Spectral-Fluctuations Test of the Quark-Model Baryon Spectrum

César Fernández-Ramírez

Instituto de Estructura de la Materia, CSIC
Madrid, Spain

Baryons: Historical Perspective

- $\Delta(1232)$

  Anderson, Fermi, Long, Nagle, Phys. Rev. 85 (1952) 936

- Proliferation of baryons
- Non-relativistic quark models

  Gell-Mann, Zweig, Greenberg, Dalitz, Karl, Koniuk, Isgur, (60's and 70's)

- Relativistic quark models

  Capstick, Isgur, PRD 34 (1986) 2809

  Bonn model, EPJA 10 (2001) 309; 395; 447

- Effective QCD-inspired models

  Page, Swanson, Szczepaniak, PRD 59 (1999) 034016

  Llanes-Estrada, Cotanch, PLB 504 (2001) 15; NPA 697 (2002) 303

- Lattice QCD

  Bernard et al., PRD 64 (2001) 054506

The Problem of Missing Resonances

- Experiments vs quark models $\Longrightarrow$ missing states
- Experimental effort

  v.g. at JLab (Halls B & C)
- Lattice QCD and effective QCD-inspired models
- We apply spectral statistics techniques to test quark models and survey the problem of missing resonances

The Low-Lying Baryon Spectrum

EXP: Yao et al., JPG 33 (2006) 1

CI: Capstick, Isgur, PRD 34 (1986) 2809

L1 & L2: Löring et al., EPJA 10 (2001) 309; 395; 447

Classical Chaos

- Study of trajectories
  - $\longrightarrow$ Integrable systems
    - Predictable
    - Trajectories restricted to a manifold within the phase space
  - $\longrightarrow$ Chaotic systems
    - Unpredictable
    - Chaoticity: Trajectories fill in the phase space

Quantum Chaos

- It is not possible to define trajectories
- Study of spectra $\rightarrow$ Fluctuations
- Spectra can be split in a smooth and a fluctuating part
  $\rho(E) = \rho_s(E) + \rho_f(E)$
- Unfolding removes the smooth part
- Universality of fluctuations in chaotic and integrable systems
- Short distance correlations

Spectral Statistics

- It studies how an ordered sequence of numbers (v.g. an energy spectrum) matches an statistical theory
- Two kinds of statistics
  - Nearest neighbors
  - Long distance correlations
- Applied to study the spectrum of many-body systems
- Statistical methods are a powerful tool to study the energy spectrum of quantum systems
- Methods are improved over the last years: Analysis of systems with low number of levels are presently reliable and problems such as the hadron spectrum can be faced

Spectral Fluctuations

- Spectra can be split in a smooth and a fluctuating part $\rho(E) = \rho_s(E) + \rho_f(E)$
- Universality of fluctuations in chaotic and integrable systems
- This allows to consider the system as a black-box without considering the underlaying interaction
- Fluctuations are extracted from the spectrum through an unfolding procedure
- Nearest Neighbors Spacings Distribution is the most utilized

  $\rightarrow$ Distance between two consecutive levels with the same symmetries (quantum numbers)

Integrability and Chaoticity in Quantum Systems

- Statistical properties of the energy-level fluctuations are universal and determine whether a system is chaotic or integrable
- Integrable and chaotic systems display different fluctuation pattern
- The sequence of spacings $\{s_i\}$ for an integrable system can be considered as a sequence of independent random variables
- Integrable systems display a non-correlated sequence of levels
- Chaotic systems are characterized by a correlation structure described by RMT (standard set)
- Paradigms of integrable and chaotic systems

$\longrightarrow$ Quantum billiards

Fluctuations and Integrable Spectra

→ Integrable systems (uncorrelated)

- Fluctuations follow Poisson distribution

Berry, Tabor, Proc. R. Soc. London A 356 (1977) 375

- Uncorrelated systems
- Example: Random noise
- Example: Harmonic oscillator (Mean field)

