b} J^{\mu*}(\omega, \mathbf{q}) J^\nu(\omega, \mathbf{q}) .$$

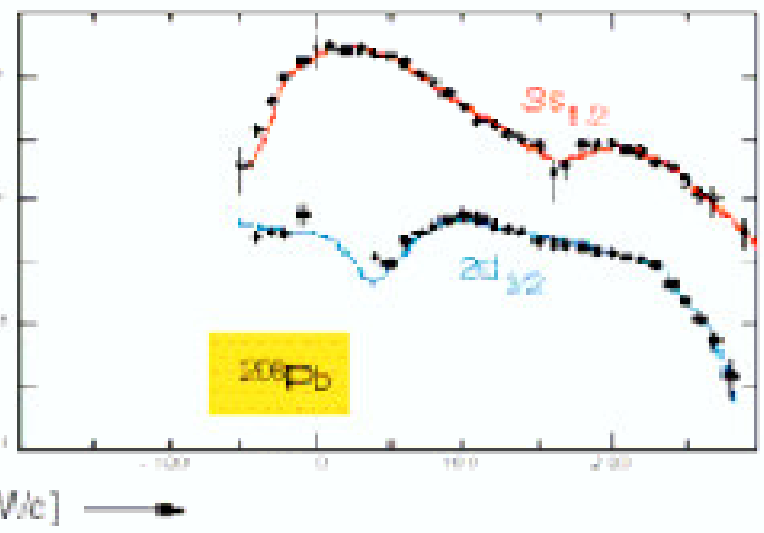
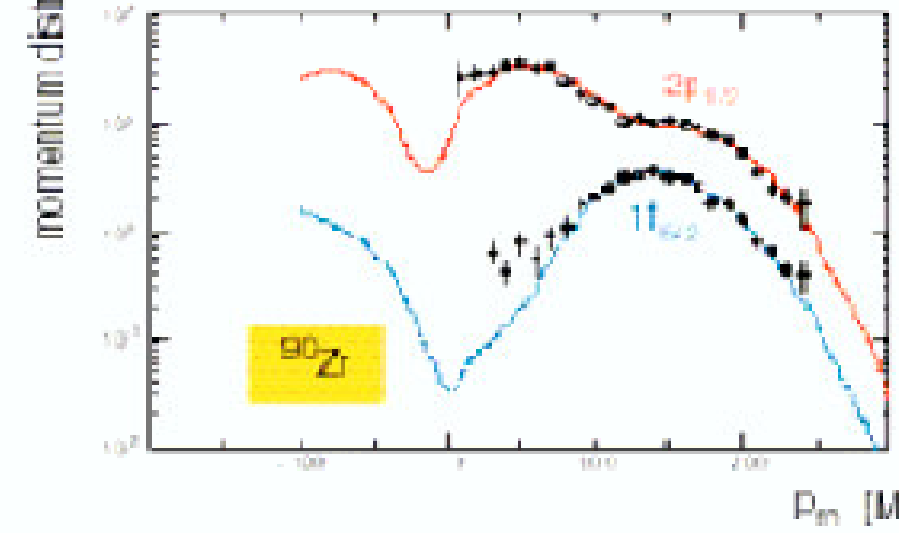
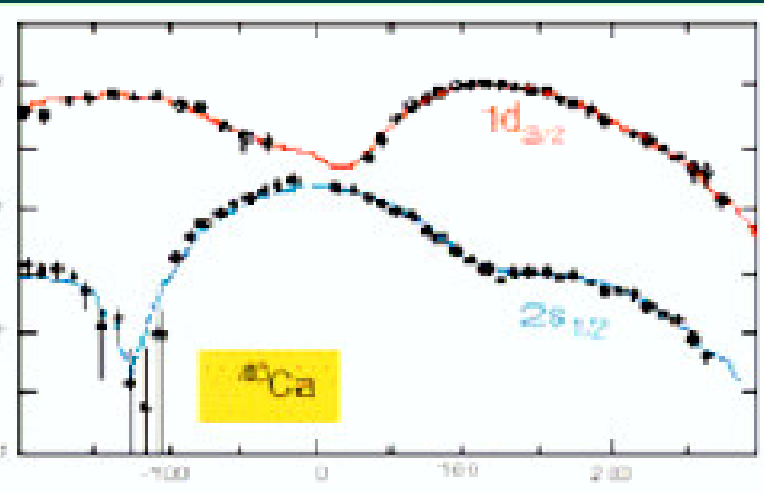
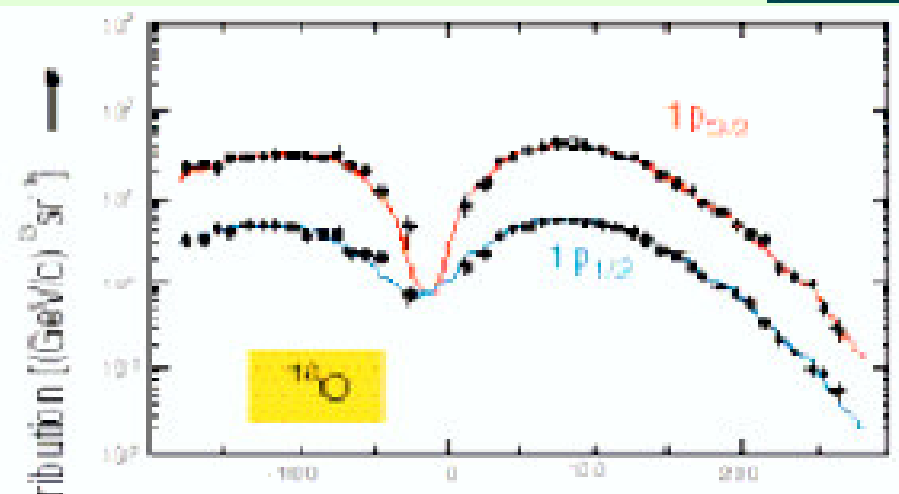
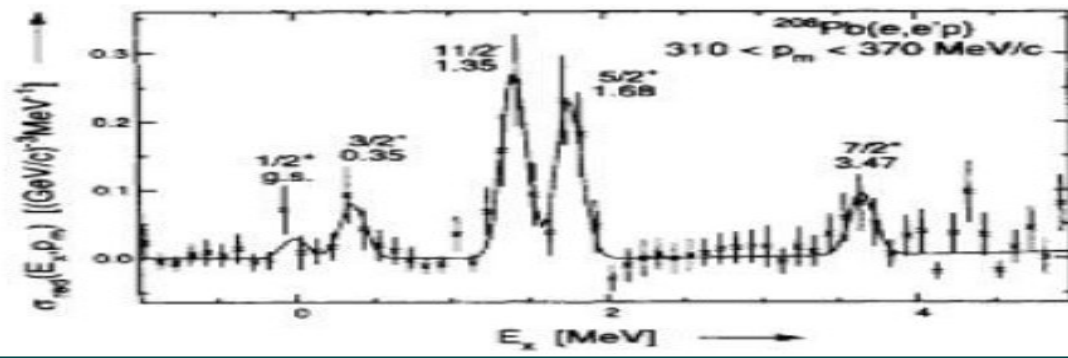
## Testing RDWIA with $A(e,e'p)A-1$ REACTIONS

- Look at exclusive  $(e,e'p)$  reactions at the top of the quasielastic peak ( $x=1$ )
- Best place to justify the use of Impulse Approximation and Mean Field models plus one body operators
- EM interaction is well known

# Mean Field Model of Nuclei



- fermion system at low energies
- suppression of collisions by Pauli exclusion
- independent particle motion
- shell structure
- mean field approximation



$p_M$  [MeV/c]  $\longrightarrow$



## Physics Motivation

Deviations from independent particle motion for orbits near the Fermi surface are attributed to effects beyond mean field (correlations) which reveal their presence in two ways:

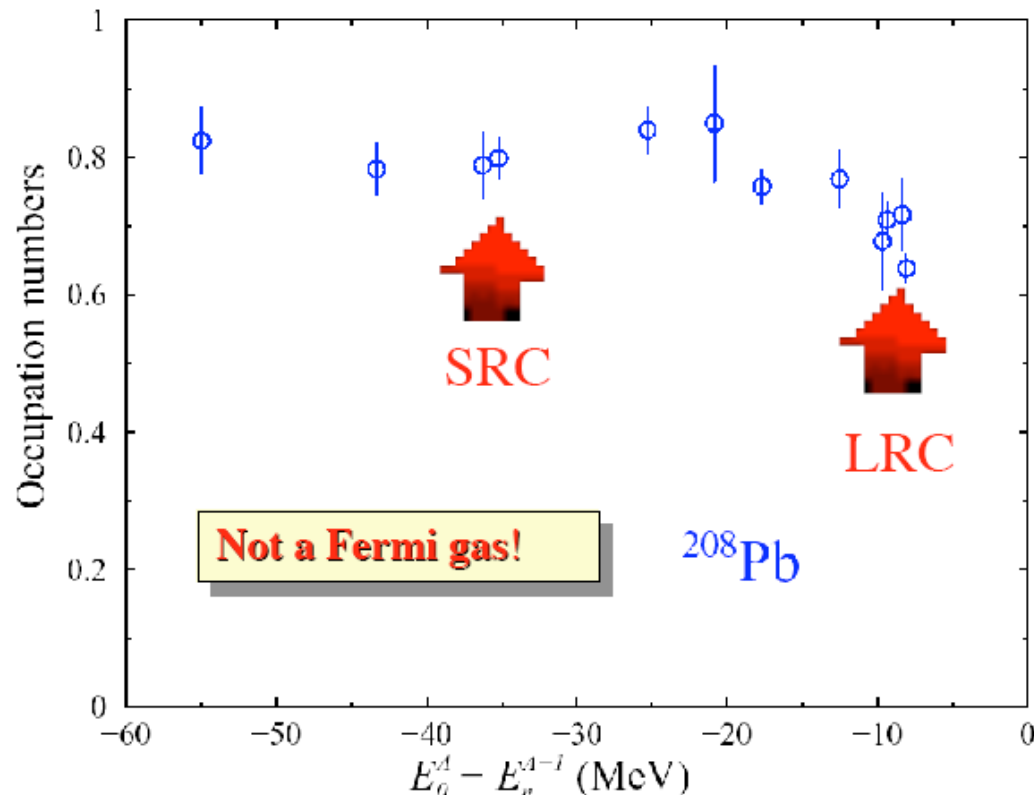
- (i) *Changes in the occupation and spectroscopic factors with respect to mean-field predictions*
- (ii) *Changes in the momentum distribution of particles, particularly at high momentum and binding energies*

**V.R. Pandharipande, I. Sick and P.K.A. deWitt Huberts**, Independent particle motion and correlations in fermions systems, **Reviews of Modern Physics** **69** (981) **1997**

## Physics Motivation

- *The  $(e, e'p)$  reaction at quasielastic kinematics and under exclusive conditions, for the outermost shells, becomes one of the most powerful and cleanest test of the mean field and the correlations needed to supplement it*
- *$^{208}\text{Pb}$  is the most suitable candidate to employ the mean field prediction, and thus it has been measured in the past in order to determine spectroscopic factors, mainly in parallel kinematics and for moderate values of  $Q^2$  ( $Q^2 \ll 1 \text{ (GeV/c)}^2$ )*

- Shell model (mean field) calculations: the shape of the experimental cross-section is well described, but the measured spectroscopic factors are below the mean field prediction. How large/small must be the spectroscopic factors?



**About 30% depletion is observed for states near the Fermi level. This cannot be explained only with short-range correlations**

**Shape at moderate  $p_m$  and parallel kinematics is well understood**

**Long range correlations are predicted to be visible at large  $p_m$**

1 M. van Batenburg (thesis, 2001) & L. Lapikás from  $^{208}\text{Pb} (e, e'p) ^{207}\text{Tl}$



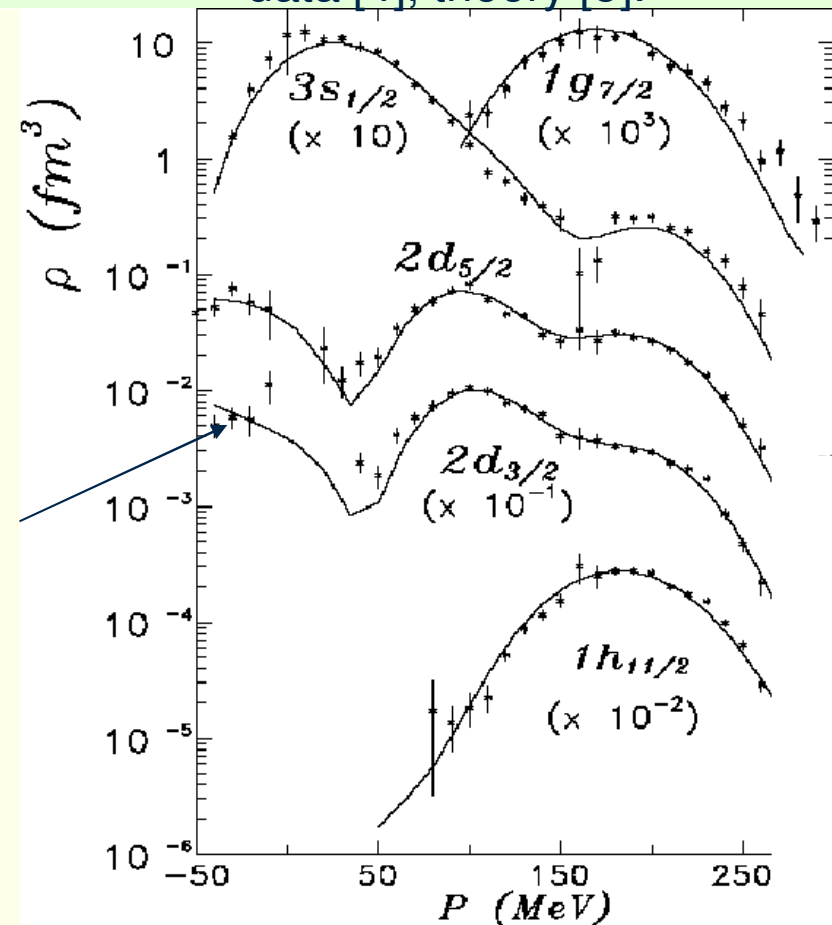
## Still open issues: (i) possible dependence on $Q^2$ of the spectroscopic factors?

### $^{12}\text{C}(e,e'p)$ data over wide range of $Q^2$ .

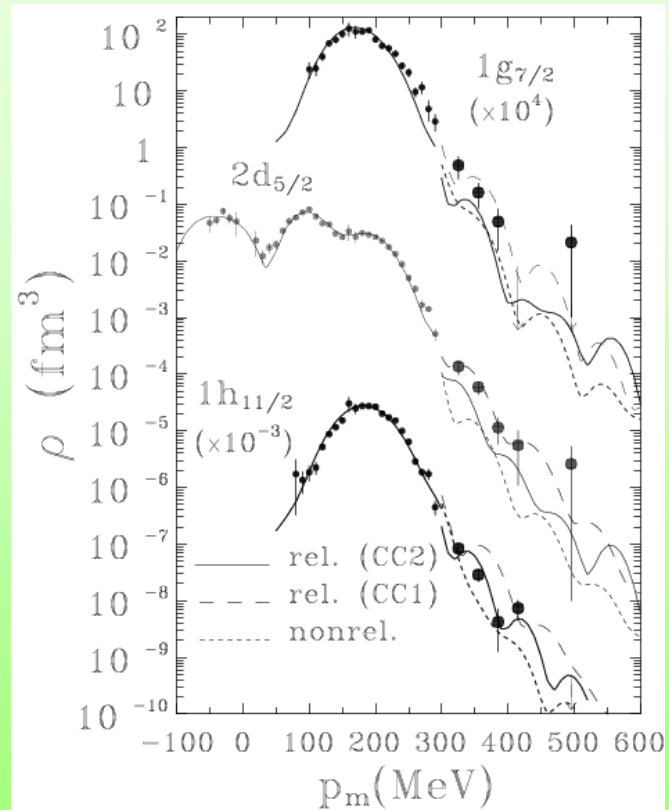
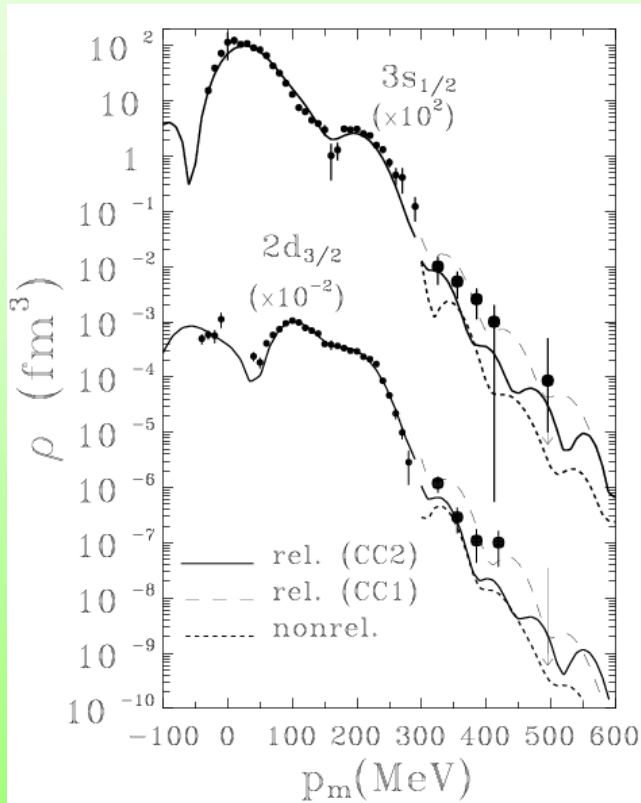
There appears to be a  $Q^2$  dependence to the spectroscopic factors observed in this reaction [L. Lapikas et al. PRC 61, 064325 (2000)]. This interpretation has been disputed and the  $Q^2$  dependence attributed to the way of introducing SRC [H. Müther and I. Sick, PRC70 041301R].  $^{208}\text{Pb}(e,e'p)$  has been studied in the past at low momentum transfers and spectroscopic factors for the valence shells in the range of 0.6 to 0.7 have been reliably extracted at parallel kinematics at low  $Q^2$ .

***A measurement at several high values of  $Q^2$  will directly address the question of momentum transfer dependence of the spectroscopic factors***

data [4], theory [5].



# Open issues: (ii) Long range correlations and cross-sections at high $p_m$ ( $> 300$ MeV/c)



**$X_B = 0.18$**

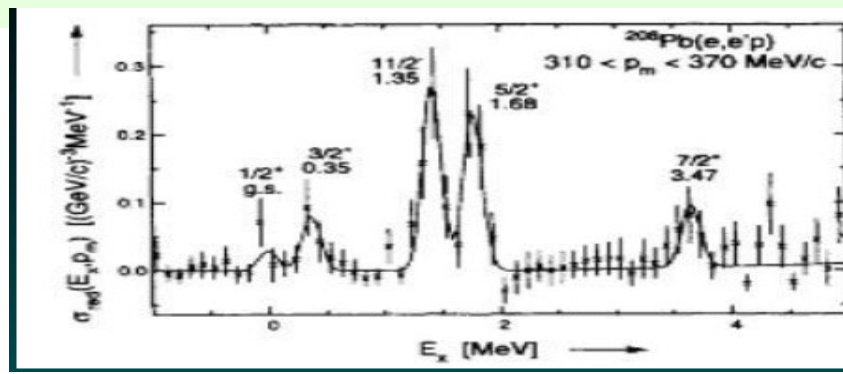
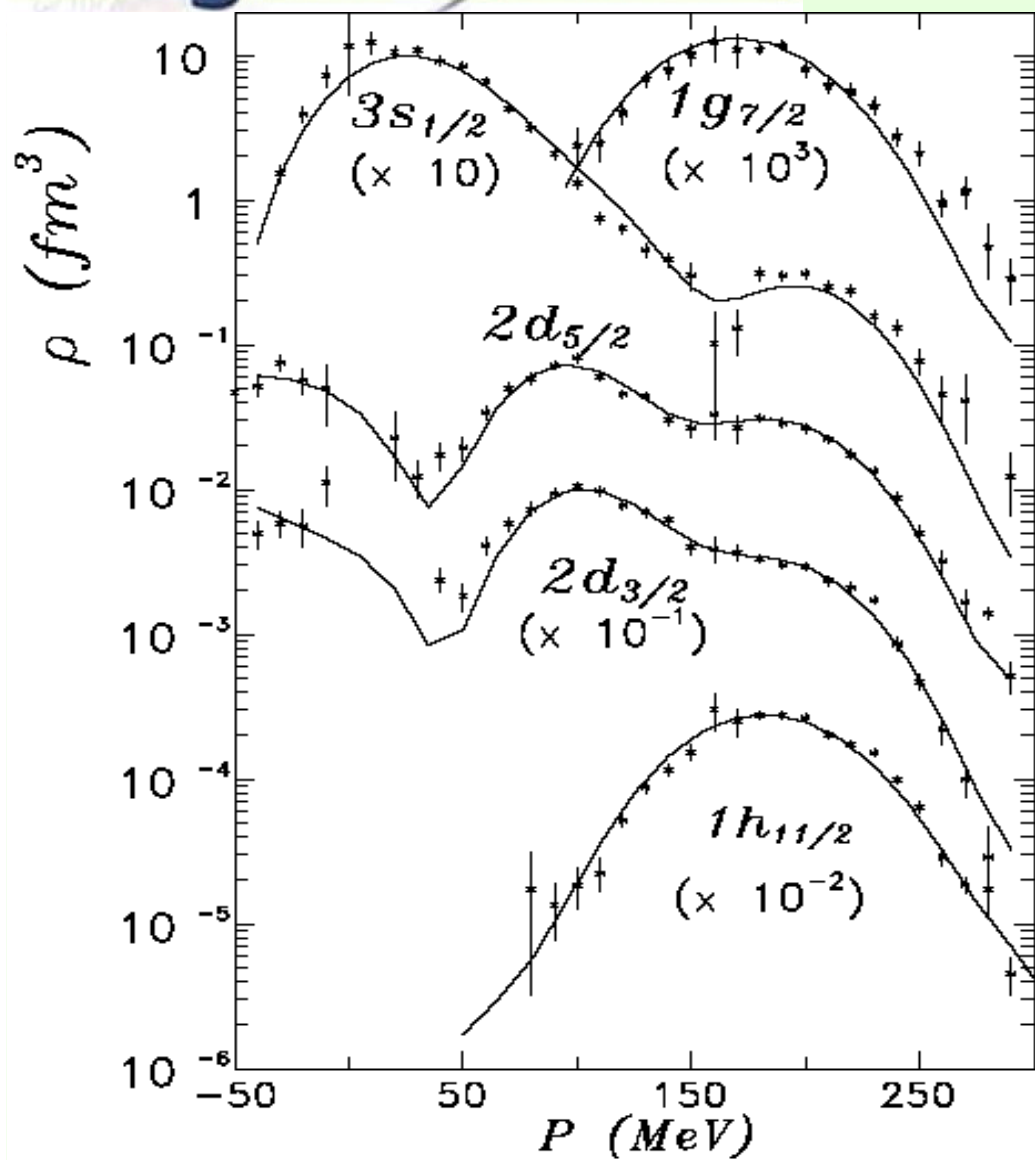
E. Quint, thesis, 1988, NIKHEF  
I. Bobeldijk et al., PRL 73 (2684)1994

Rel. Theory: PRC 48 (2731) 1994, PRC 51 (3246) 1996

J.M. Udias et al.

If long range correlations are the reason for the small spectroscopic factors, then they may produce some visible effect at high missing momentum. An experiment was performed at NIKHEF-K to measure the large momentum region, but the kinematics was far from  $X_B=1$ . Additional strength was indeed found, but this can be explained either via long-range correlations [I. Bobeldijk] or by relativistic effects in the mean field model.

# Reasonably good agreement with data in parallel kinematics

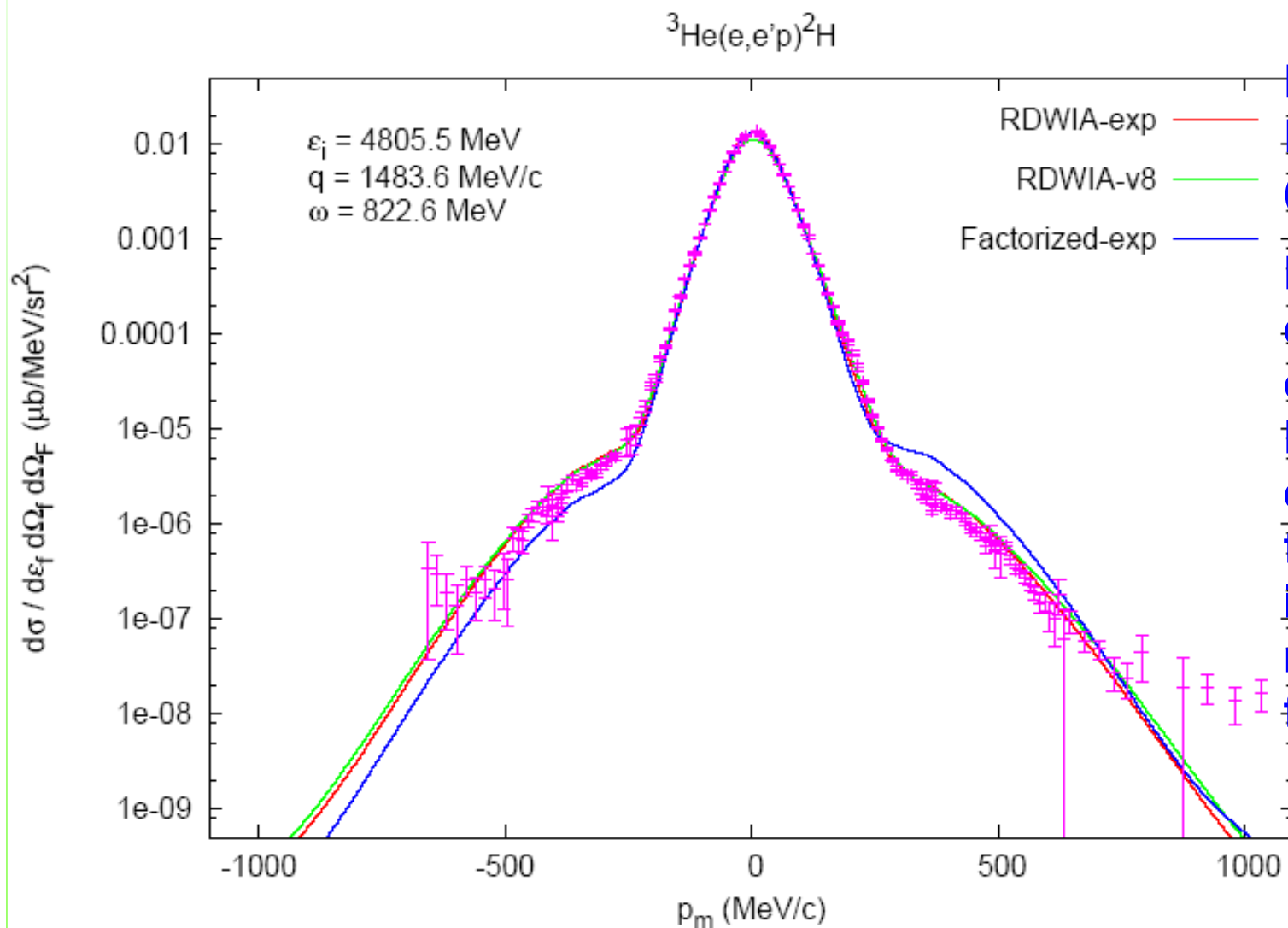


	$3s_{1/2}$	$2d_{3/2}$	$1h_{11/2}$	$2d_{5/2}$	$1g_{7/2}$
Non rel. (Ref. [41])	50 %	53 %	42 %	44 %	19 %
Non rel. (Ref. [42])	55 %	57 %	58 %	54 %	26 %
Rel. (Refs. [40, 6])	70 %	72 %	64 %	60 %	30 %

Relativistic analyses provide larger scale factors, due to 'Darwin term' (PRC 51 (1995) 3246)

[41] E. Quint et al. (1988).  
 [42] I. Bobeldijk et al. PRL 73 (1994) 2684.

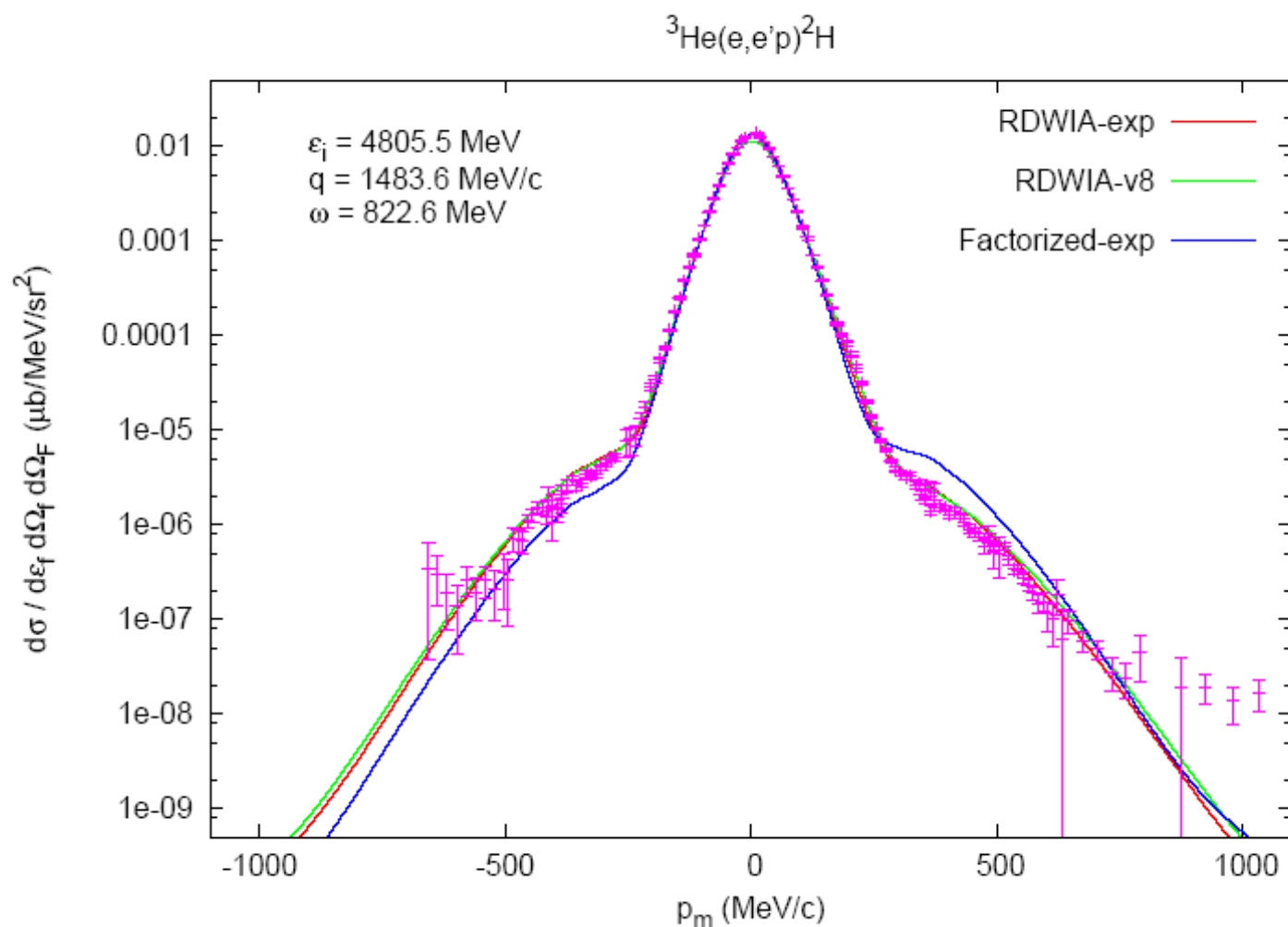
Breakdown of factorization will be seen at demanding kinematics (q- $\omega$  constant, high momentum)



**Data: M.M. Ravchev, PRL 94 (2005) 192302**

**Full theoretical calculation of the overlap from Faddeev calculations. No free parameters in this results, not even the sp. factors**

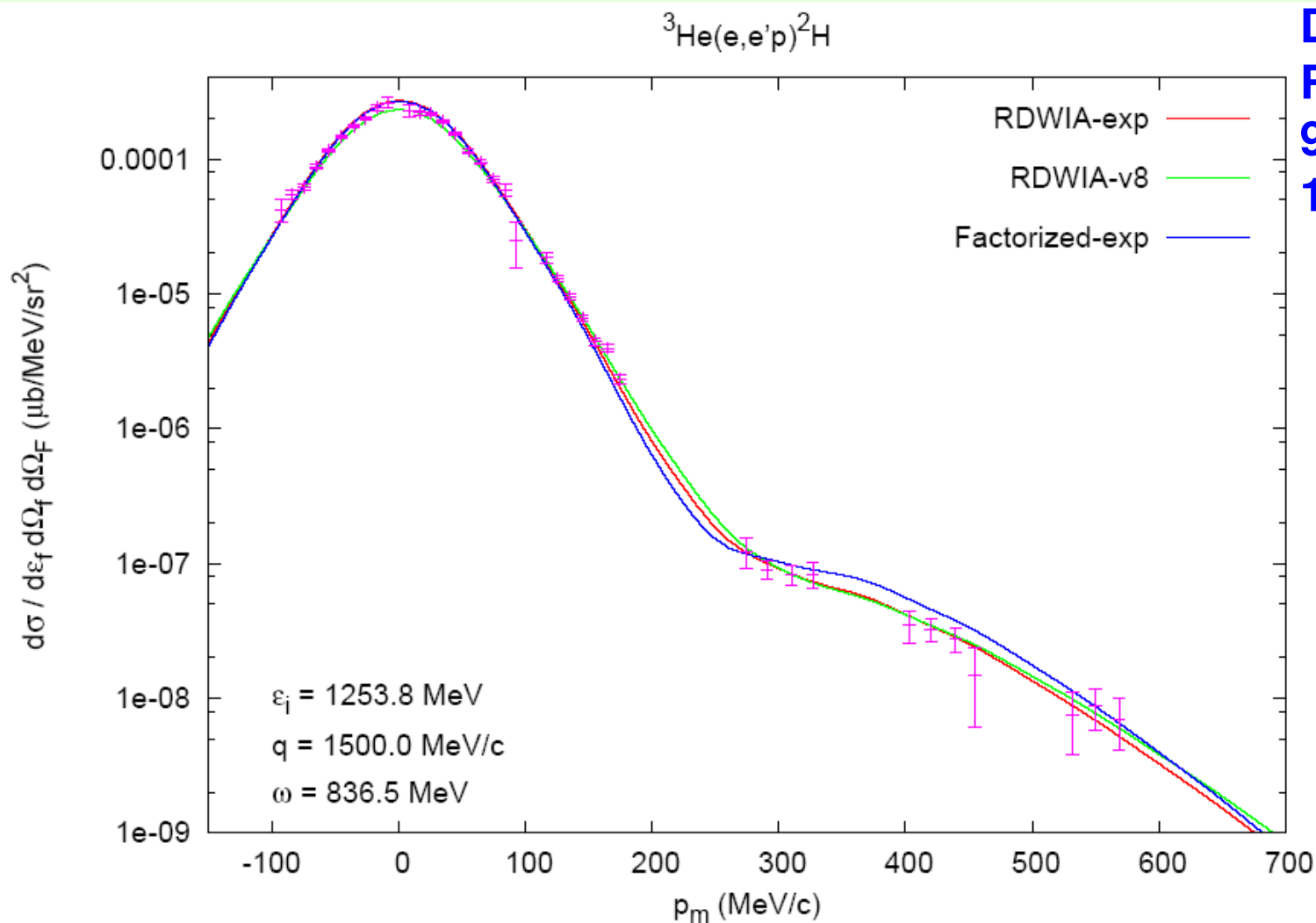
Breakdown of factorization will be seen at demanding kinematics ( $q$ - $\omega$  constant, high momentum)



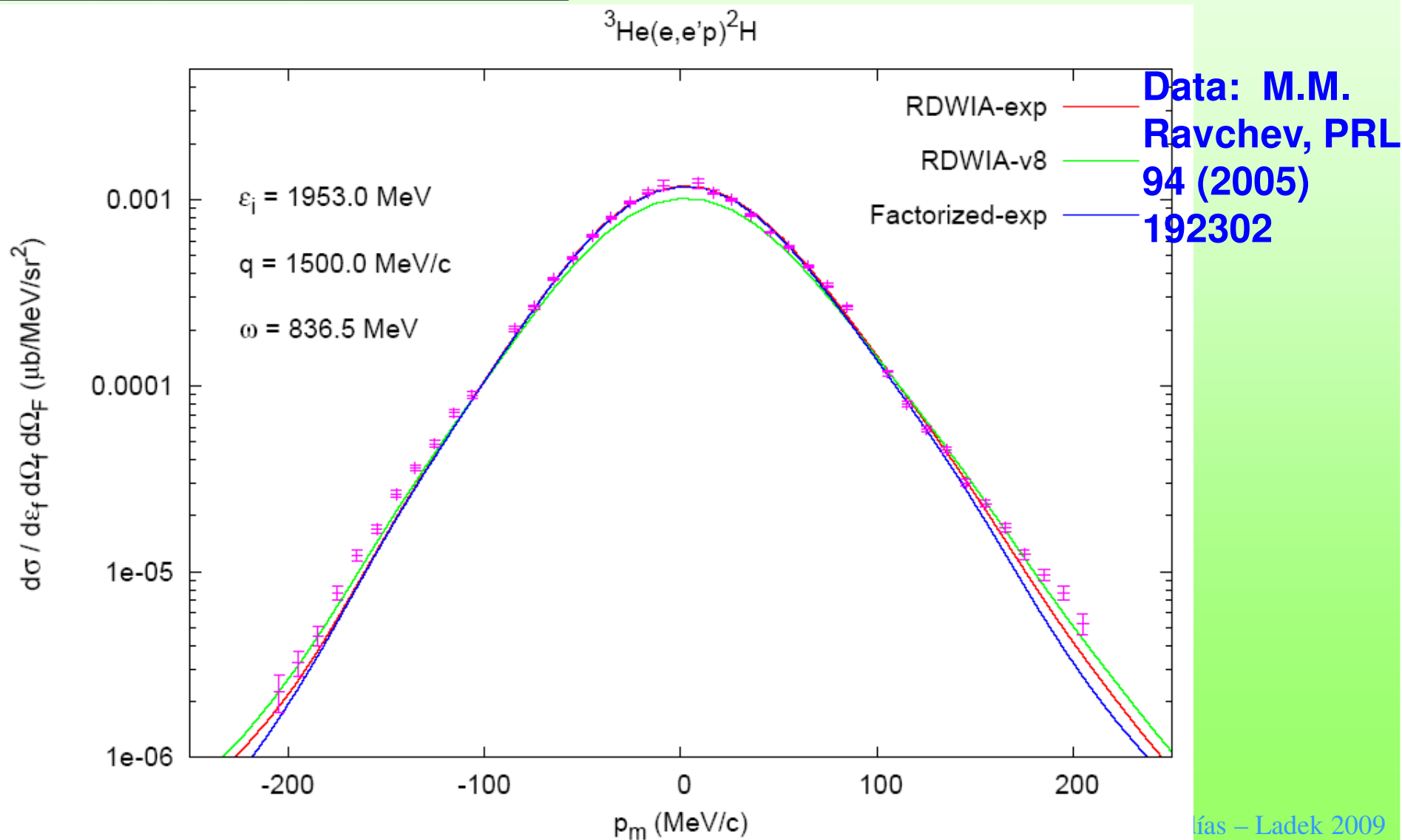
**Data: M.M. Ravchev, PRL 94 (2005) 192302**

Breakdown of factorization will be moderately seen at moderately demanding kinematics (moderately high  $p_m$ )

Data: M.M. Ravchev, PRL 94 (2005) 192302



Breakdown of factorization will hardly be seen at very moderate kinematics (modest  $p_m$ )



# Transparency from (e,e'p)

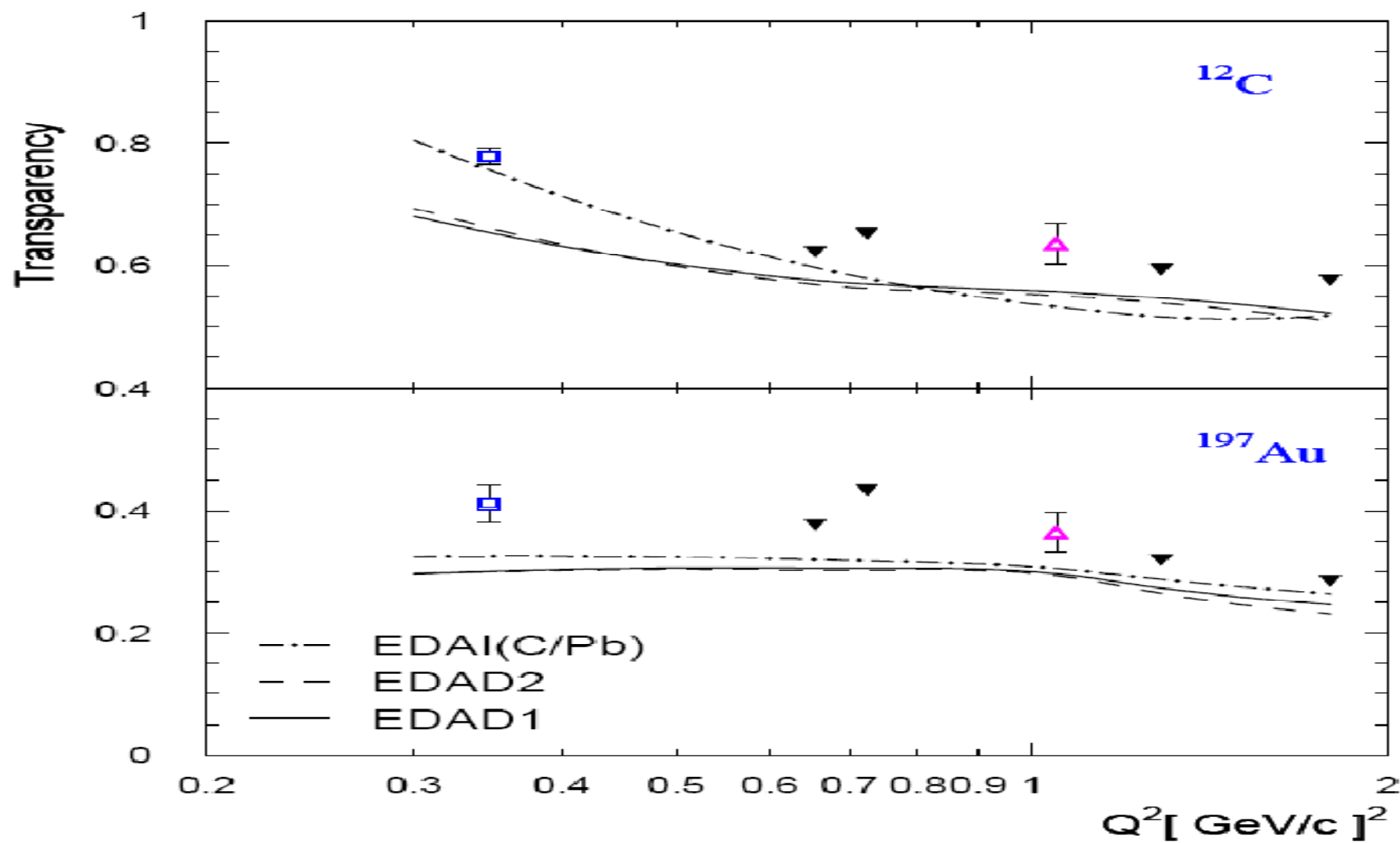
$$T_{exp}(Q^2) = \frac{\int_{\Delta^3 p_m} d\vec{p}_m \int_{\Delta E_m} dE_m S_{exp}(\vec{p}_m, E_m, \vec{p}_F)}{c_A \int_{\Delta^3 p_m} d\vec{p}_m \int_{\Delta E_m} dE_m S_{PWIA}(\vec{p}_m, E_m)}$$

$$S_{exp}(\vec{p}_m, E_m, \vec{p}_F) = \frac{\frac{d^5 \sigma^{exp}}{d\Omega_p d\epsilon' d\Omega_{\epsilon'}}(e, e'p)}{K \sigma_{ep}}$$

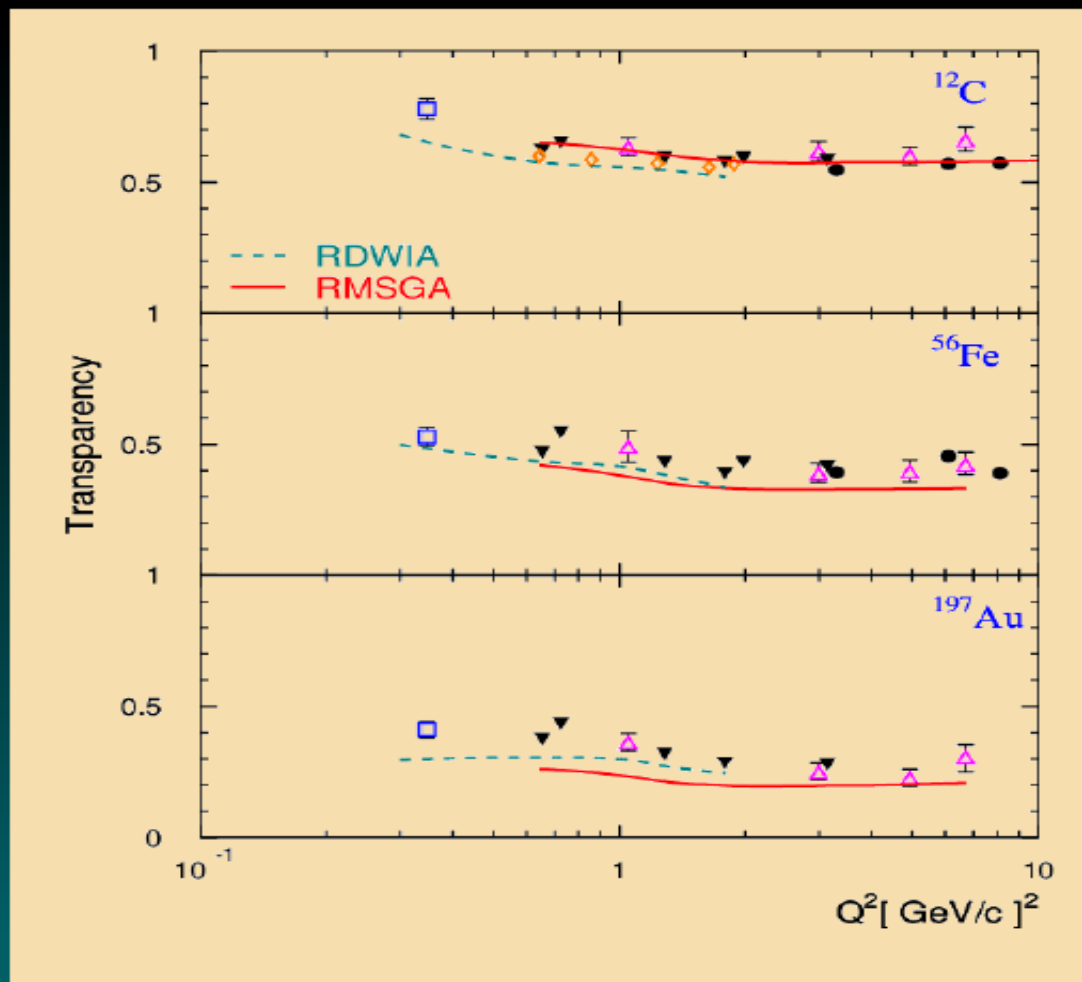
$$T_{theo}(Q^2) = \frac{\sum_{\alpha} \int_{\Delta^3 p_m} d\vec{p}_m S^{\alpha}(\vec{p}_m, E_m, \vec{p}_F)}{c_A \sum_{\alpha} \int_{\Delta^3 p_m} d\vec{p}_m S_{PWIA}^{\alpha}(\vec{p}_m, E_m)}$$



# Comparison to data: too little transparency



## Our results for Nuclear Transparencies



- Good description for  $^{12}\text{C}$
- Slight understimation for heavier nuclei
- Good description of the  $Q^2$  dependence of data
- Better agreement between RMSGA and RDWIA for 'intermediate' nuclei