Fluctuations and Chaotic Spectra

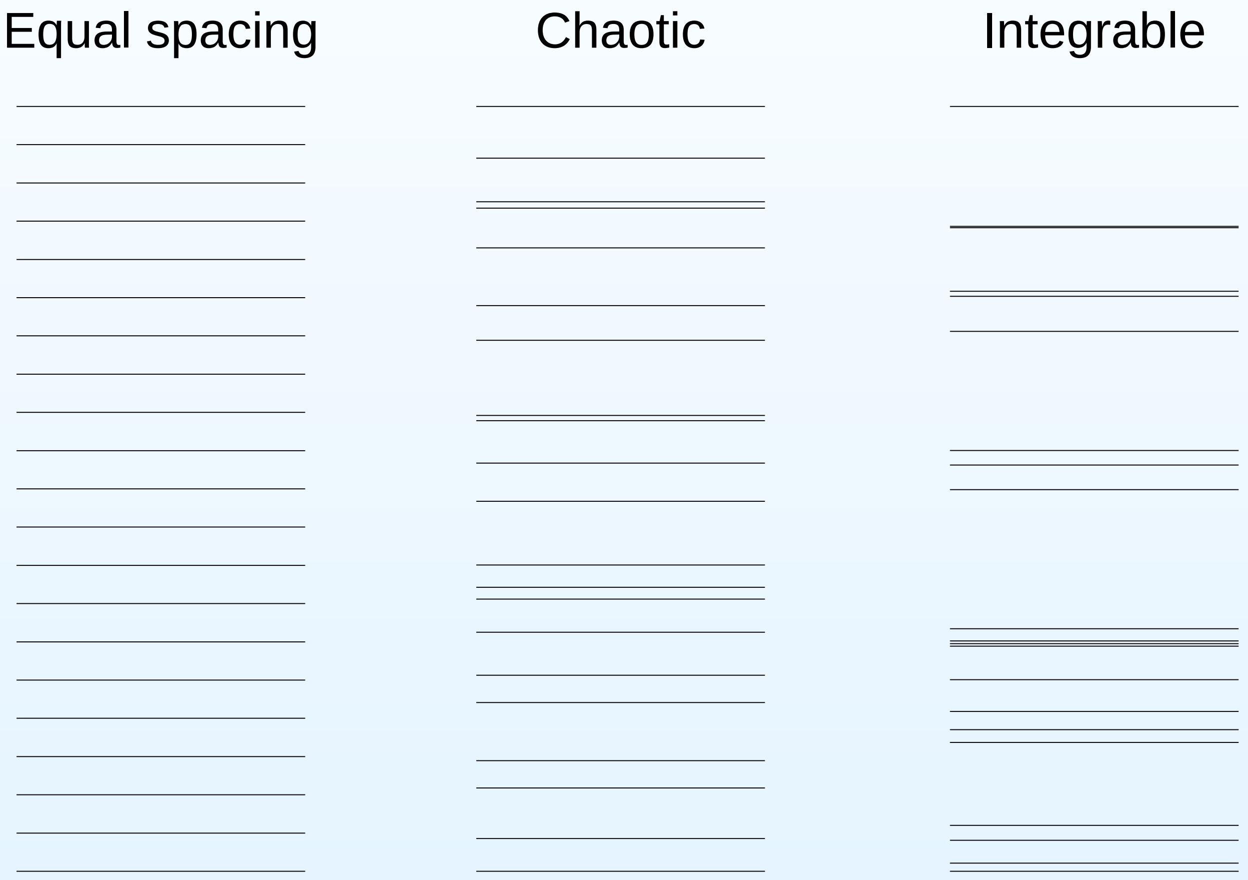
$\longrightarrow$ Chaotic systems (correlated)

- Flutuations follow Wigner surmise

Bohigas, Giannoni, Schmit, PRL 52 (1984) 1

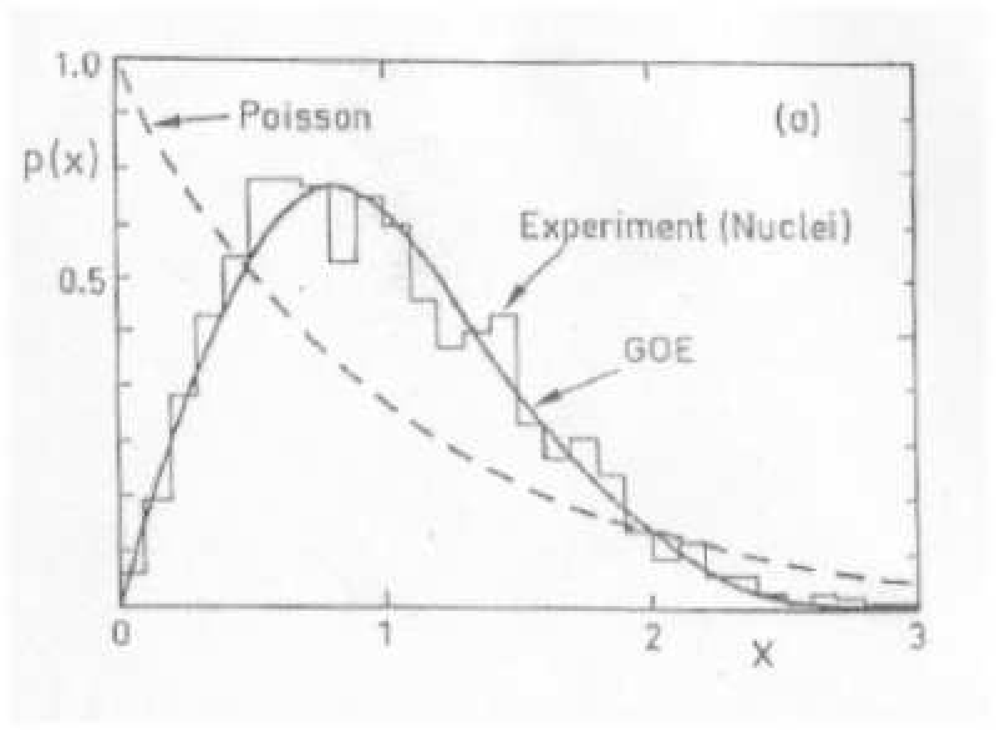
- Example: Nuclei
- Standard for chaotic systems: Random Matrix Theory (useful for statistical studies)