# Results for neutrino-nucleus cross sections

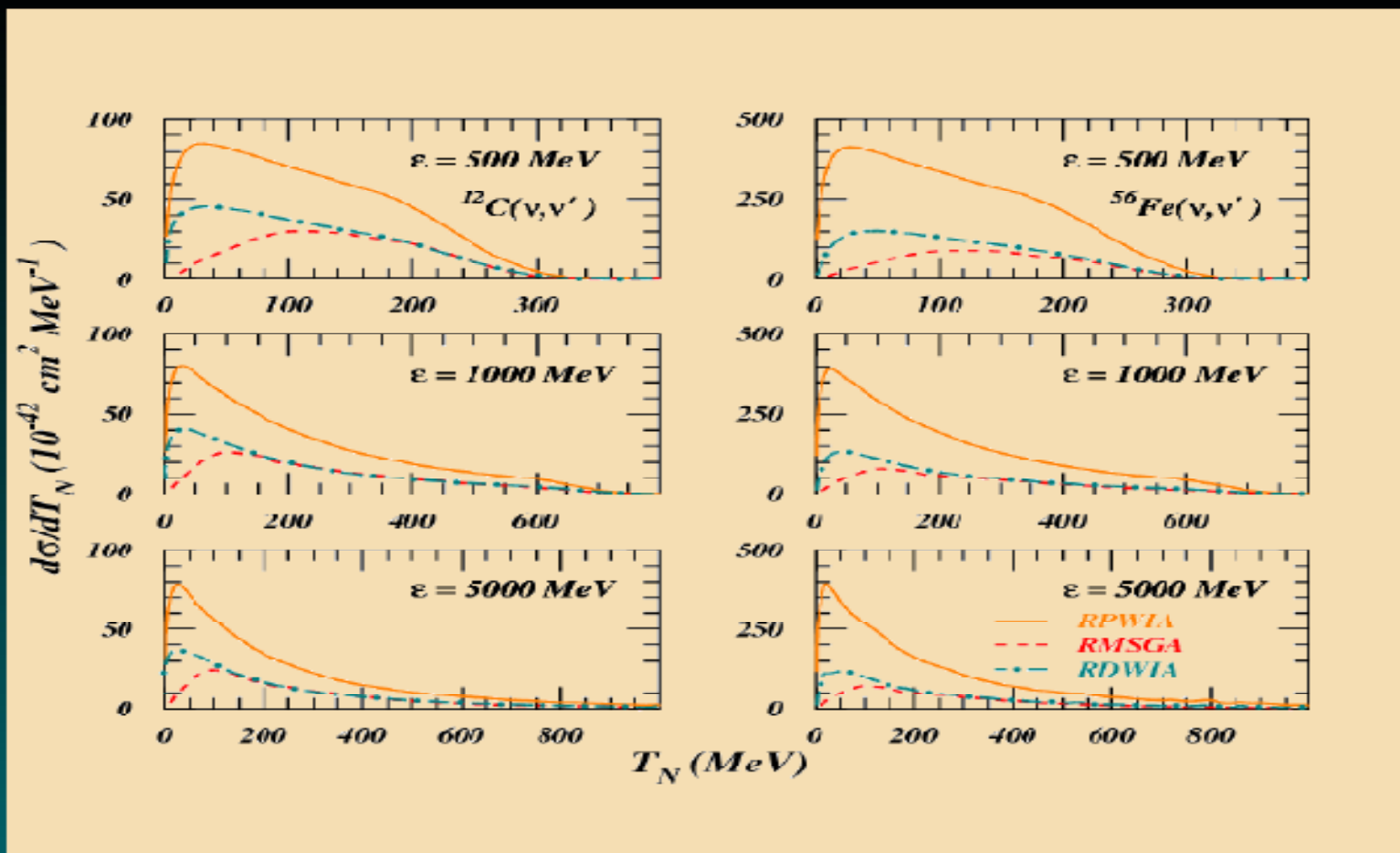
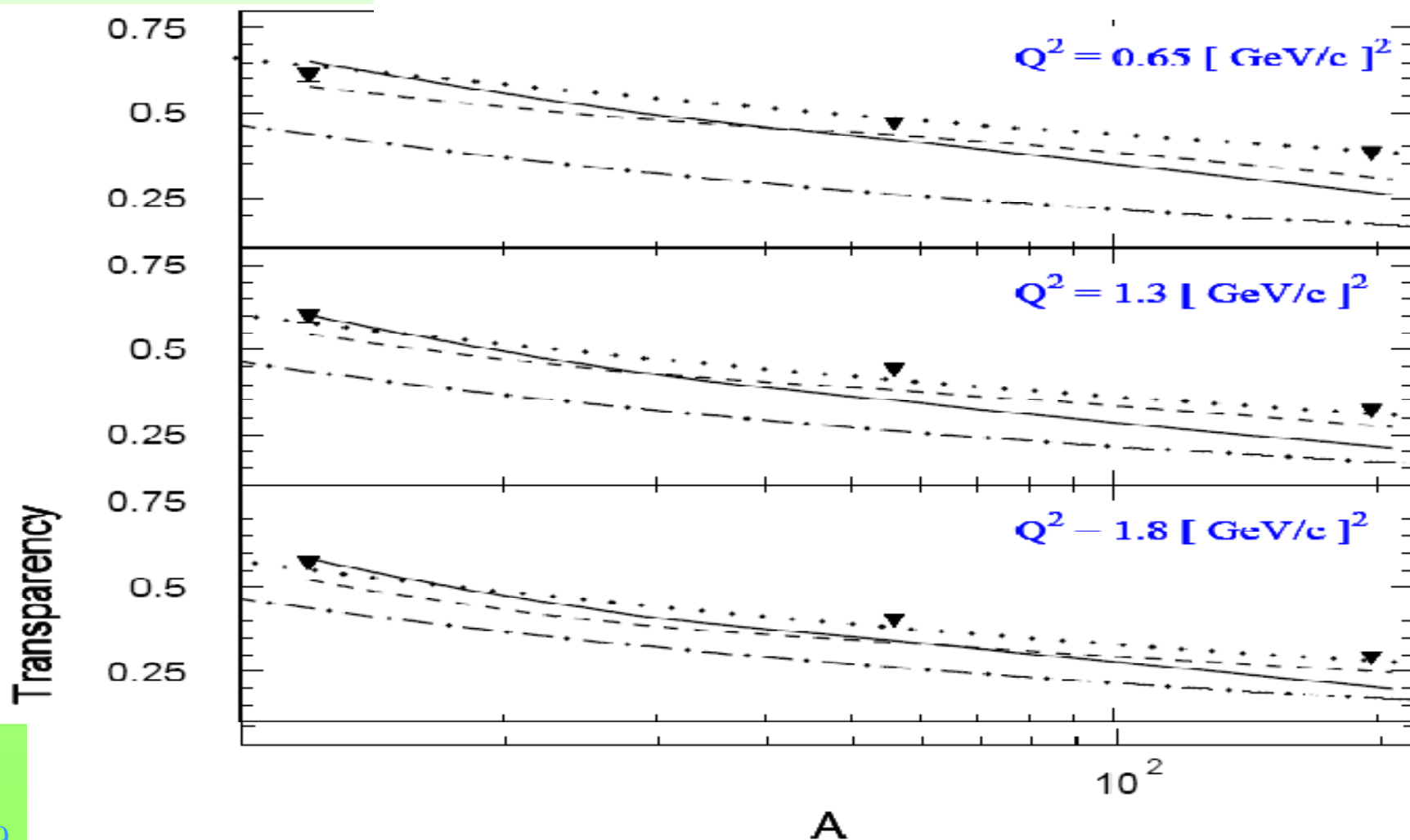




Fig. 4. The  $A$ -dependence of the nuclear transparency at five values of the four-momentum transfer  $Q^2$ . The solid (dashed) curves are RMSGA (RDWIA) calculations. The dotted curves represent the  $A^{-\alpha(Q^2)}$  parametrization, while the dot-dashed curve gives  $A^{-1/3}$ . Data are from [7,8](solid triangles) and [4,5](open triangles).



## What do we observe?

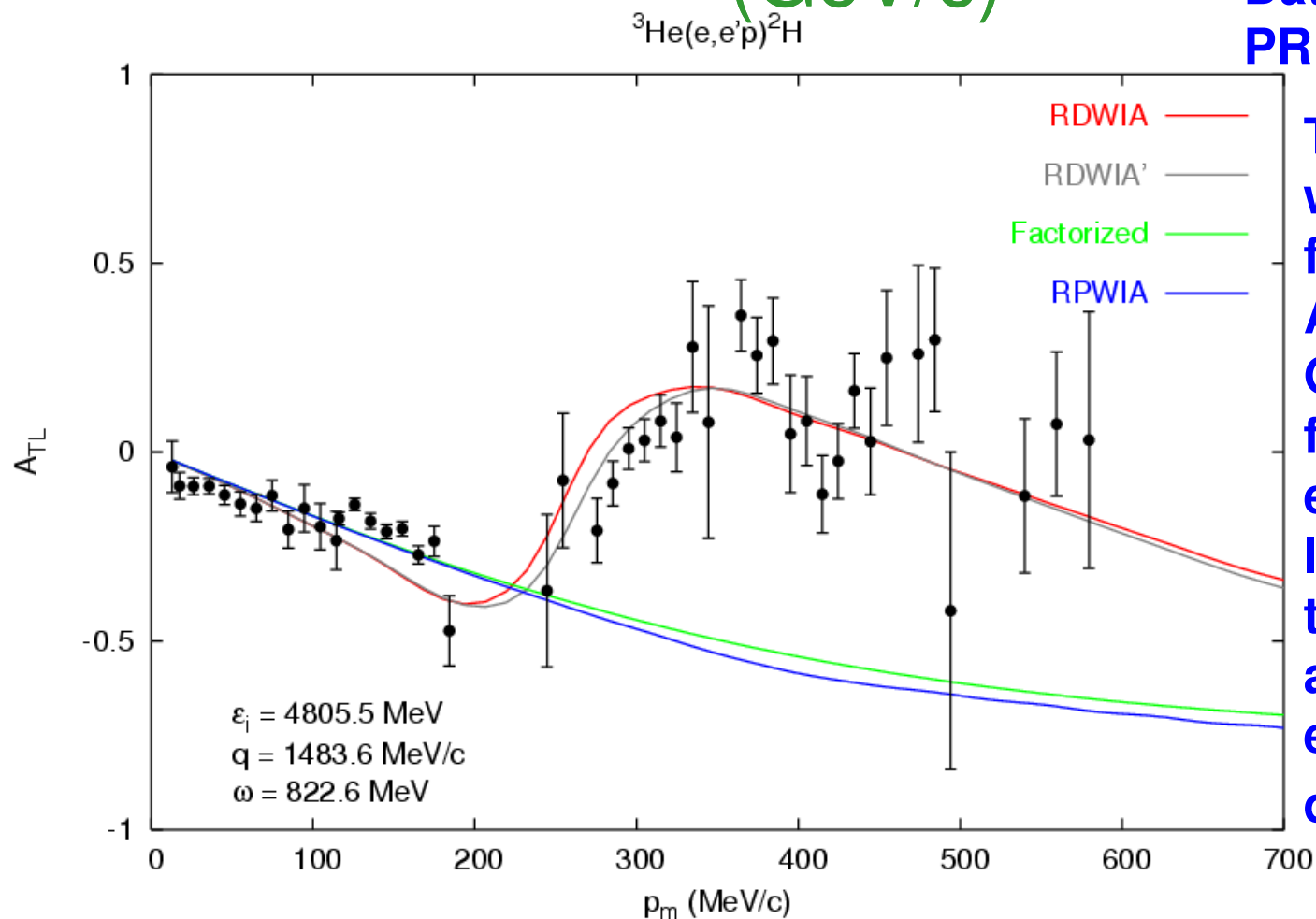
- Transparencies are underpredicted by phenomenological optical potentials
- Glauber estimations are about right ( $^{12}\text{C}$ ) or overpredict (for larger nuclei) the transparency
- The  $A$ -dependence is well described by the phenomenological optical potential

## What do we conclude?

- Transparency ( $e, e'p$ ) experiments, after all, are non-exclusive in a complicated and experiment-dependent way. They include more than just the elastic proton propagation channel
- Optical potential results ***represent just an absolute lower bound to the transparencies***
- The  $A$ -dependence is well described by the phenomenological optical potential
- Question: what can we use for non-exclusive experiments?

Breakdown of factorization will clearly be seen in  $A_{TL}$  at  $Q^2$  larger than, say,  $0.5$   $(\text{GeV}/c)^2$

Data: M.M. Ravchev, PRL 94 (2005) 192302

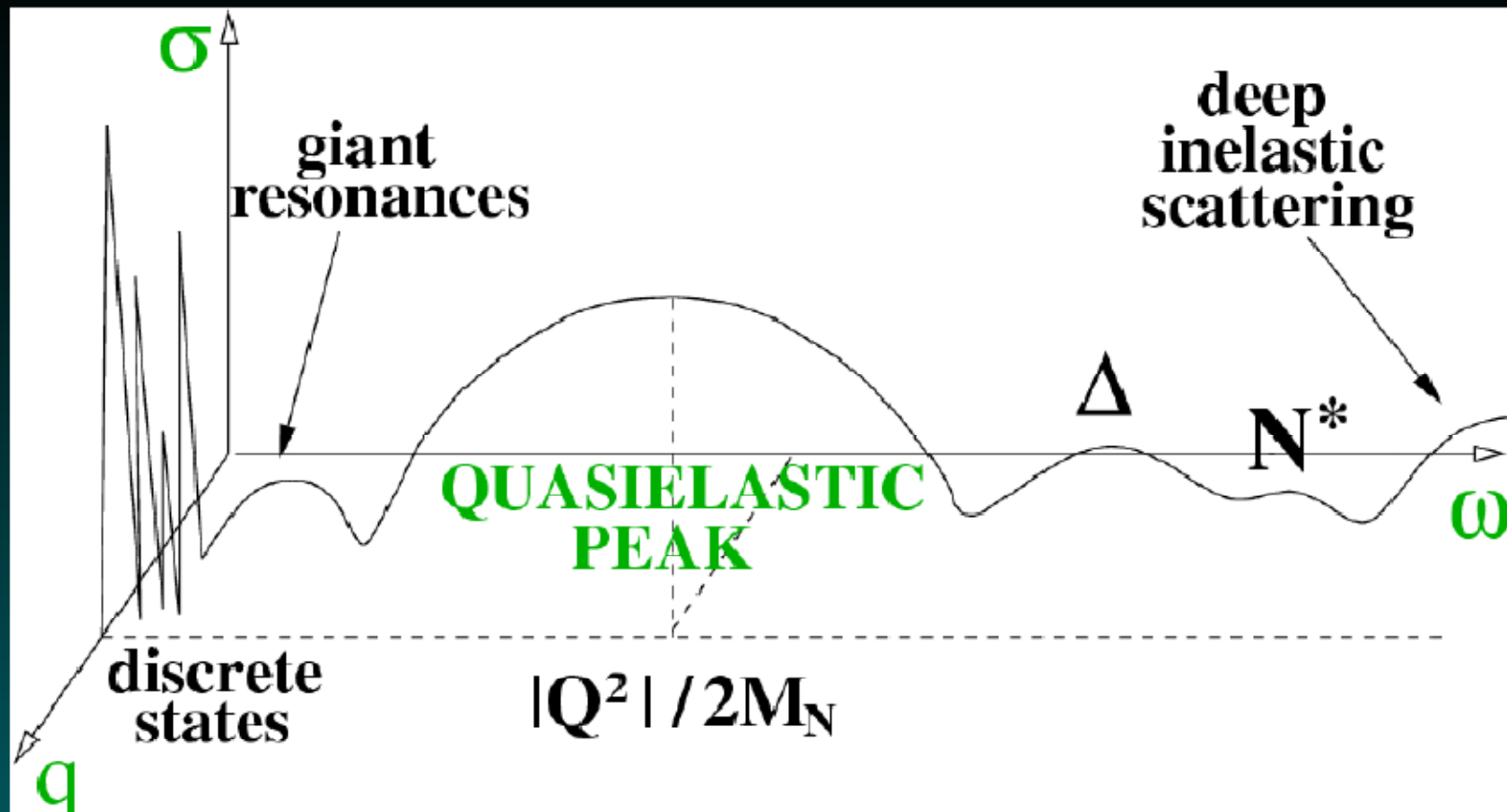


Theory: relativized wave function from Fadeev with AV8' interaction. Optical potential from RIA-IA1 with effective NN lagrangian fitted to  ${}^4\text{He}(p,p)$  data and using experimental density



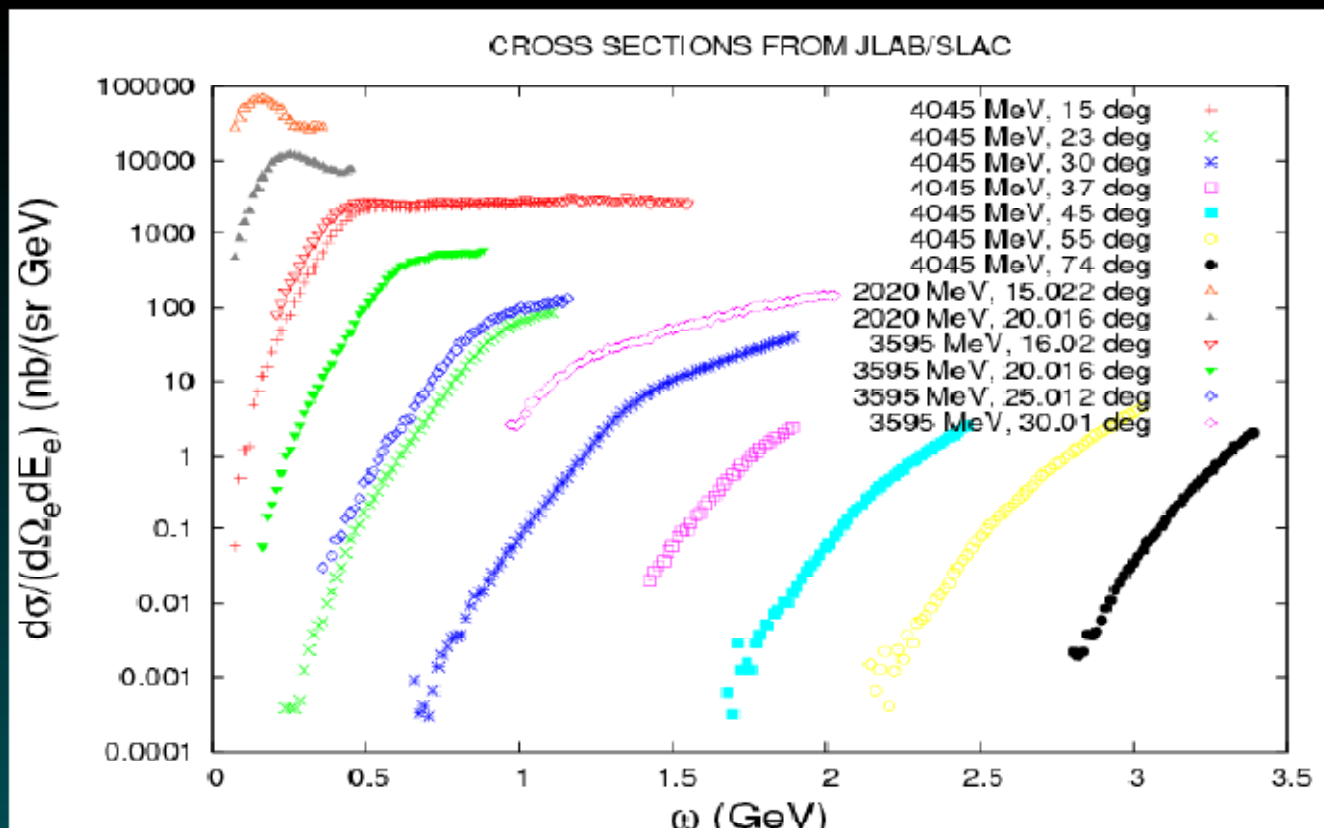
# Survey of neutrino-nucleus interactions

## Inclusive electron scattering on nuclei



Many things may happen to the nucleus, depending on the values of  $q$  and  $\omega$

## Inclusive $^{12}\text{C}$ quasielastic electron data



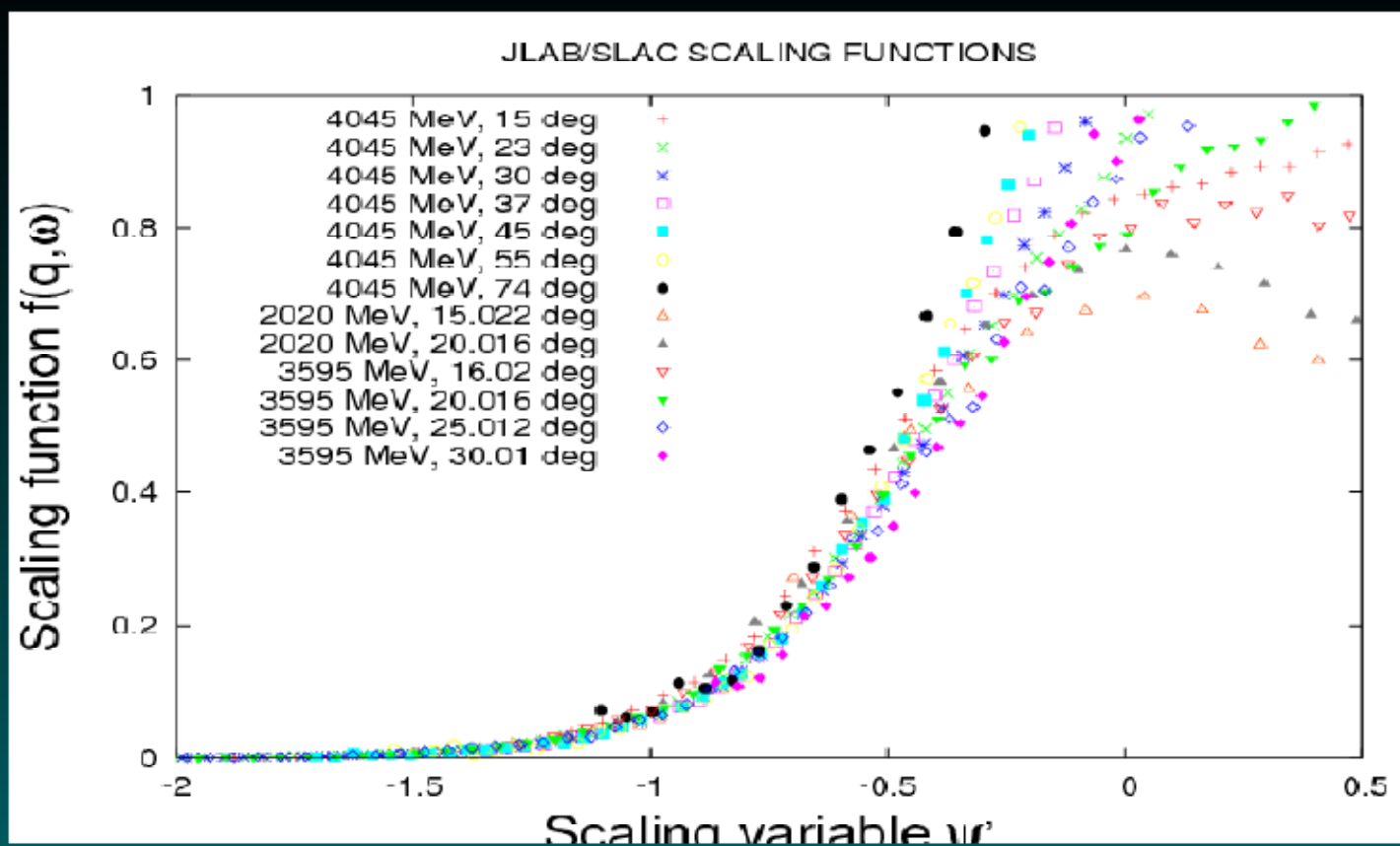
Day *et al*,  
**PRC48(1993)1849**

Arrington *et al*,  
**PRL82(1999)2056**

$0.5 < q < 4 \text{ GeV}/c$

$$f(q, \omega) \propto \frac{\left[ \frac{d\sigma}{d\Omega_e dE_e} \right]}{\bar{\sigma}_{\text{electron-nucleon}}}$$

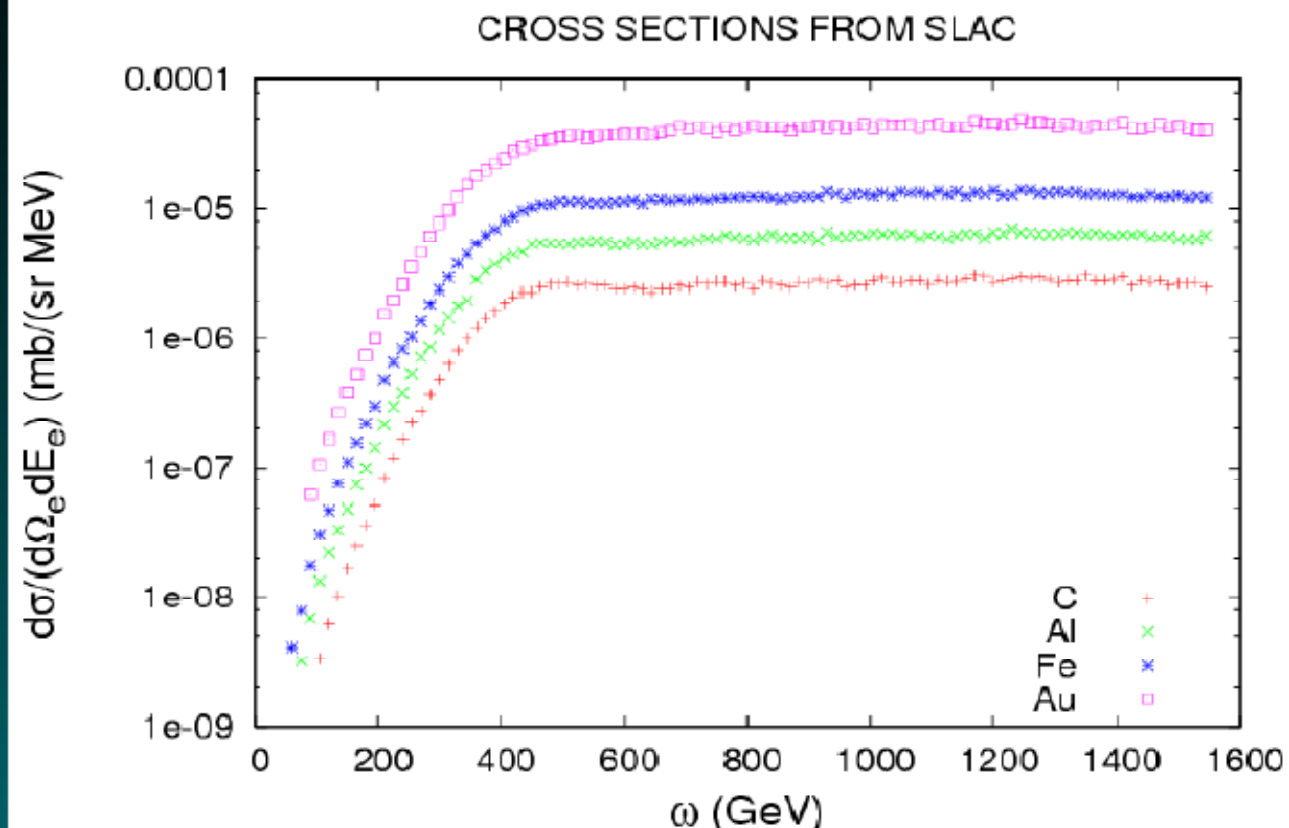
# Inclusive $^{12}\text{C}$ quasielastic electron data SCALING BEHAVIOR



Quite good scaling for negative scaling variable (y-scaling). Large violations for large energy transfers due to the transverse response

## More inclusive quasielastic electron data

Same transferred momenta,  
different targets (C, Al, Fe, Au)

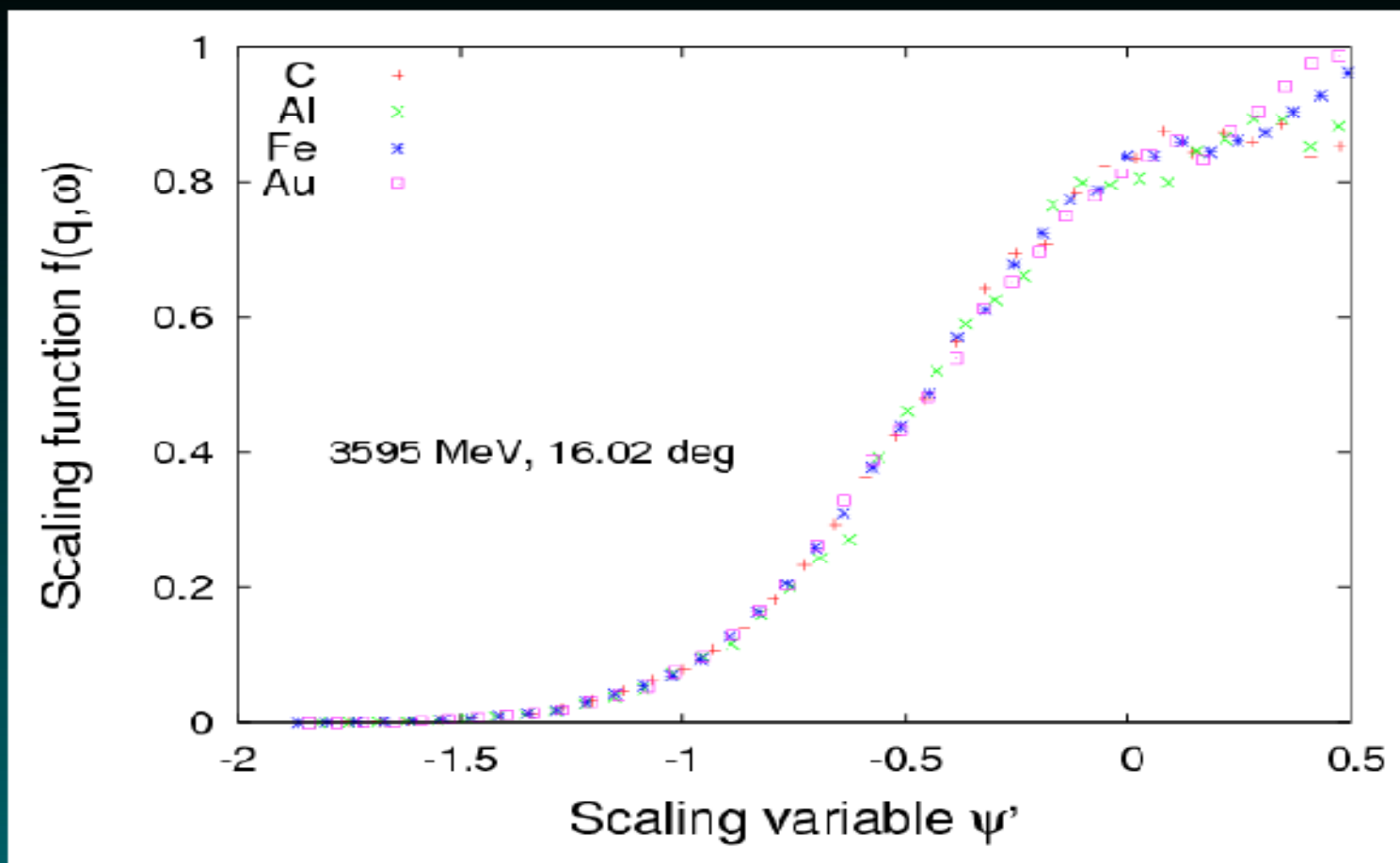


*Day et al,*  
**PRC48(1993)1849**

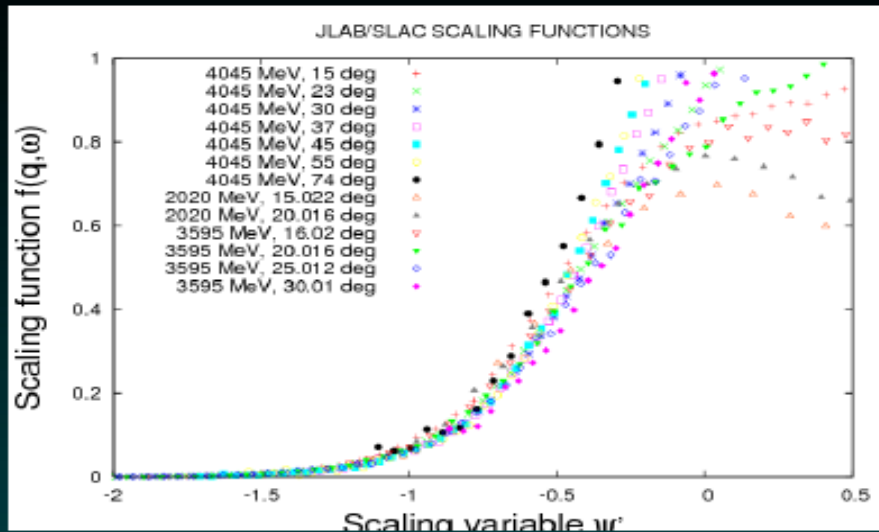
$q \approx 1 \text{ GeV}/c$   
 $E_e = 3,6 \text{ GeV},$   
 $\theta_e = 16^\circ$

## Inclusive quasielastic electron data at $q \approx 1$ GeV/c

### SCALING BEHAVIOR

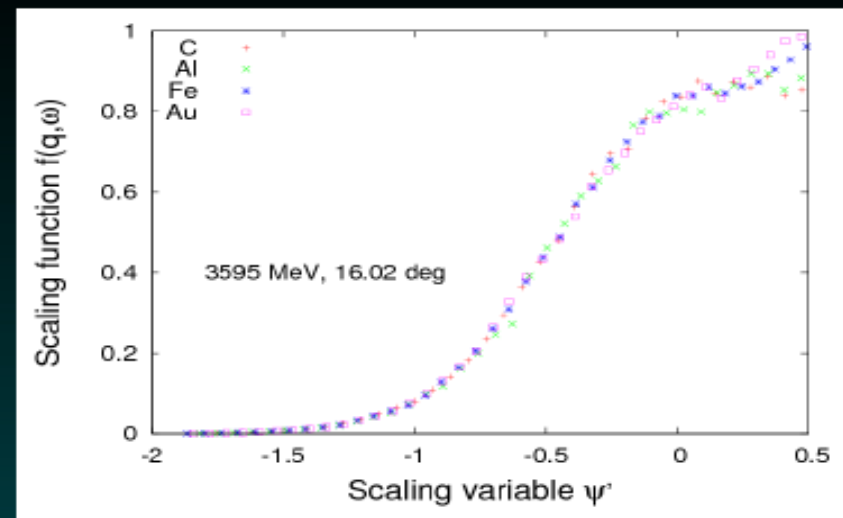


**Same target ( $^{12}\text{C}$ ),  
different transferred momenta**



**FIRST KIND SCALING**

**Same transferred momenta,  
different targets (C, Al, Fe, Au)**



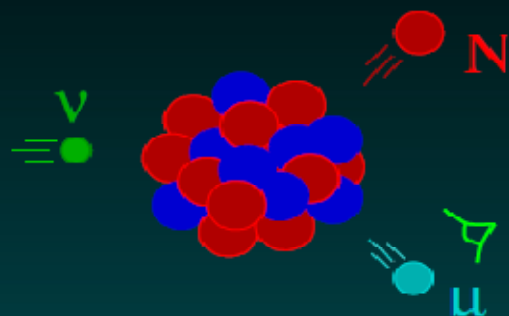
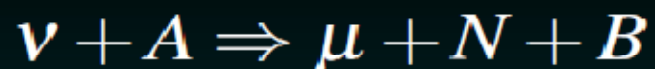
**SECOND KIND SCALING**

**FIRST (y-scaling) + SECOND = SUPERSCALING**

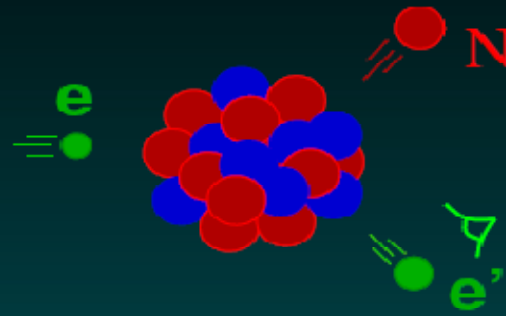
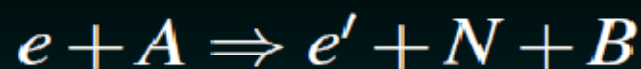
Day *et al*, Ann. Rev. Nucl. Part. Sci. **40** (1990) 357,  
Donnelly and Sick, Phys. Rev. C **60** (1999) 065502,  
Donnelly and Sick, Phys. Rev. Lett. **82** (1999) 3212

## Quasielastic ( $e, e'$ ) versus ( $\nu, \mu$ ) (Charged-Current neutrino reactions)

**Neutrinos**



**Electrons**



**Electron and neutrino INCLUSIVE scattering are very related, one should check models of neutrino scattering against the large amount of inclusive electron data...**

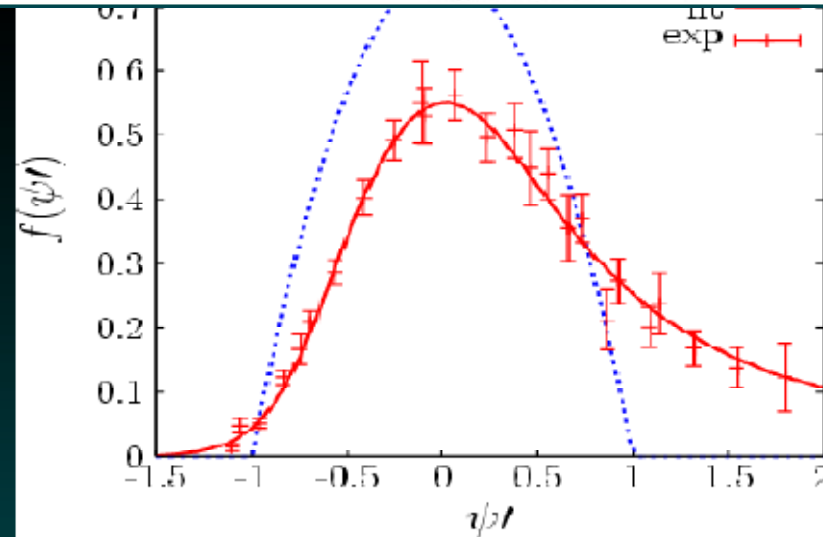
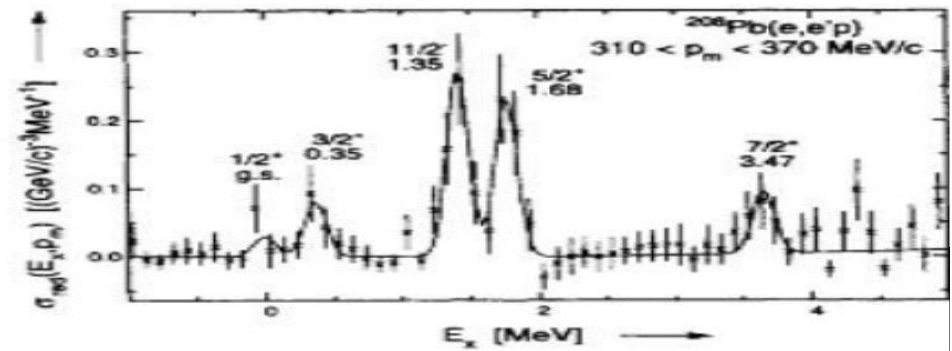
**How well compares the Relativistic Fermi Gas to QE inclusive electron data?**



# RFG is a lousy modelling of nuclear dynamics

**Relativistic Fermi Gas:**  
 Perfect superscaling behaviour  
 (Alberico *et al*,  
 Phys. Rev. C 38, 1801 (1988))

**Experimental data:**  
 Good superscaling behaviour,  
 although not perfect



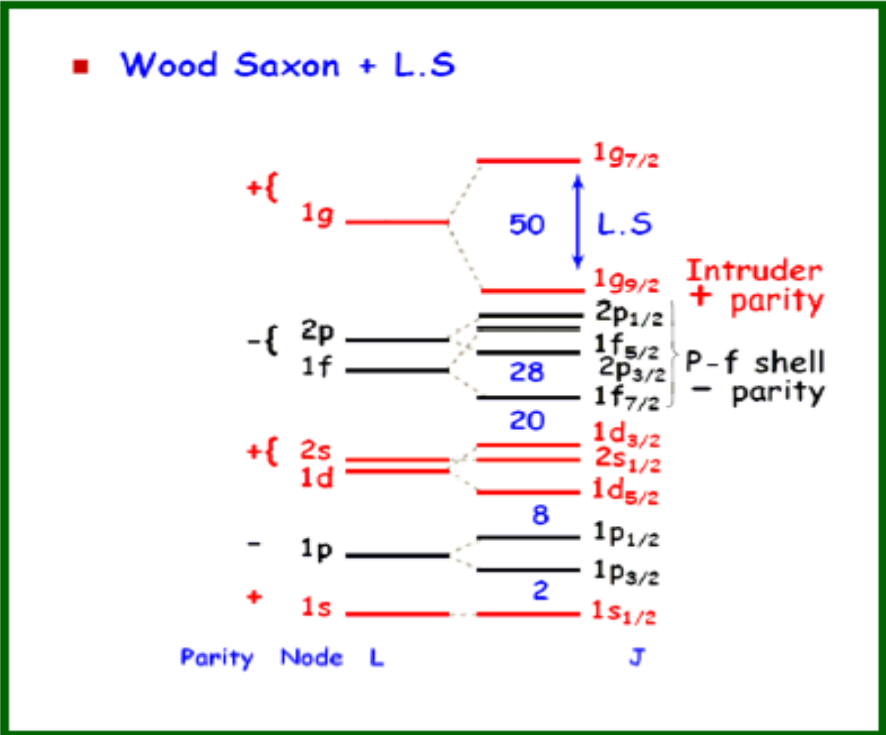
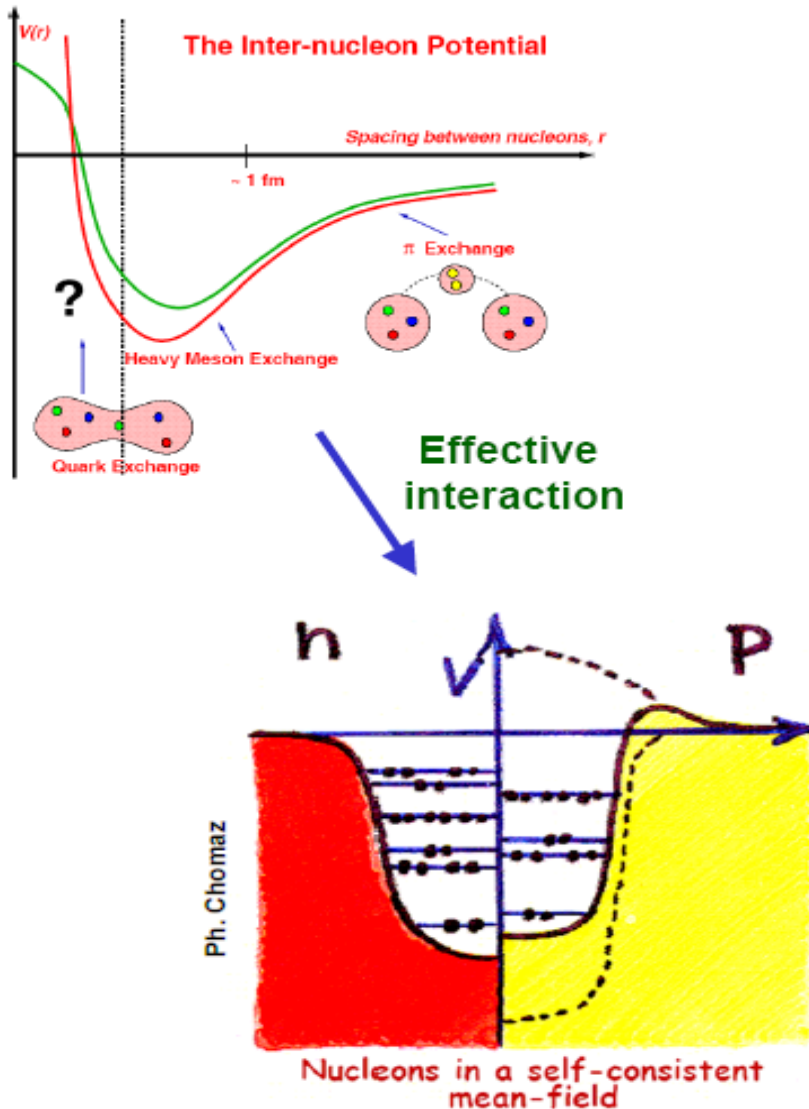
The RFG superscales, and data also superscale, but the scaling functions of data and of RFG disagree  $\Rightarrow$

**The RFG lacks important initial and final state nuclear dynamics effects, even at high energies!**

In Miniboone, the nucleon axial mass and Pauli blocking are modified to fit their event distribution, rather than using a better model...

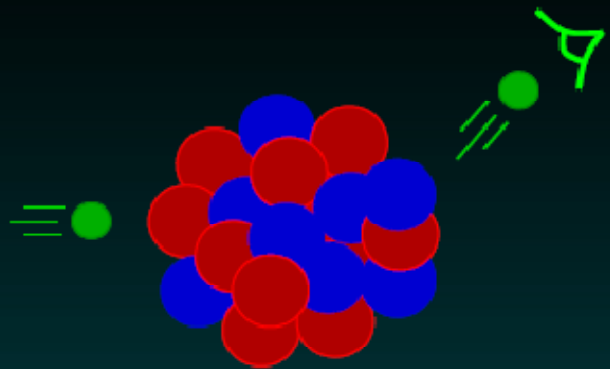


# Mean Field Model of Nuclei



- fermion system at low energies
- suppression of collisions by Pauli exclusion
- independent particle motion
- shell structure
- mean field approximation

## Some general ideas about scaling in inclusive scattering



- Requires a **weakly interacting probe** and a **composite target**
- The probe must scatter from **one of the bound constituents** of the target

$$F(q, \omega) = \frac{\left[ \frac{d\sigma}{d\Omega_{probe} dE_{probe}} \right]}{\overline{\sigma}_{probe-constituent}}$$

At high  $q$  this function depends on a combination of  $q$  and  $\omega \Rightarrow$  **SCALING**

**Instead of employing direct neutrino-nucleus modeling,  
why not USE SCALING TO PREDICT CHARGE-CHANGING  
NEUTRINO-NUCLEUS CROSS SECTIONS?**

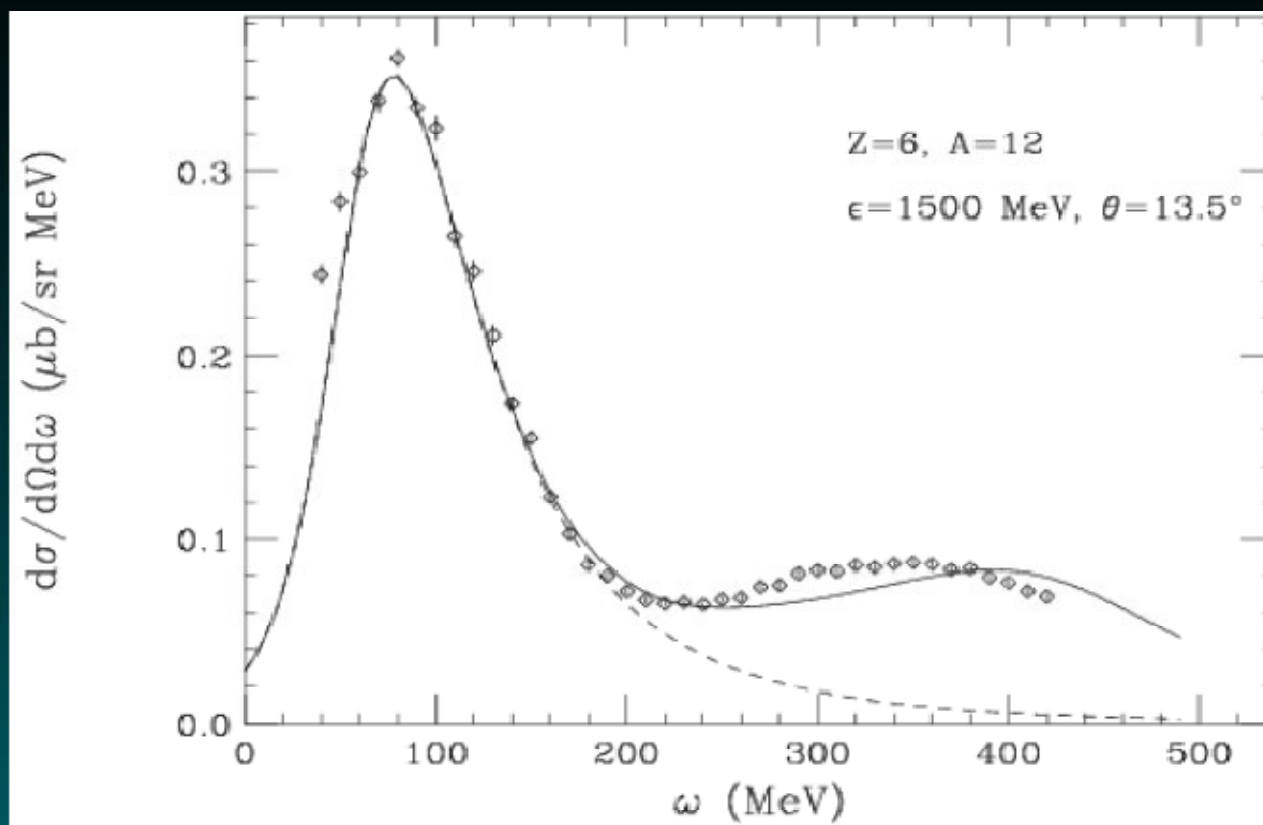
(Amaro *et al*, Phys. Rev. C71, 015501(2005))