Examples of Integrable and Chaotic Spectra



Spectral Fluctuations and Nuclei

Experimental spectrum in nuclei follows RMT (Wigner surmise)



Bohigas, NPA 751 (2005) 343c

Symmetries in the Baryon Spectrum

- I isospin
- J spin
- π parity
- We drop strangeness due to $SU(3)$ invariance
- From the full spectrum we extract sequences of levels with given $I\,(J^\pi)$

Unfolding Procedure

- $\rho(E) = \rho_s(E) + \rho_f(E)$
- Unfolding allows to extract the fluctuating part from the level density
- We choose the simplest unfolding prescription
- $S_i = E_{i+1} - E_i$
- We rescale using its average value

$$s_i = S_i / \langle S \rangle$$

- Nearest Neighbor Spacings (NNS)
- This procedure assumes an energy independent behavior of the smooth part of the density $\rho_s(E) = 1/\langle S \rangle$

Pascalutsa, EPJA 16 (2003) 149

Nearest Neighbor Spacing Distribution (NNSD)

- Poisson: integrable / uncorrelated

$$P(s) = \exp(-s)$$

- Wigner: chaotic / correlated

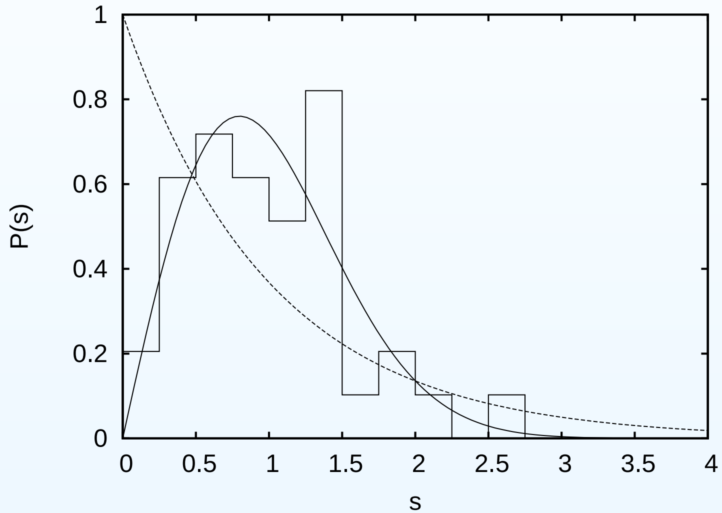
$$P(s) = \frac{\pi s}{2} \exp\left(-\frac{\pi s^2}{4}\right)$$