Starting point of this SuperScaling approach: both electron and neutrino scattering share the **same universal scaling function** *under similar kinematics*:

$$\text{Scaling function} \propto \frac{\left[ \frac{d\sigma}{d\Omega_{eN} dF_e} \right]}{\bar{\sigma}_{eN}} \Rightarrow \frac{d\sigma}{d\varepsilon_{\mu} d\Omega_{\mu}} \propto \bar{\sigma}_{\nu N} \text{ Scaling f.}$$

**electron data  $\Rightarrow$  (CC) neutrino predictions**

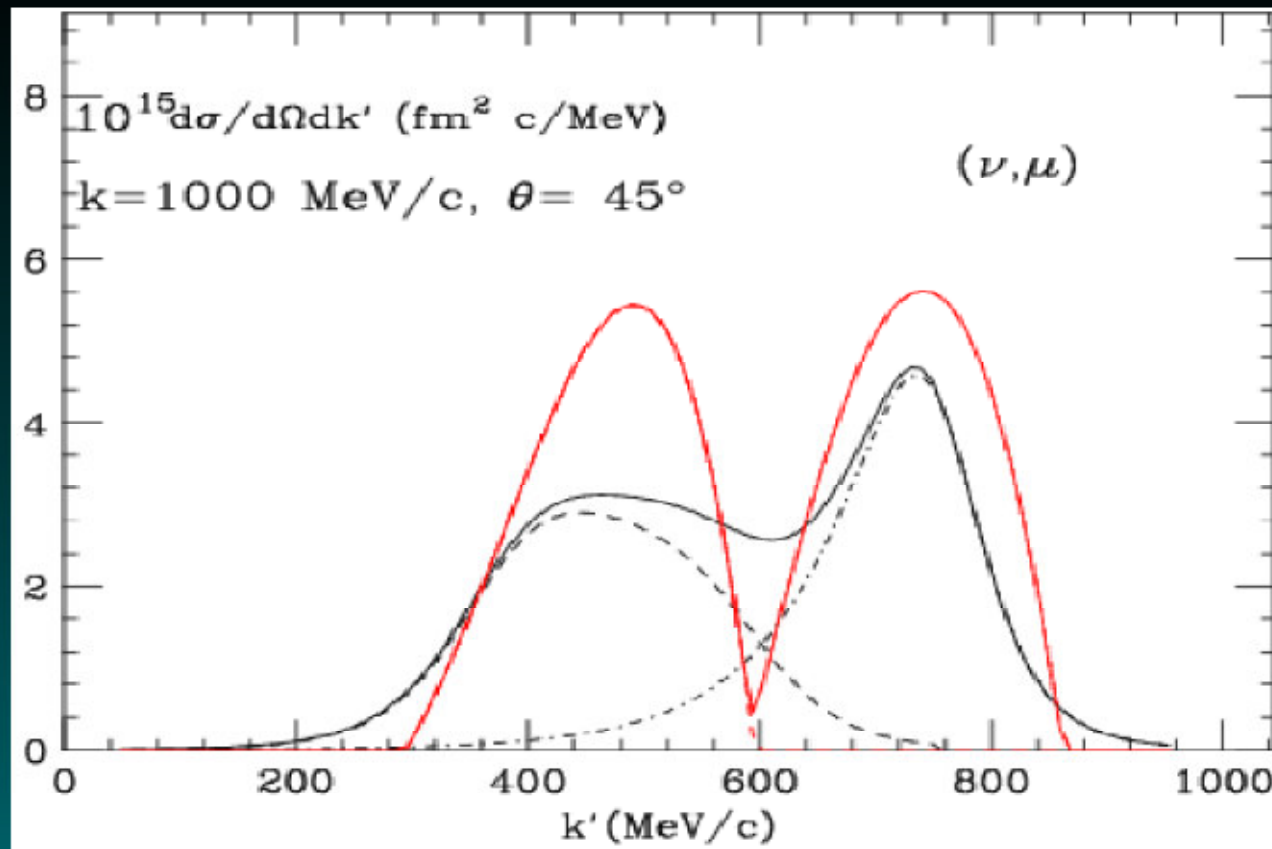
## An example of these predictions based on (QE + $\Delta$ ) scaling ideas...



**Advantage: very simple to implement!**

It is straightforward to extend the scaling ideas to the delta region and pion production

**An example of these *neutrino* predictions based on nuclear information from the experimental electron data...**

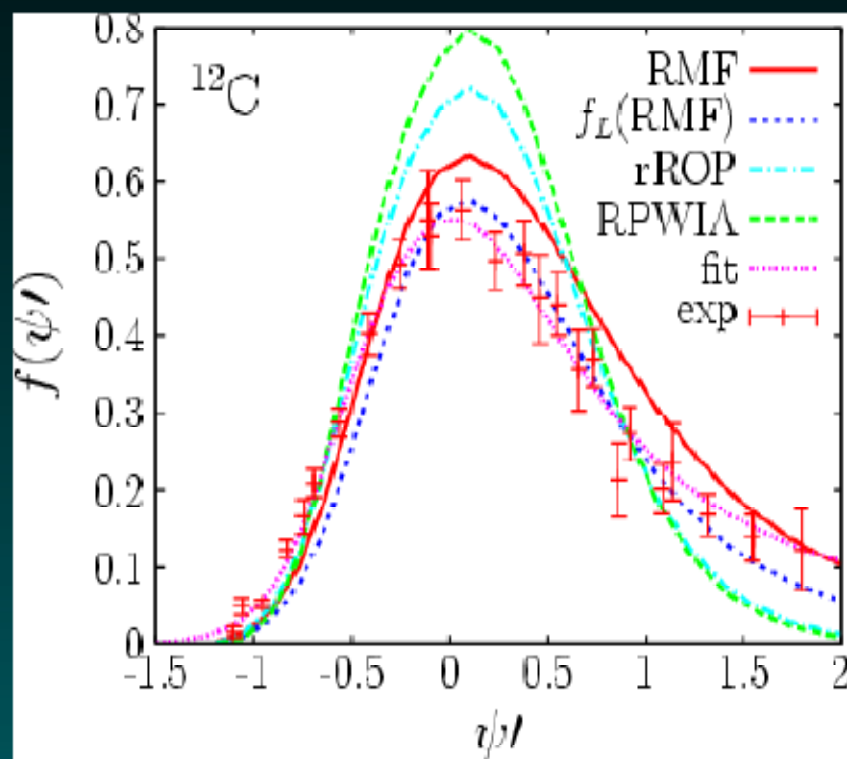


**Advantage: very simple to implement!**

**...not many models have been able to reproduce the experimental scaling function...**

**The RELATIVISTIC IMPULSE APPROXIMATION + RELATIVISTIC MEAN FIELD (RIA-RMF) for describing the bound and ejected nucleon does, both in magnitude and shape**

(Caballero et al,  
 Phys. Rev. Lett. **95**, 252502 (2005),  
 Phys. Rev. C **74**, 015502 (2006)...)



## y-scaling variable?

$y$  is the minimum initial momentum of the nucleon allowed by the kinematics.

$$y \approx \sqrt{\omega(2M_N + \omega)} - q$$

$$\text{If } y = 0 \Rightarrow$$

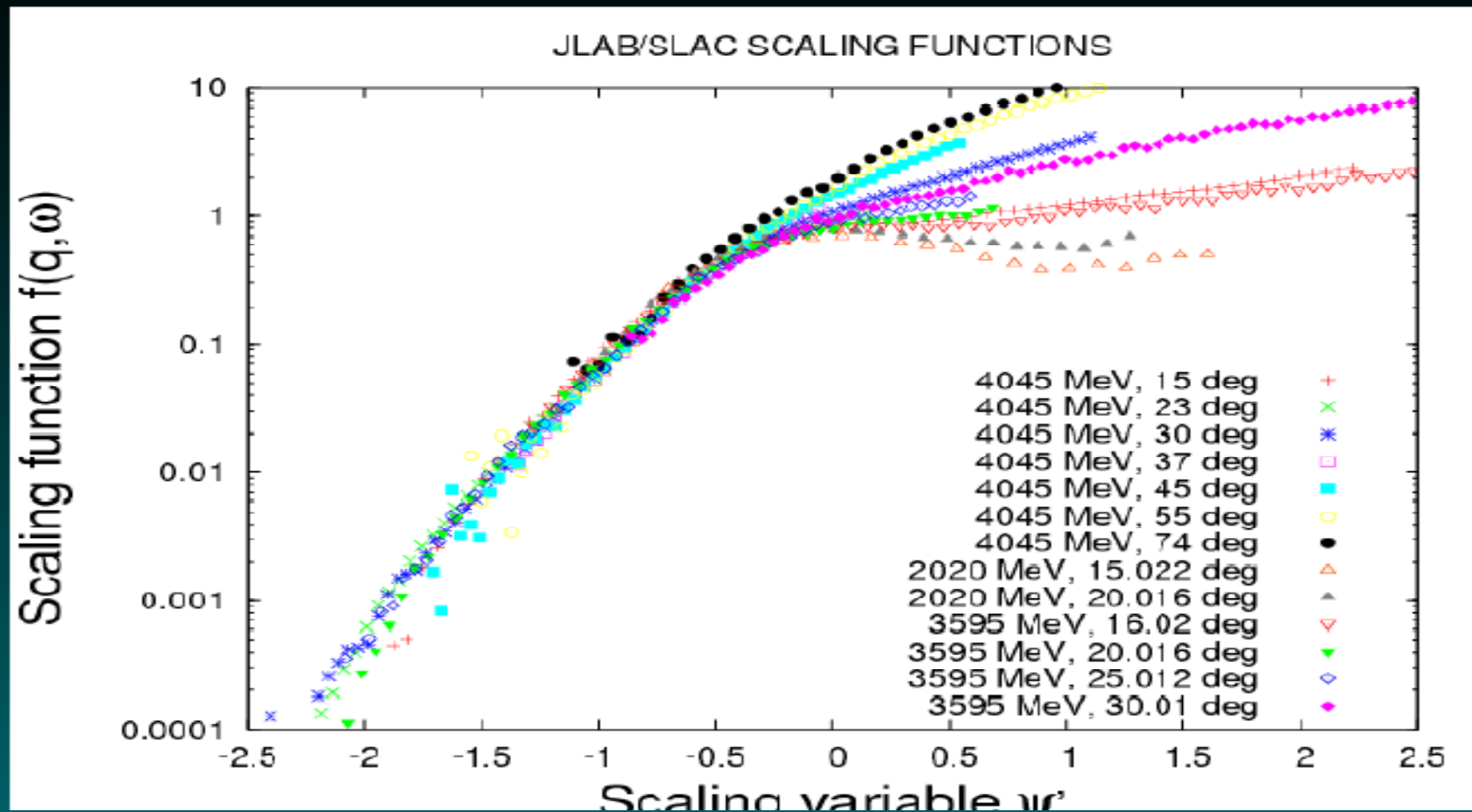
$$q^2 = \omega(2M_N + \omega) \Rightarrow$$

$$|Q^2| = 2M\omega$$

$y$  and Bjorken  $x$  scaling variables are closely related to each other! One has binding energy and nucleus recoil corrections

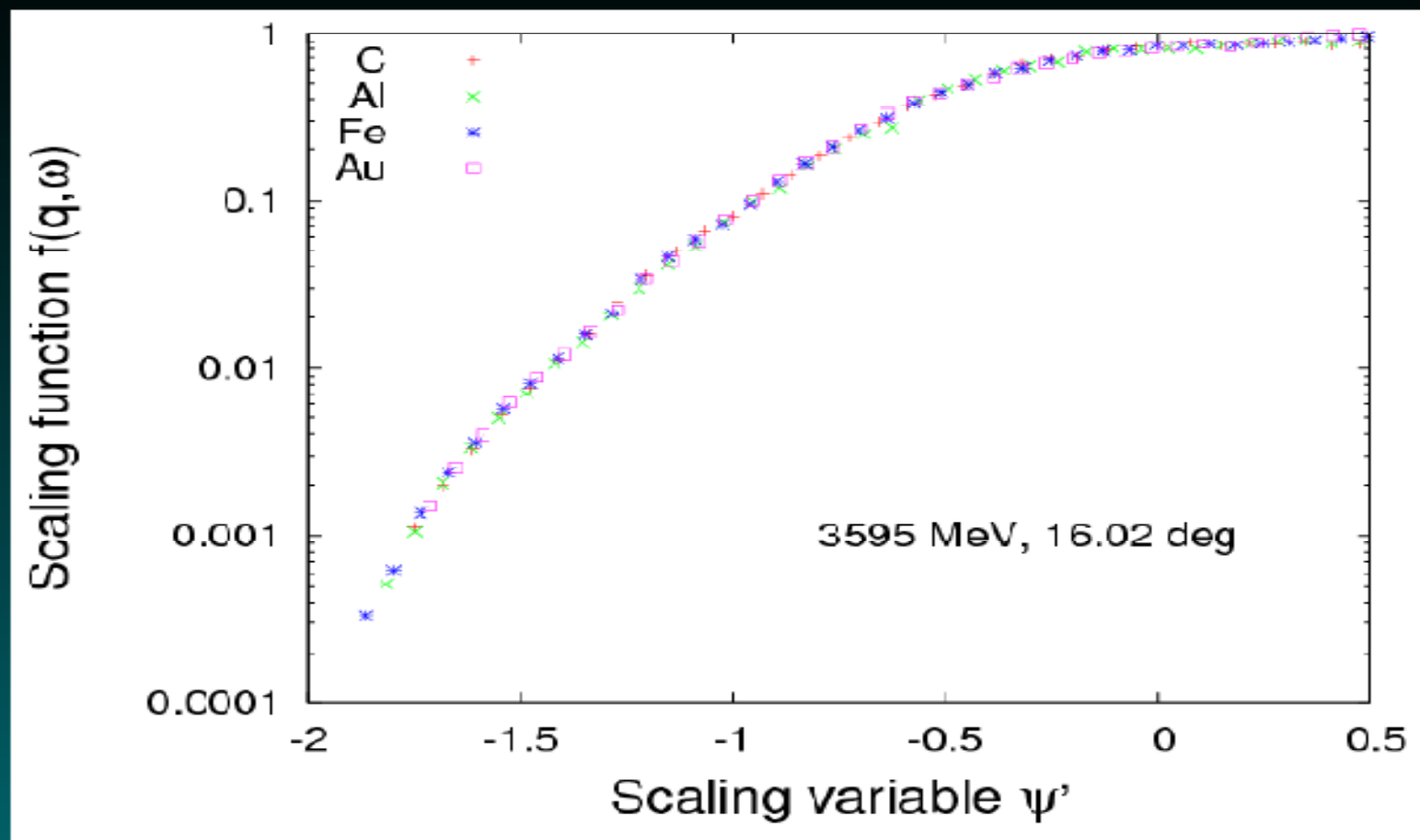
# Inclusive $^{12}\text{C}$ quasielastic electron data

## SCALING BEHAVIOR IN LOG SCALE



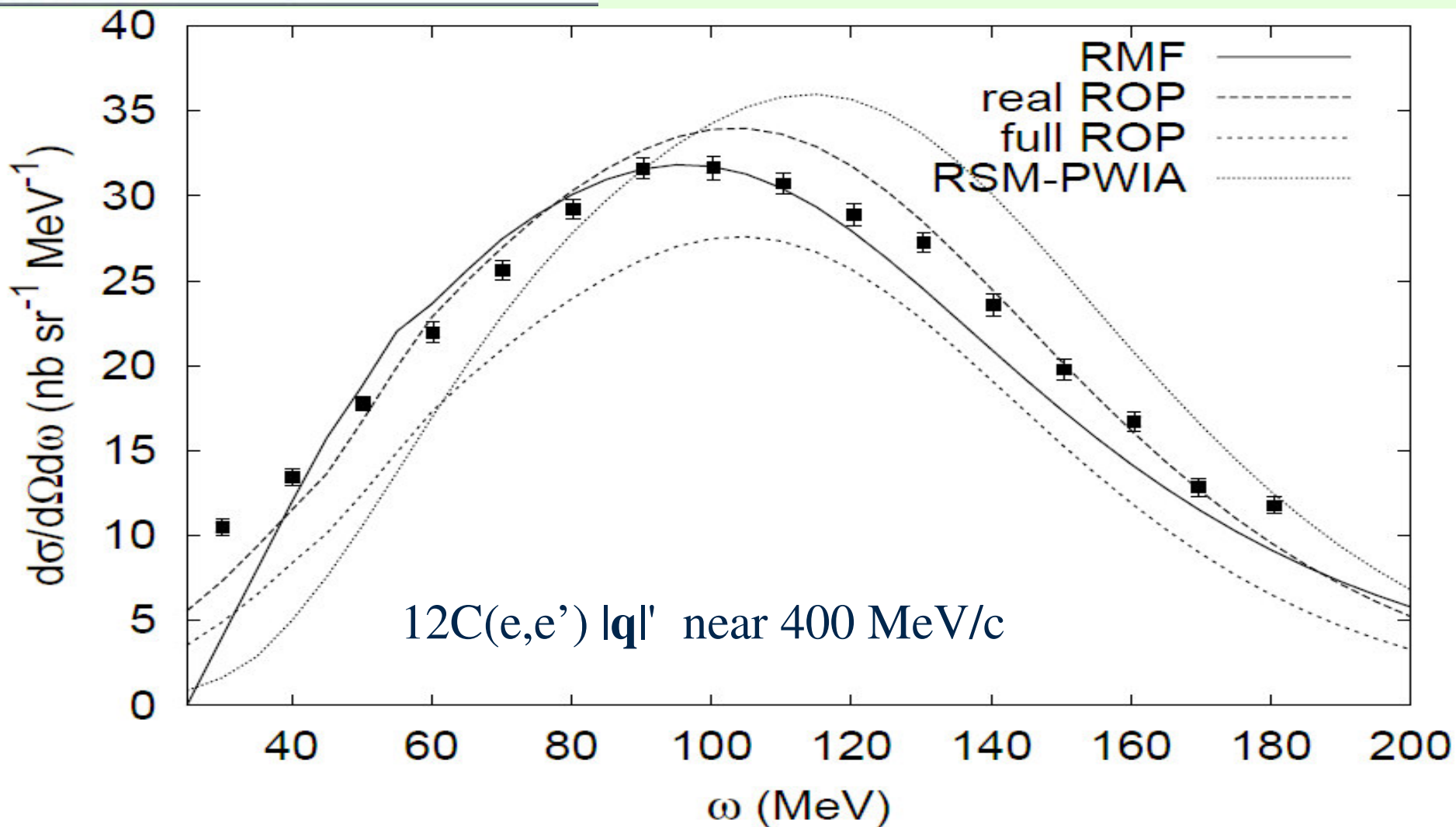


## Inclusive quasielastic electron data at $q \approx 1 \text{ GeV}/c$ SCALING BEHAVIOR IN LOG SCALE

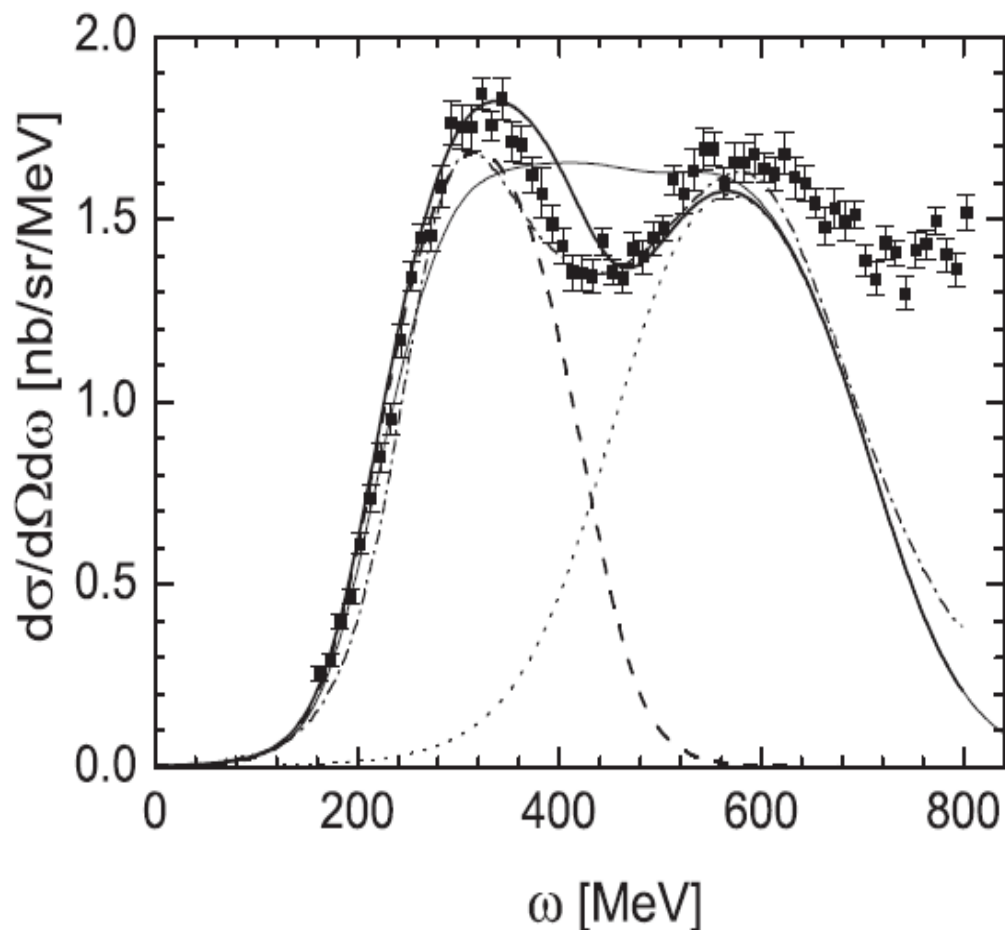




Use RMF to describe propagation of the ejected proton  
Good approximation for inclusive results. Extensive comparison to *inclusive* (e,e') data from moderate momentum transfers to about 2 GeV/c

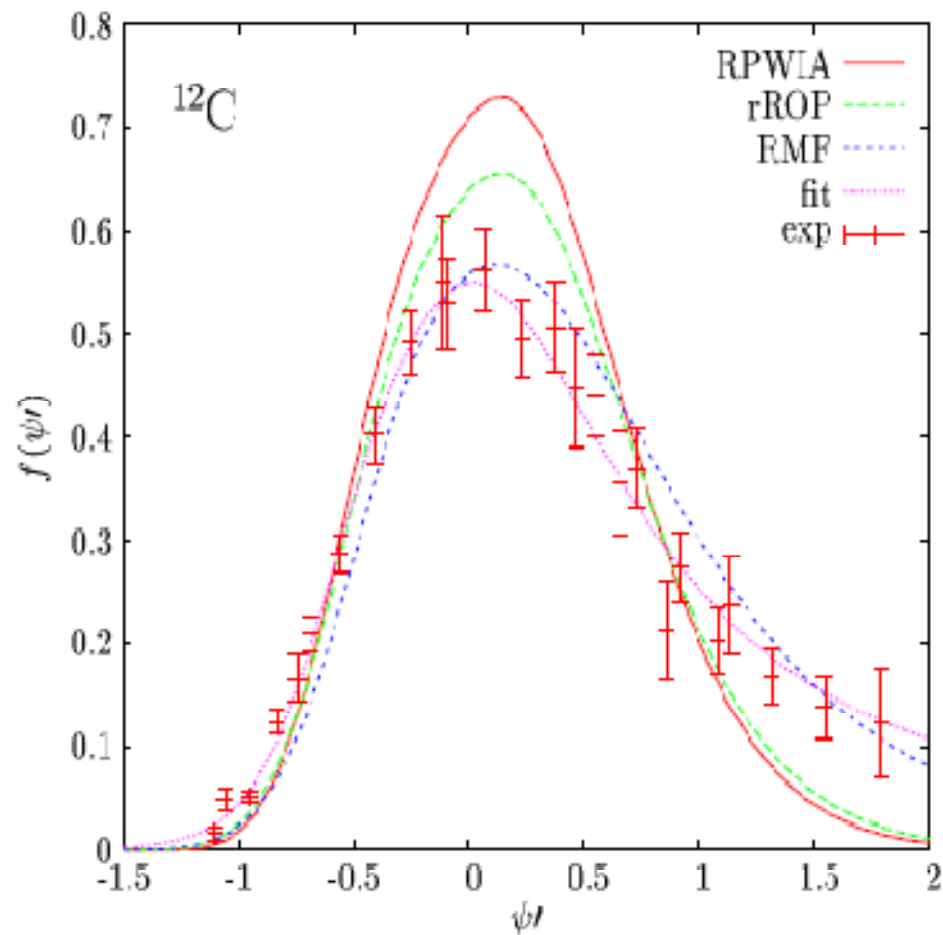


But at higher momentum transfer, it is more complicated to extract the pure nucleonic contribution



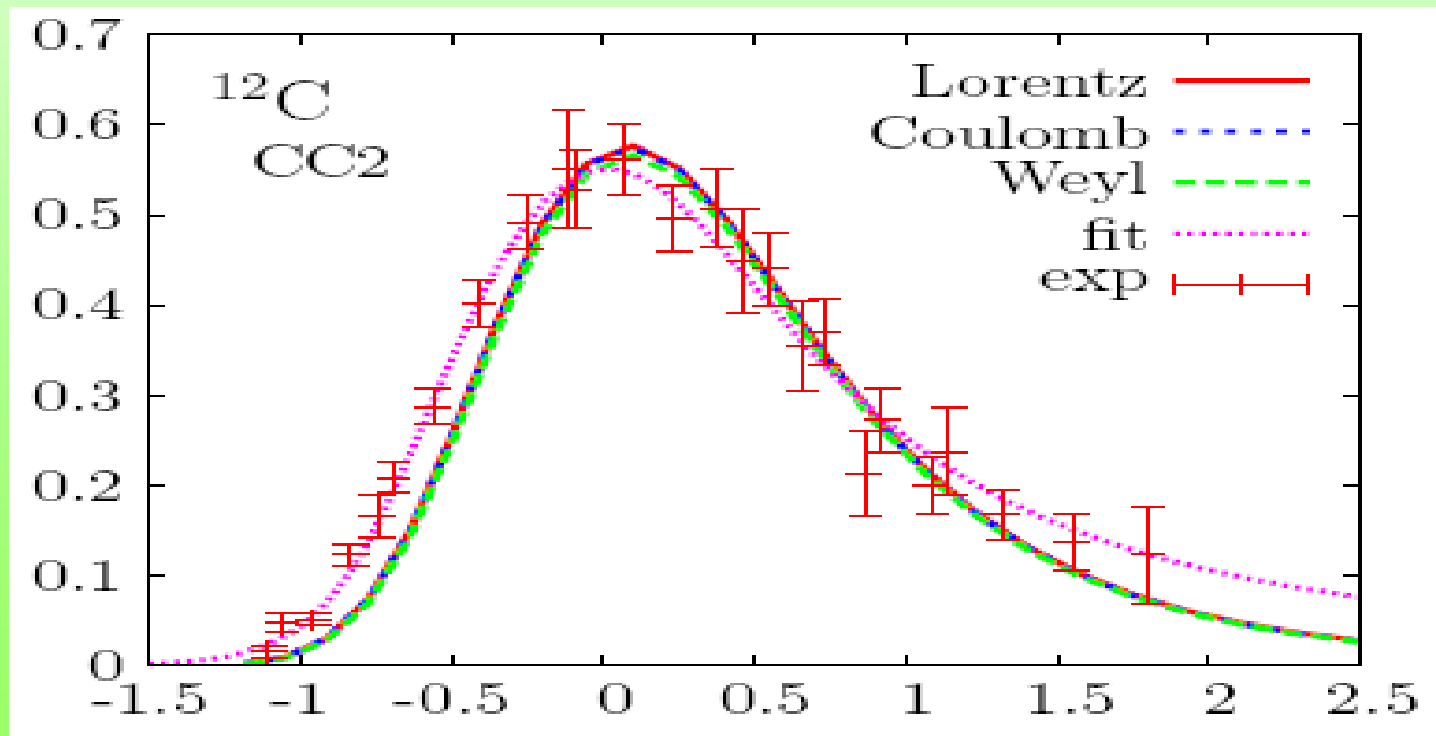
Comparison of theory to data is most easily done at the QE (and delta resonance) peak, but it is very difficult in the region of the dip. This is a problem for theory, not for experiment, that can measure the dip region with the same accuracy as the QE or delta resonance zones. Calls for combined effort that addresses both nucleonic and delta d.o.f.

inclusive electron scattering on  $^{12}\text{C}$  at  $\epsilon = 1299$  MeV and  $\theta = 15^\circ$

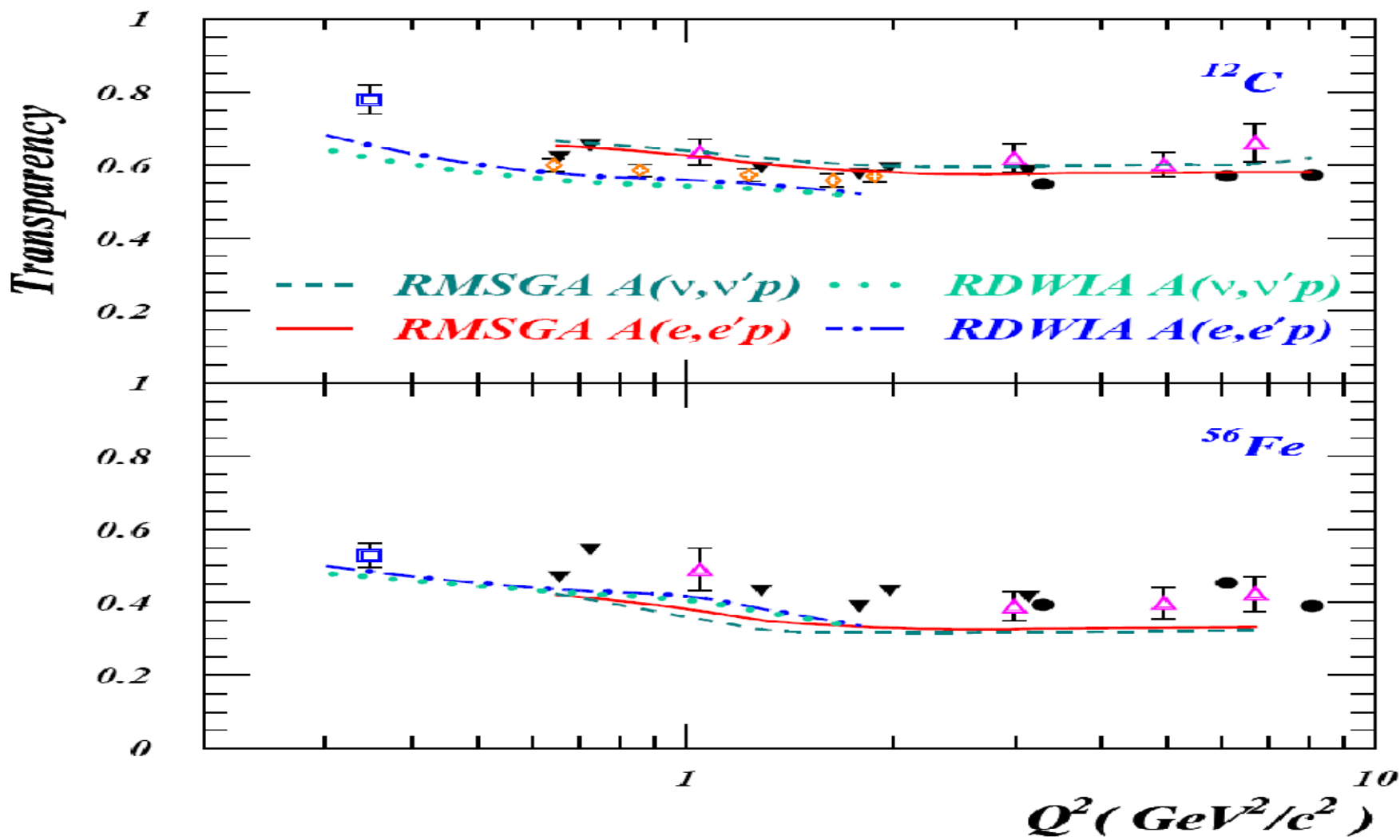


- $|q|=1$  GeV/c
- We isolate the pure nucleonic response by comparing to the L-scaling function
- More symmetrical responses are ruled out. RMF compares better with data

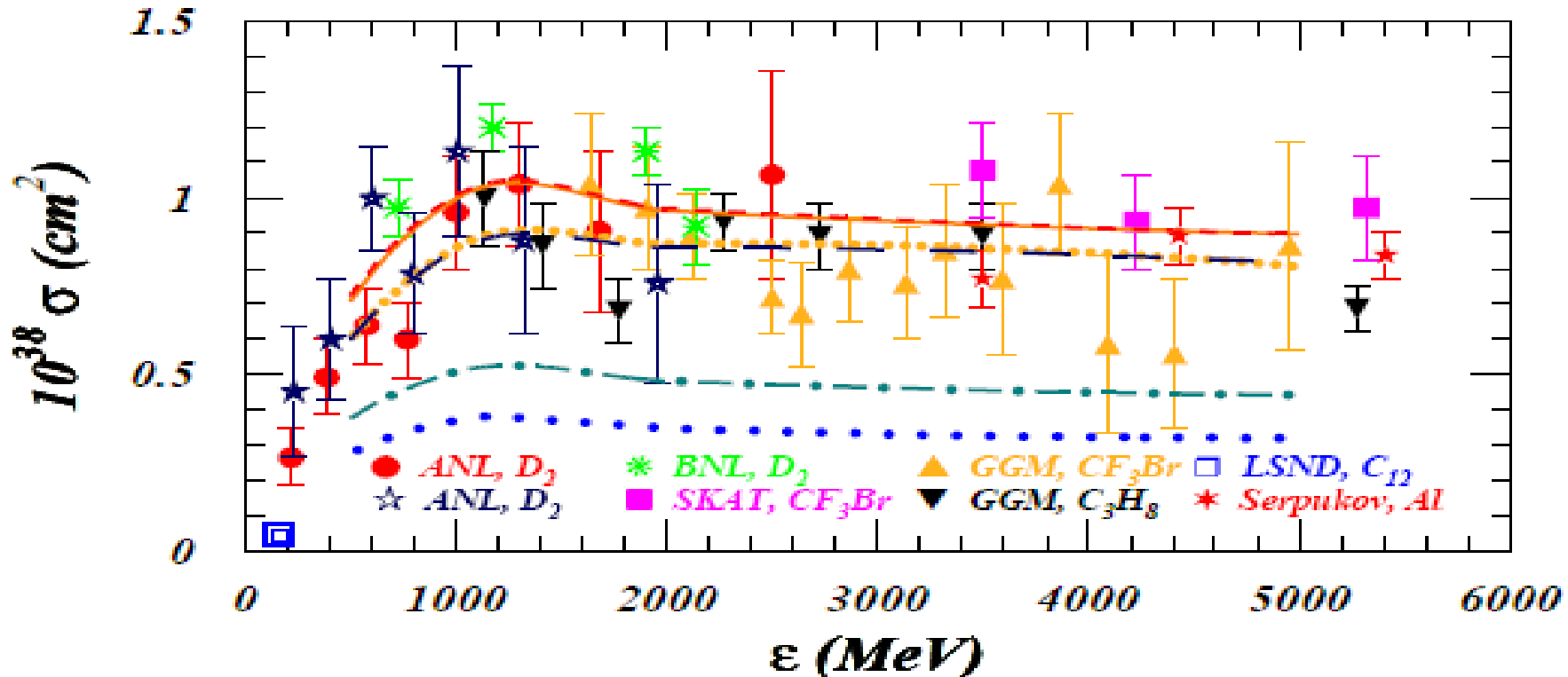
The theoretical analysis indicates that the universal superscaling function is, to a very large extent, independent on the probe. This comes from actual calculations for electron, charged and neutral currents



Or, we can look for a different kind of experiment  
neutrino vs. electron transparencies



Both RMF and OP predictions for nucleonic contributions to CC and NC neutrino scattering can be compared to experiment. Pions (almost completely) rejected



Total CC predictions (per nucleon) for non-pionic 'quasielastic' charged current reactions ( $\nu, \mu^-$ ) obtained: a) without FSI interactions (red curve). With FSI interactions within RMF for  $^{12}C$  and  $^{56}Fe$  (dotted orange and long dashed black lines, respectively). 'Pure' elastic contribution is shown by dot-dashed (green,  $^{12}C$ ) and long dotted (cyan,  $^{56}Fe$ ) curves. Data from several experiments and targets are also plotted. 10% effect of FSI can be observed, even at 5 GeV. No A dependence is seen in the data, consistent with a fully inclusive experiment and in agreement with RMF predictions.

# Summary

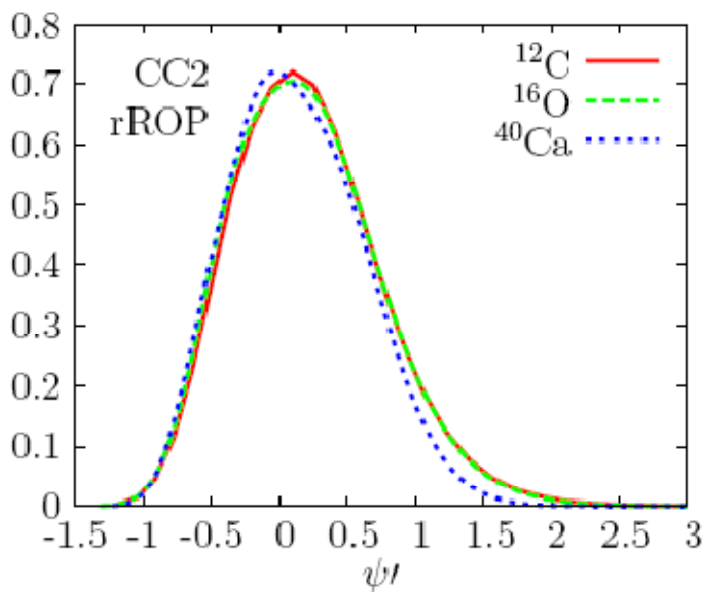
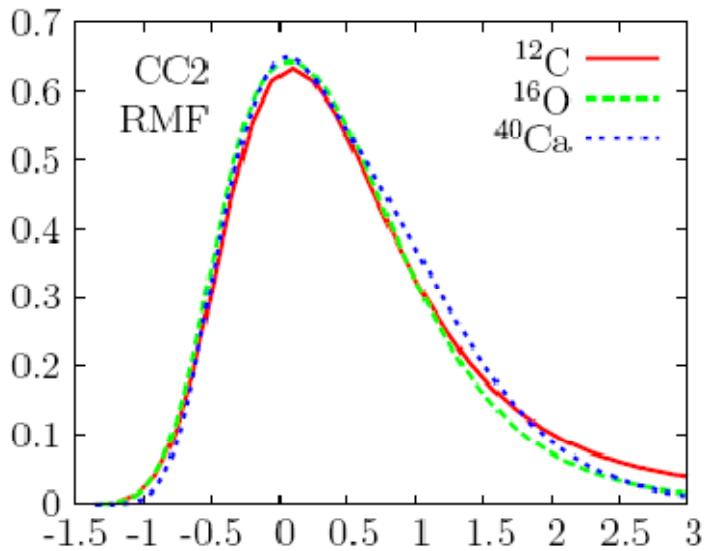
- (2) RDWIA with the RMF (computing the nucleon self-energy in the relativistic mean field approach) is successful in describing TRULY inclusive ( $e, e'$ ) or neutrino-nuclei experiments. These experiments display no  $A$ -dependence in the cross-section (normalized to ONE nucleon), indicating a closure mechanism
- (3)  $A$ -dependence of transparencies appears because they are not fully inclusive experiments. Optical potential predictions are just a lower bound for transparencies, saturated only if they corresponded to fully exclusive measurements (with full occupancies)
- (4) In the comparison to transparencies from experiment, care must be taken to the exclusive/inclusive actual experimental combination





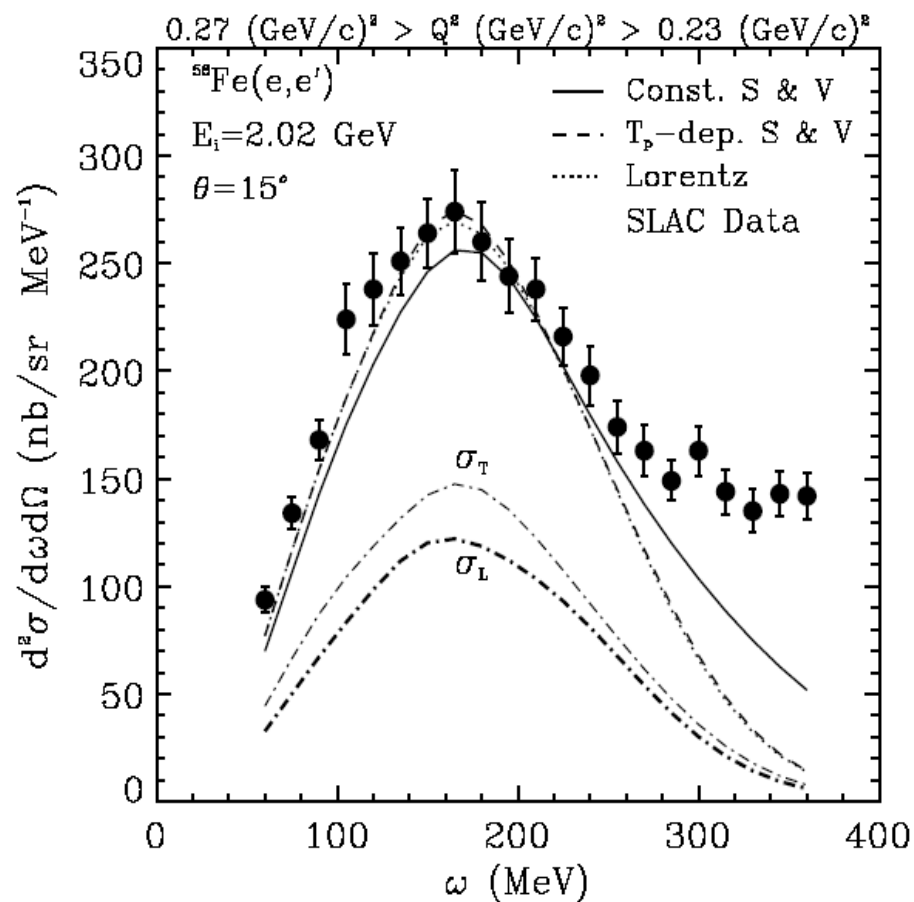
## Summary: what have we learnt from exclusive measurements??

- **Relativistic impulse approximation (RIA) + RMF is simple and capable of explaining many different experimental results, including polarization measurements**
  - **Improved experimental information with:**
    - improved statistics
    - larger A coverage ( $^{208}\text{Pb}$ ,  $^{16}\text{O}$  (e00102),  $^4\text{He}$ ,  $^{12}\text{C}$ )
    - $x=1$
    - different  $Q^2$  values
- is arriving in the next few months, what would allow to disentangle relativistic effects and/or long range correlations**



Both scaling of first and second kind are clearly observed in the predictions of theoretical models based upon OBE+IA. Even when these models are (un)factorized or when they include important FSI interactions among nucleons. This has to do with the properties of the (distorted) nuclear response in general and not with the properties of the probe (provided OBE). Within the RMF, differences of the scaling function obtained from different probes are mostly due to spinor distortion. Thus, the scaling features observed in electron scattering are also expected in neutrino (charged and current) scattering

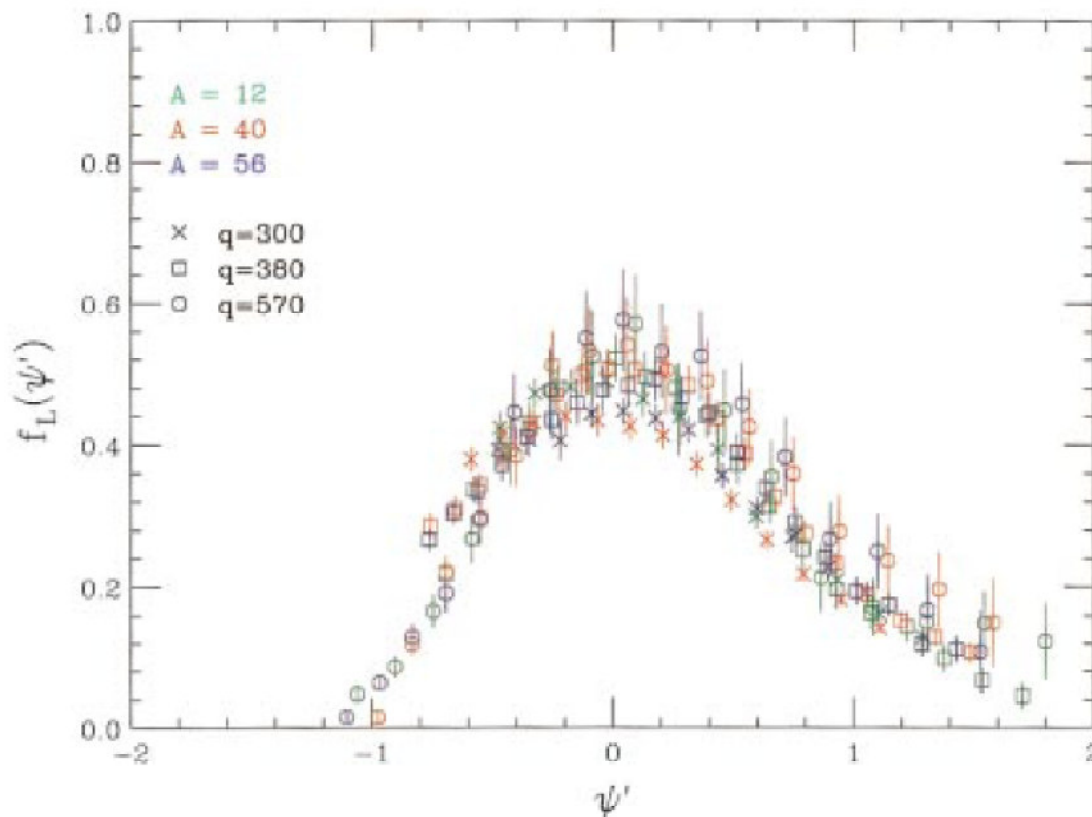
## Inclusive (e,e') reactions: comparison with data



Constant potentials (RMF) produce an asymmetric cross-section with increased strength (tail) at large  $\omega$  (K.S. Kim and L.E. Wright PRC 68 (2003) 027601)

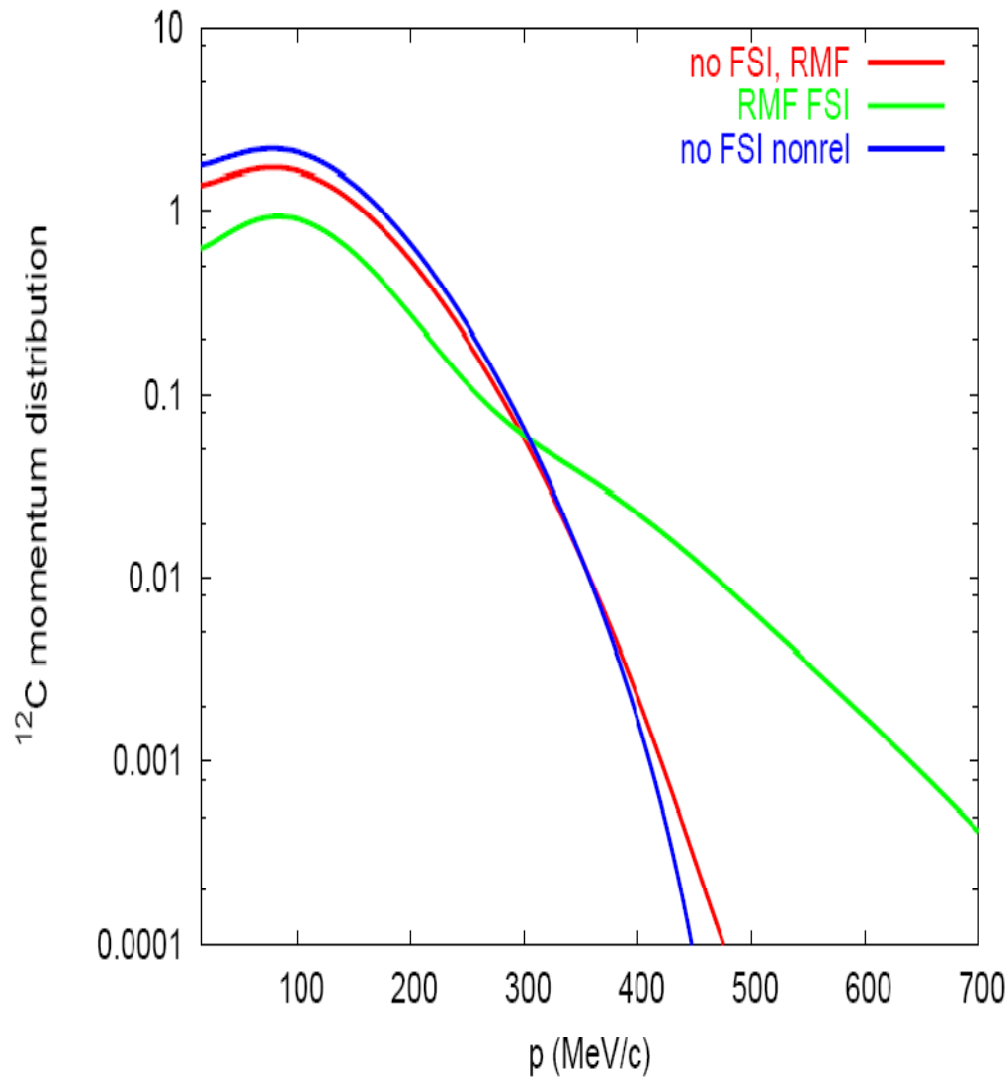
Comparison with data is not conclusive due to the delta peak contributing into the quasielastic region

Both scaling of first (mild) and second (good) kind are clearly observed in the inclusive electron scattering data. This supports the theoretical predictions indicating that off-shellness of the nucleons in nuclei and even strong FSI do not destroy scaling

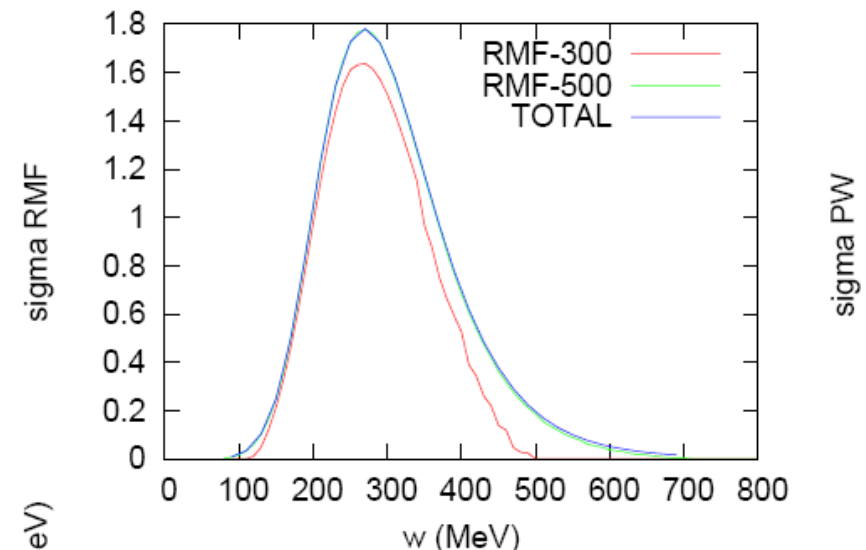


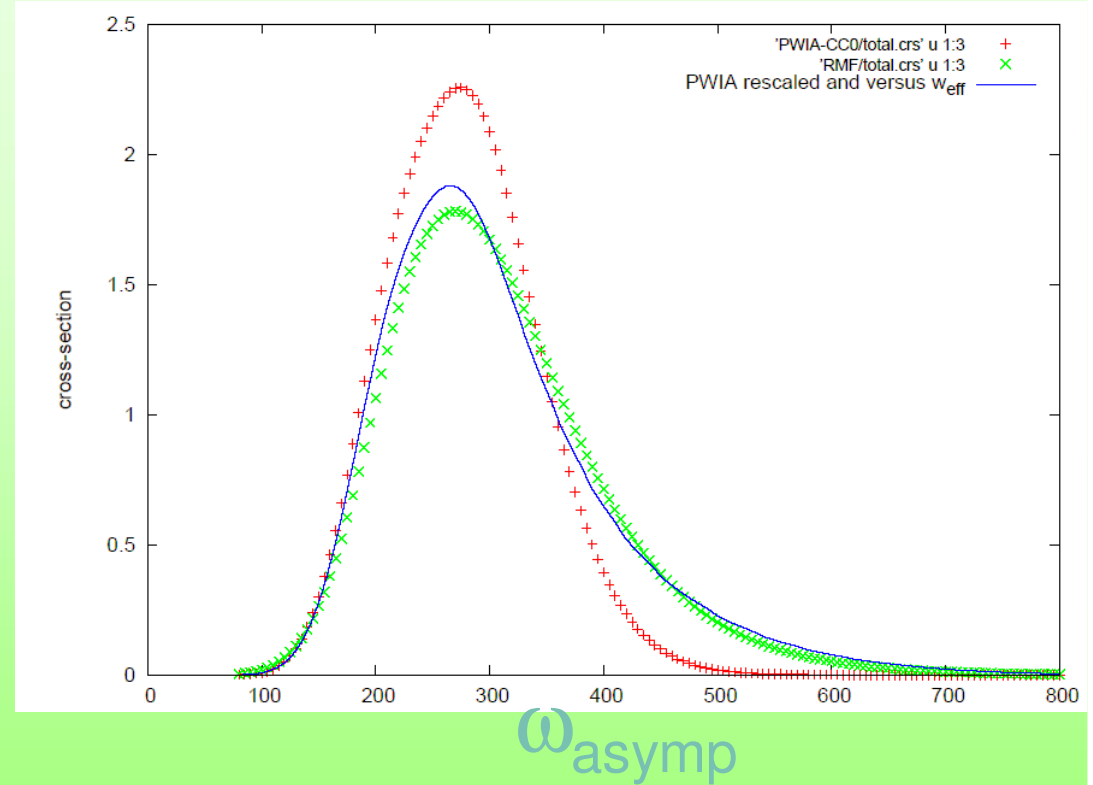
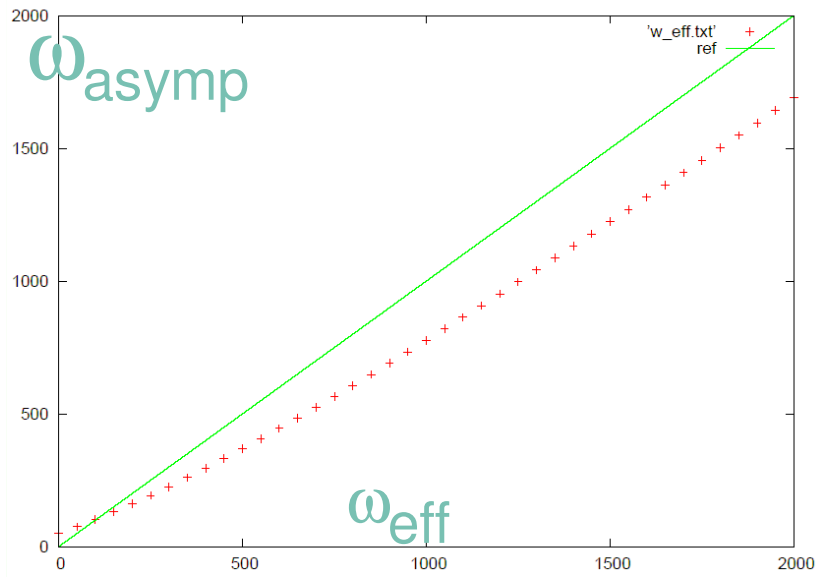
This allows to extract a (*exceedingly convenient!*) universal superscaling function from the inclusive electron scattering data

It is clear that the longitudinal data exhibit a large asymmetric tail



The asymmetry is generated by the additional strength in between 300 and 500 MeV/c. This is, in the RMF, due to FSI





The asymmetry can be generated by means of effective kinematic that takes into account the strong potentials



# Modeling inclusive lepton-nucleus reactions within the relativistic mean field approximation

In collaboration with many people:

E. Moya, Javier R. Vignote, J.A. Caballero, T.W. Donnelly, E. Amaro, M. Barbaro, C. Maieron, C. Martínez-Pérez, P. Lava, J. Ryckebusch, J. López Herráiz, Y. Umino and others

- Over the last years, the ‘simple’ relativistic impulse approximation has been employed with success to describe inclusive  $(e,e')$  scattering at the quasielastic peak, provided  $Q^2$  is large enough, say greater than  $0.2 \text{ (Gev/c)}^2$  (K.S. Kim and L.E. Wright PRC 68 (2003) 027601, PRC 67, 054604 (2003) , Y. Jin, D.S. Onley and L.E. Wright, PRC 45 (1992) 1333, C. Maieron et al, PRC68 (2003) 048501)
- Try to understand the reasons for this success and use the same modeling to describe neutrino reactions



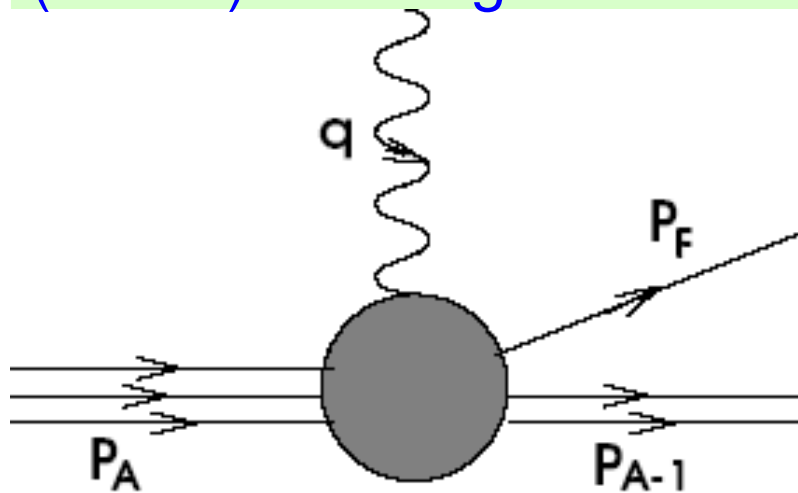


# ELECTRON INDUCED REACTIONS IN HEAVY, MEDIUM AND LIGHT NUCLEI (after 100 years of relativity)

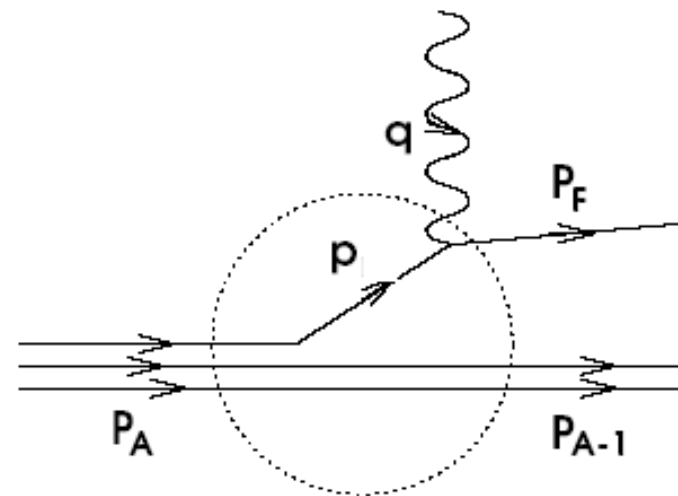
- **Overview of the model**
- **Mean Field and Impulse Approximation**
- **Asymmetry in the  $(e,e')$  cross-sections**
- **Predictions for neutrino CC**
- **Conclusions**

# OVERVIEW OF THE MODEL (ingredients)

Simple: One photon (boson) exchange:



Even simpler: Impulse Approximation



$$J_N^\mu(\omega, \vec{q}) = \int d\vec{p} \bar{\psi}_F(\vec{p} + \vec{q}) \hat{J}_N^\mu(\omega, \vec{q}) \psi_B(\vec{p})$$

Complex nuclei!!!. If we can not treat them exactly, let's be simple!!!!. Take the simplest ingredients: Mean Field

# Relativistic mean field RMF

- Use Dirac equation with local potentials, obtained with a lagrangian fitted to reproduce saturation properties (binding energy, density) of nuclear matter, or radii and mass of selected nuclei
- RMF does saturate, even if no Fock terms are introduced (Dirac Hartree) and without explicitly considering correlations. This makes for a very simple modeling
- Generally speaking, introducing Fock terms or correlations shifts the saturation point, but after fitting of the parameters of the model, the saturation point is set wherever we want it

## Relativistic mean field RMF (II)

- By choosing the parameters of the lagrangians to reproduce the saturation point at the mean field level, some effects of correlations have been taken into account
- The relativistic mean field incorporates at the same time repulsive (vector exchange) and attractive (scalar exchange) terms, resulting in effective vector and scalar potentials. The explicit appearance of strong repulsive term helps emulating correlations
- Nonlocalities (dependences of the potentials or the effective mass on the density, the energy or the momentum) as well as other effects introduced by correlations or Fock terms, are recovered from the relativistic formalism when performing the nonrelativistic reduction, even if the relativistic equations and potentials are local
- Of course, RMF also include relativistic effects

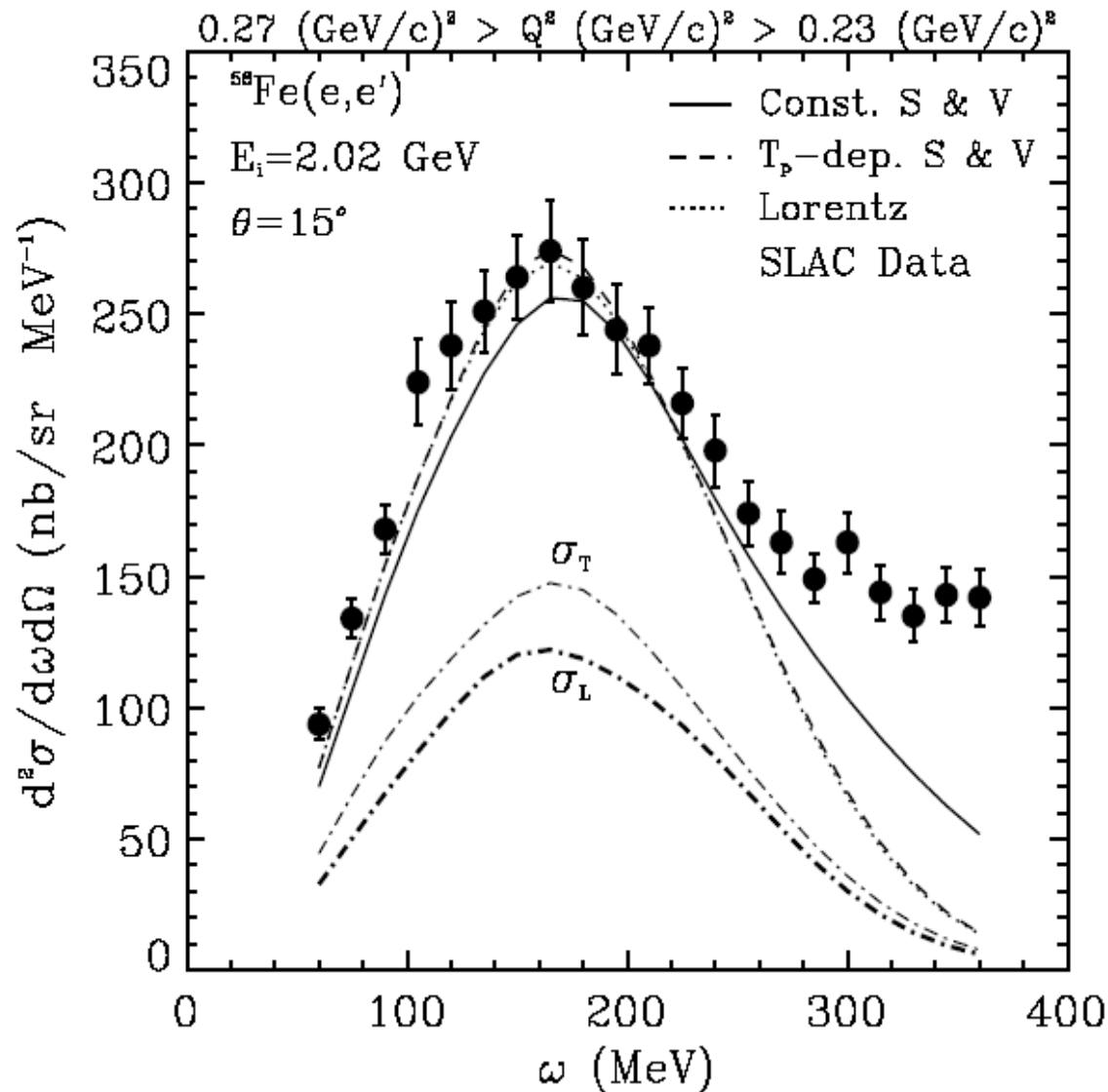
# Inclusive (e,e') reactions: RDWIA-

## RMF



- If FSI fitted to elastic channel only (optical potential approach to elastic scattering), we need to recover the lost flux into inelastic channels. At times this is done setting to zero the imaginary part of the potentials, for instance. This provides an E-dependent real potential and thus violates the dispersion relationship (**Mahaux and Sartor, Adv. Nucl. Phys. 20 (1991) 1, Horikawa et al, PRC22 (1980) 1680**)
- Another choice is the use of the same (E-independent) mean field potential for the final proton as for the bound proton so that no loss of flux introduced. This simple choice
  - i) preserves orthogonalization
  - ii) verifies continuity equation and Siegert's theorem
  - iii) fulfills the dispersion relationship

# Inclusive (e,e') reactions: comparison with the data

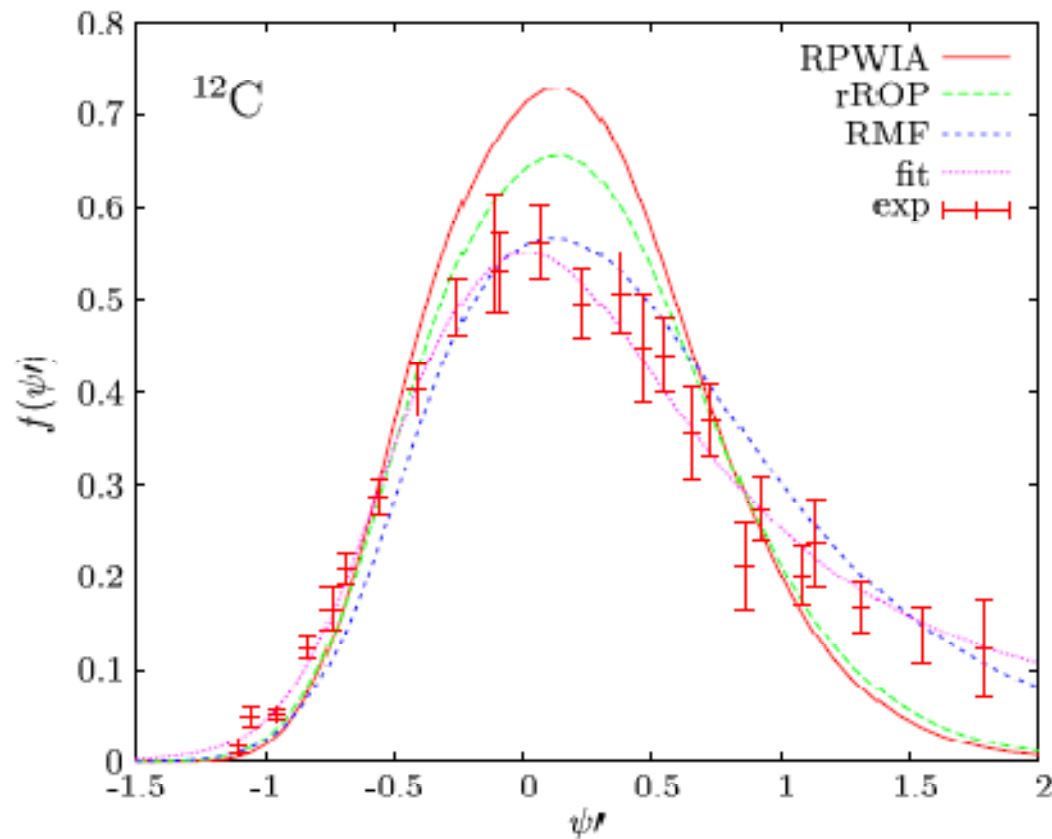


Constant potentials (RMF) produce an asymmetric cross-section with increased strength (tail) at large  $\omega$  (K.S. Kim and L.E. Wright PRC 68 (2003) 027601)

Comparison with data is not conclusive due to the delta peak contributing into the quasielastic region

# Comparison to inclusive data: Scaling analyses

(J.A. Caballero et al., PRL 95 (2005) 252502)



- $E_{\text{beam}} = 1 \text{ GeV}/c$

- $\theta_e = 45^\circ$

- We get rid of the delta complications by comparing to the pure nucleonic scaling function

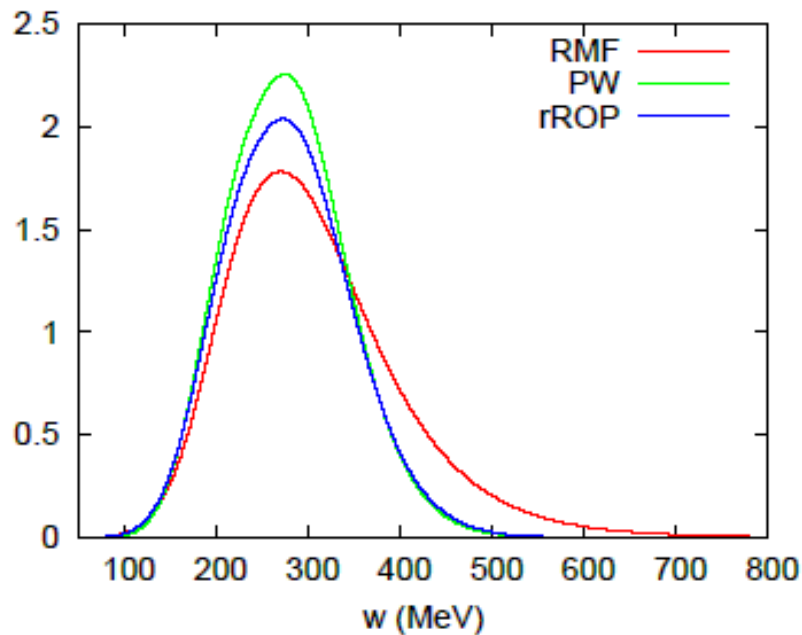
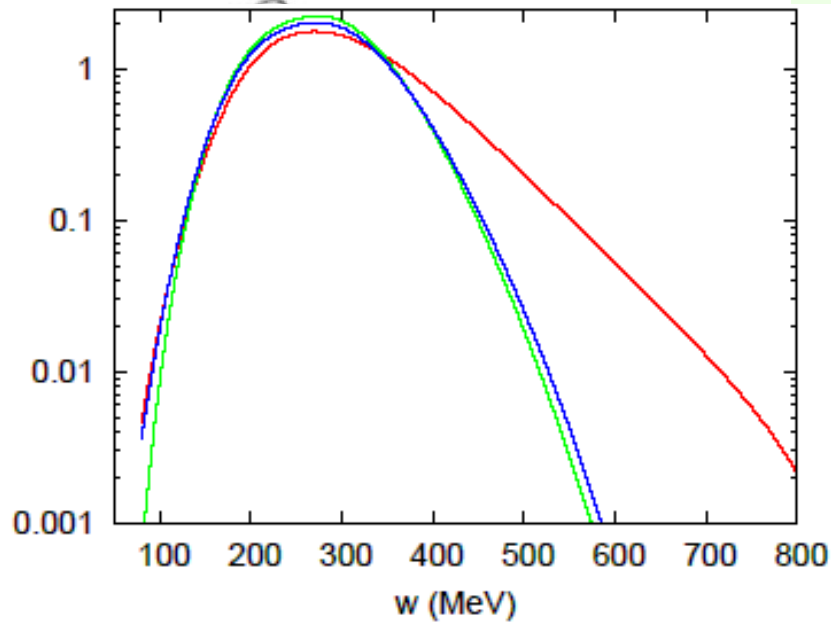
- Symmetrical responses are ruled out by experiment. RMF compares favourably with data





## to higher $\omega$ transfer: (standard thinking)

- Correlations introduce fragmentation of the strength in the *initial state*. Compared to mean field, the strength is distributed over a larger range and more uniformly. This doesn't seem to be the cause of **large** asymmetries in the cross-section
- Correlations also allow for multinucleon emission (not included in the mean field), thus giving contributions to the  $(e, e')$  cross-sections shifted by several 'separation energies' with regards to the single nucleon knockout: asymmetry



- $d\sigma/d\Omega$  in nbarn/MeV/sr,  $^{12}\text{C}$ ,  
 $e_{\text{beam}}=1$  GeV

- $\theta_e=45$  deg

- Predictions from the RMF are in agreement with the experimental scaling function

- The additional strength beyond  $\omega=300$  MeV is due, in the RMF, to FSI effects



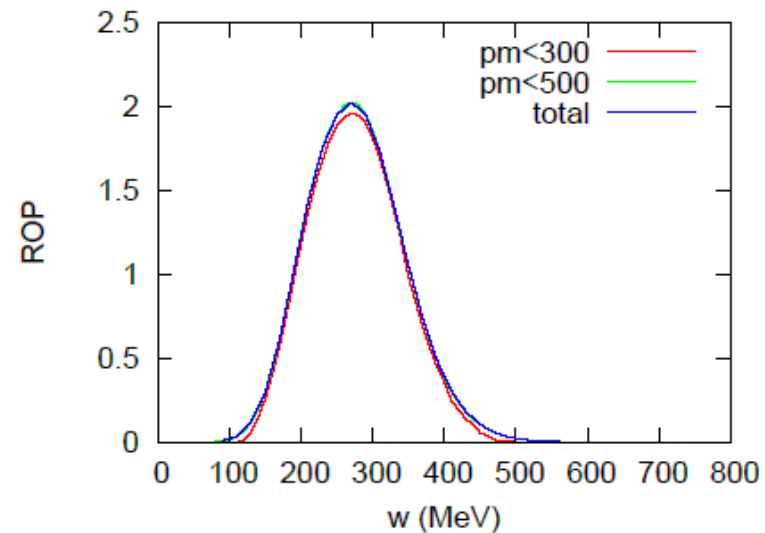
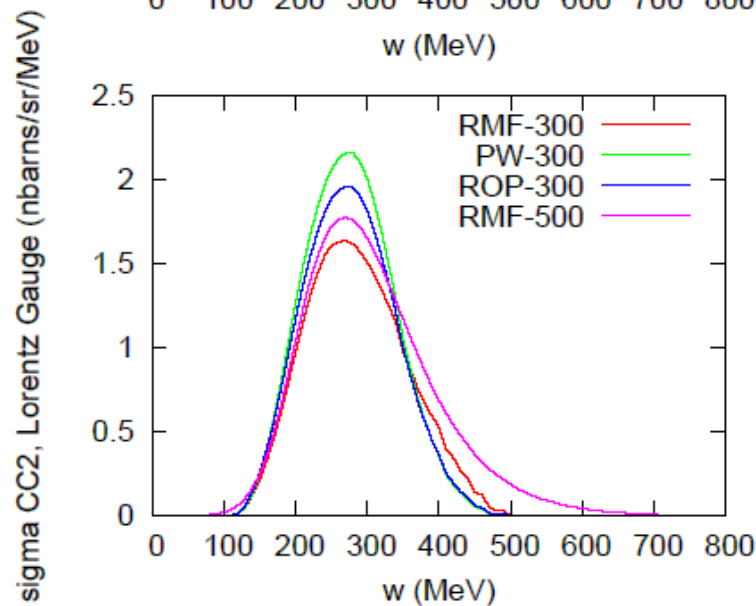
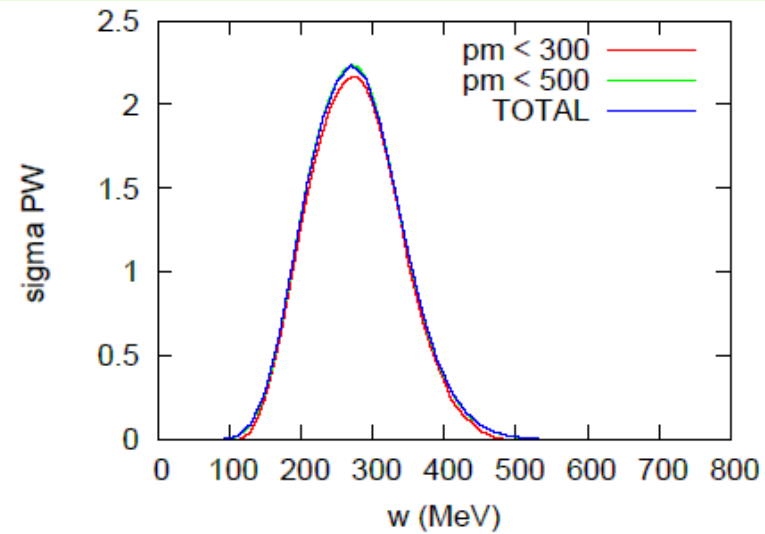
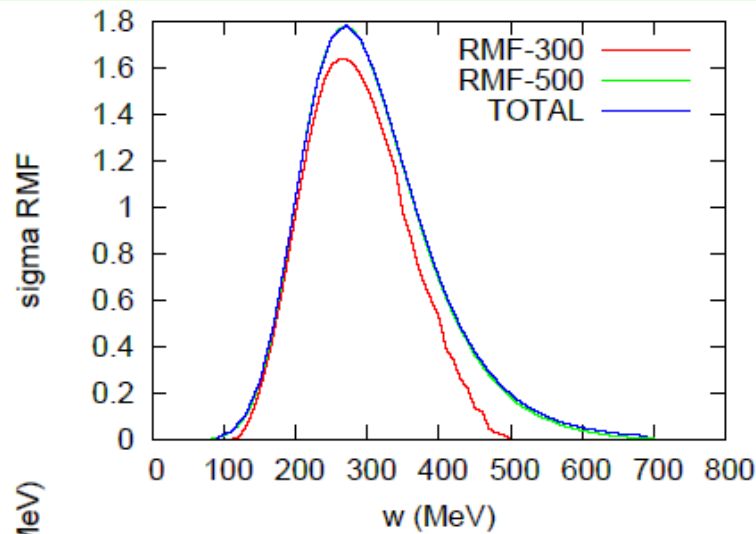
## Scalar and Vector potentials

$$(\tilde{E}\gamma_0 - \vec{p} \cdot \vec{\gamma} - \tilde{M})\psi = 0$$

$$\tilde{E} = E - V(r)$$

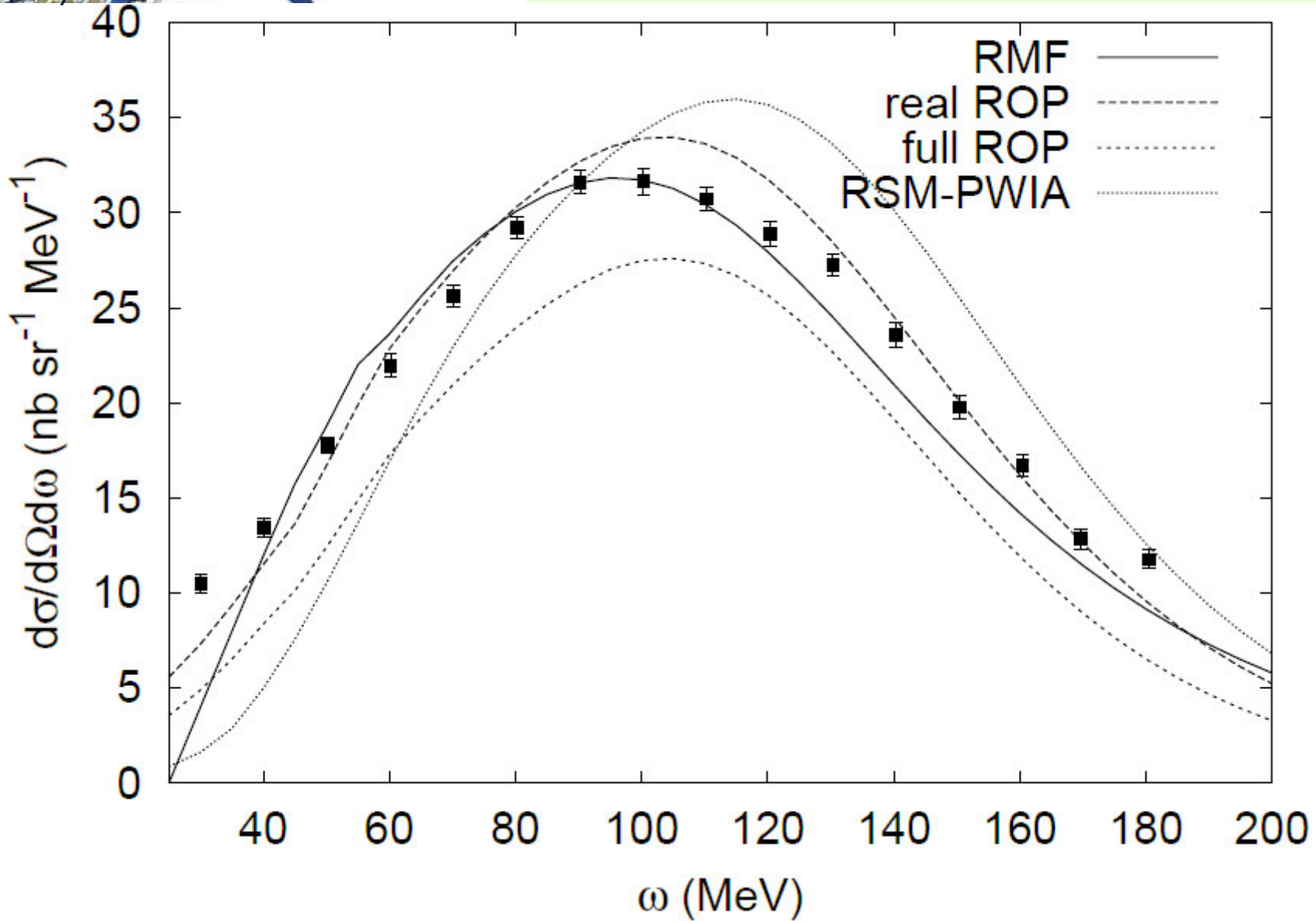
$$\tilde{M} = M - S(r)$$

- Solve a Dirac-like equation
- Bound state: Phenomenological  $\sigma$ - $\omega$  lagrangeans  
(Serot and Walecka model) at mean field level
- Final State: Inclusive, include 'every channel'. Use the same mean field potential. This is consistent with the Impulse Approximation
- Current operator: 'free' current operator prescription



- The effect is difficult to estimate *a priori* from simple approaches, because potentials are larger in the nuclear interior but they go to zero as the nucleon approaches the surface
- The full RMF calculation provides quantitative estimates and seems to be in agreement with the data
- Within RMF, the asymmetry is due to high momentum contributions to the cross-section introduced by FSI

$^{12}\text{C}(e, e')$   $|\mathbf{q}'|$  400 MeV/c. RMF results compare well with  $(e, e')$  data also at moderate momentum transfer



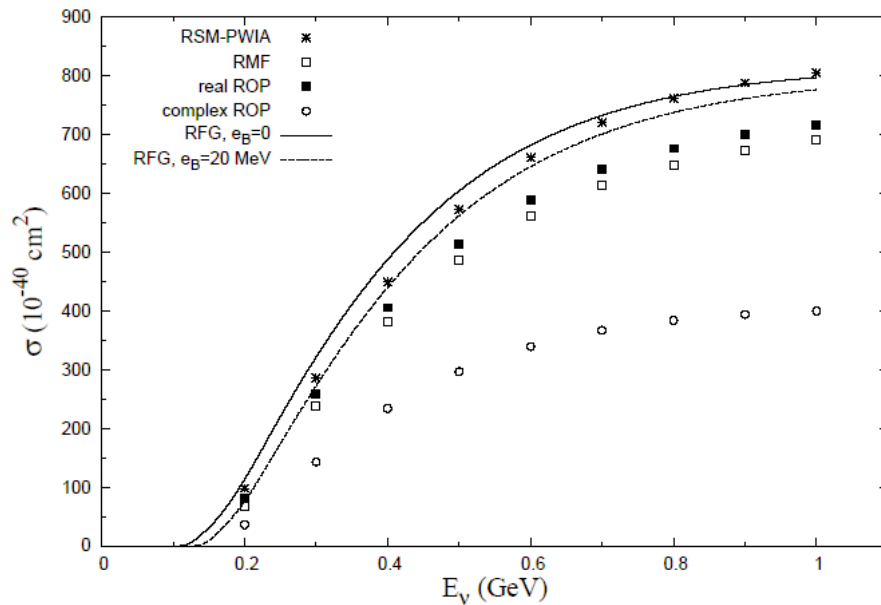


FIG. 4: Integrated cross section  $\sigma(E_\nu)$  for the quasielastic scattering of muon neutrinos on  $^{16}\text{O}$  as a function of the incident neutrino energy. The curves are calculated within the RFG model with  $k_F = 225$  MeV and binding energy  $e_B = 0$  (solid line) and  $e_B = 20$  MeV (dashed). The points correspond to RSM calculations without FSI (stars) and with FSI effects taken into account within the RMF (empty squares), real ROP (full squares) and complex ROP (circles) approaches.

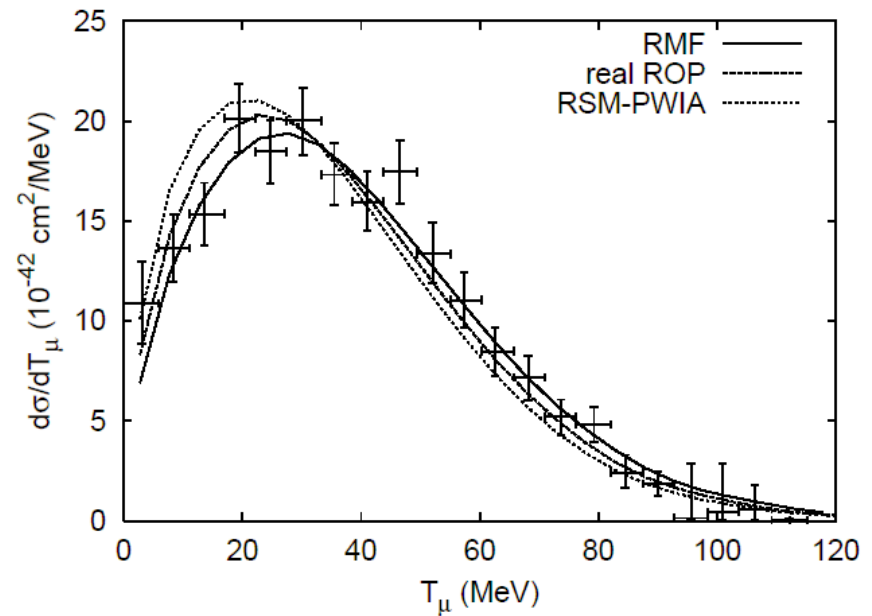
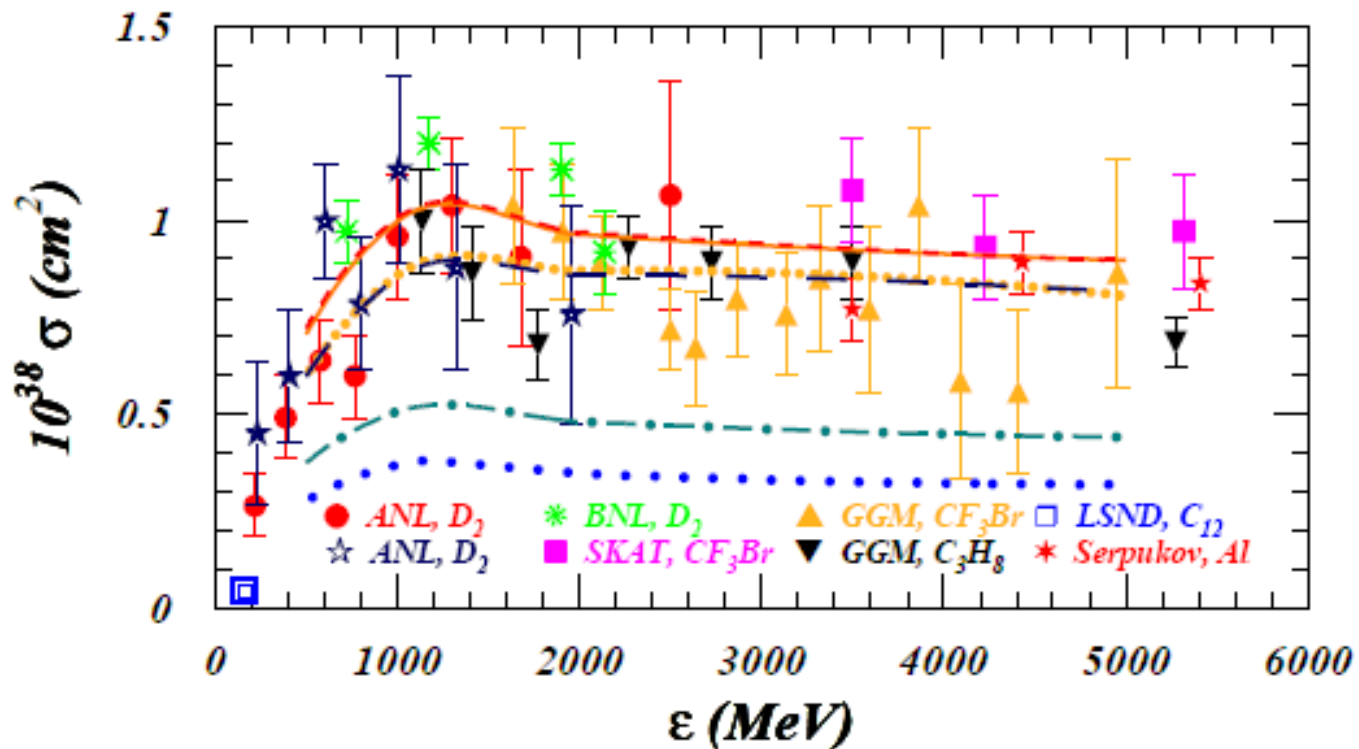


FIG. 5: Observed distribution of muon kinetic energies  $T_\mu$  compared with the flux-averaged predictions of our RSM, in PWIA (dotted line) and including FSI within the RMF (solid) and purely real ROP (dashed) frameworks. The theoretical distributions have been normalized to give the same integrated values as the experimental points, and have been folded in energy with a bin size of 5 MeV, the same employed for the experimental data. Data are from Albert et al. [12].

## Comparison to LSND $^{12}\text{C}(\nu, \mu^-)X$ data

$\sigma_{\text{RMF}} = (15 \text{ to } 16) \times 10^{-40} \text{ cm}^2$  that is 40% above data. MEC shall reduce this by no more than 10%

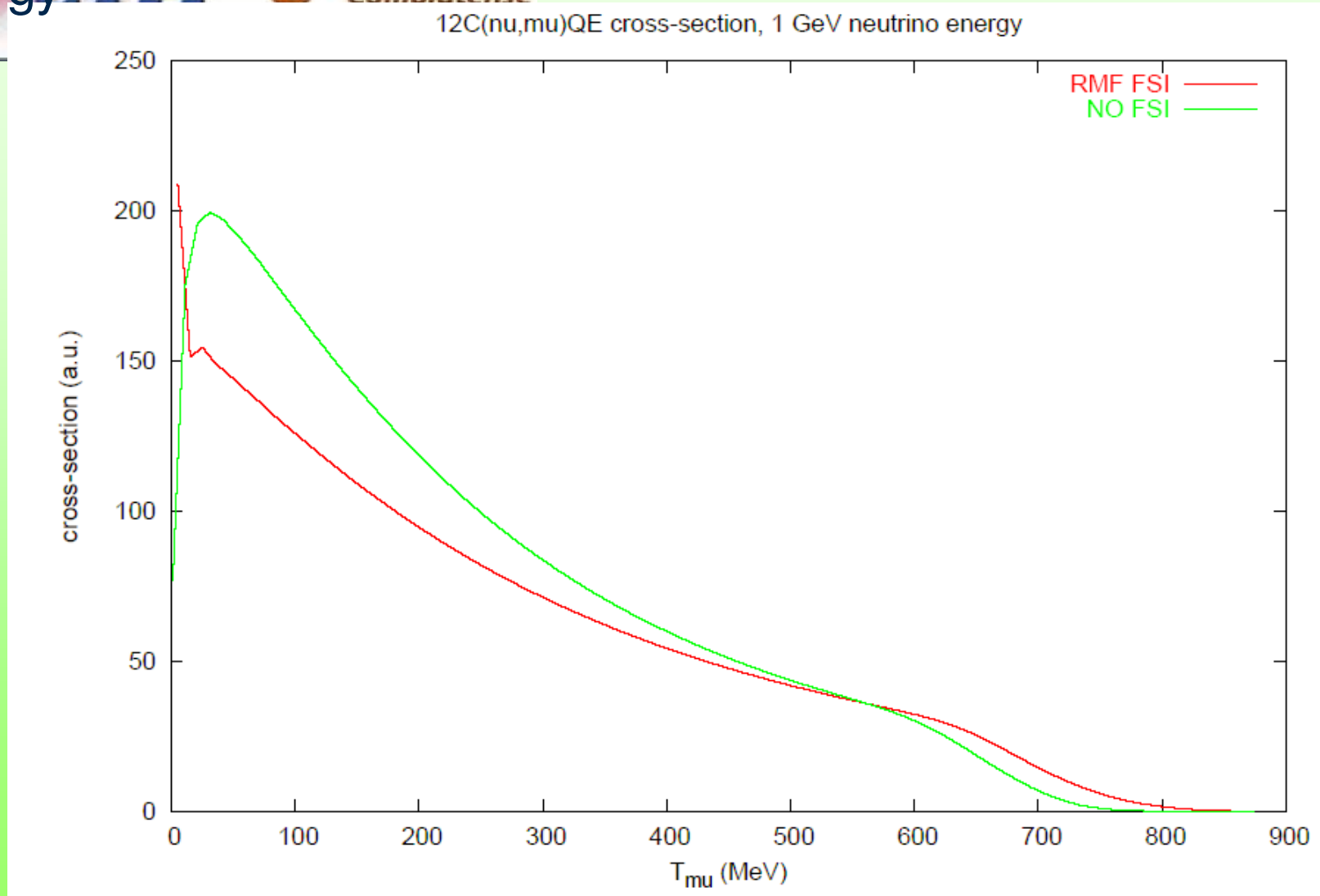
Y. Umino et al. PRC 52 (1995) 3399



Total CC predictions for ‘quasielastic’ charged current reactions ( $\nu, \mu^-$ ) obtained: a) without FSI interactions (red curve). With FSI interactions for  $^{12}\text{C}$  and  $^{56}\text{Fe}$  (dotted orange and long dashed blue lines, respectively). ‘Pure’ elastic contribution is shown by dot-dashed (green,  $^{12}\text{C}$ ) and long dotted (cyan,  $^{56}\text{Fe}$ ) curves. Data from several experiments and targets are also plotted. 10% effect of FSI can be observed, even at 5 GeV



RMF calculations, that agree with (e,e') data at intermediate energies, show that FSI has an effect even for 1 GeV neutrino energy



# Conclusions



- The relativistic formalism, based upon the Dirac equation with local (E-independent) scalar and vector potentials, may incorporate in an *effective* way correlations
- For inclusive observables, for which the kinematics and phase space are smoothed it seems that the impulse approximation and the relativistic mean field, in spite of its simplicity, do a very decent job
- The asymmetric tail of the nucleonic contribution to  $(e,e')$  within RMF, is due to the increased strength at high 'asymptotic' momentum of the initial nucleon, caused by strong FSI
- One can, in principle, also obtain asymmetry (or tail) of the  $\gamma$ -scaling response by incorporating high-momentum tails into the initial nucleon momentum distributions, and ISI effect
- FSI or ISI effects can only be disentangled within a model
- Neutrino reactions from few 100's MeV to several GeV, where the RMF shows good agreement with  $(e,e')$  data, can be predicted. FSI effects in the cross-sections are not negligible