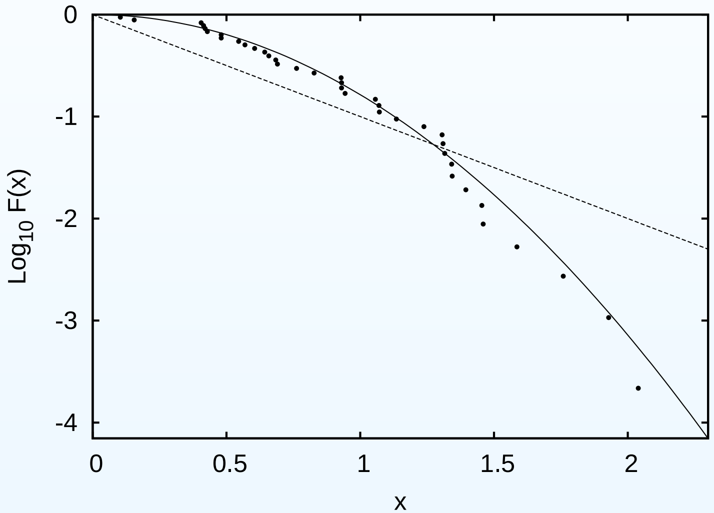
- Accumulated NNSD,

$$F(x) = 1 - \int_0^x ds\, P(s),$$

allows a better study of the tail of the distribution

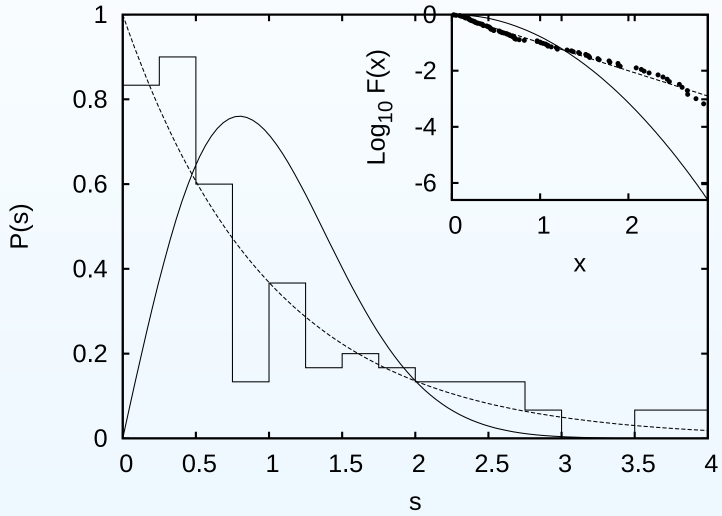
Experimental Baryon Spectrum up to 2.2 GeV

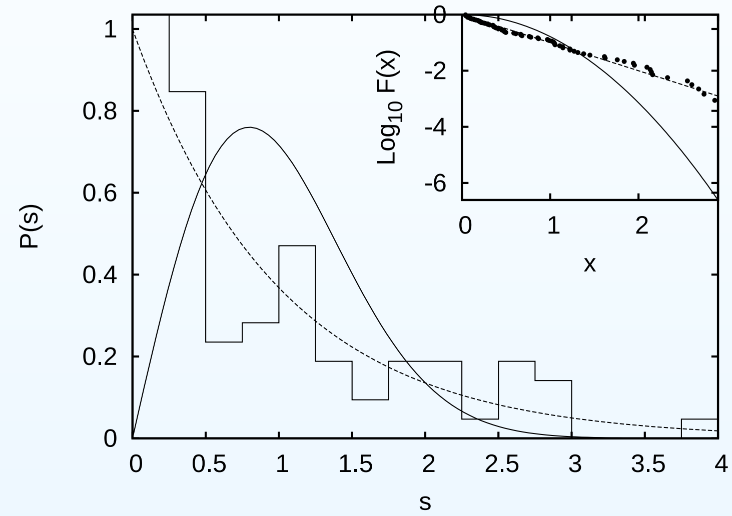


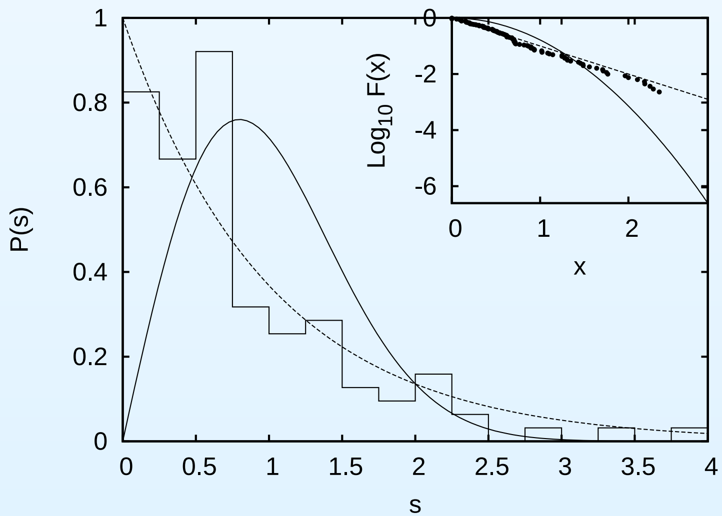


70 energy levels distributed in 15 sequences

Relativistic Quark Models







Models by

Löring *et al.*, (sets L1 & L2)

EPJA 10 (2001) 309; 395; 447

Capstick and Isgur, (set CI)

PRD 34 (1986) 2809

Direct Comparison by means of a Goodness-of-Fit Test

Wilcoxon Rank-Sum Test

Wilcoxon, Biometrics Bull. 1 (1945) 80

Mann, Whitney, Ann. Math. Stat. 18 (1947) 50

	CI	L1	L2
EXP	0.0487	0.1067	0.1036

Allows to test whether two populations of different size are statistically alike.

Unfolding

- Sometimes, very short sequences of levels
- In such cases, unfolding can provide misleading results, making spacings spuriously closer and bringing the NNSD tend to the Wigner surmise
- Unfolding can yield different effects in different spectra: We avoid a direct comparison of the spectral fluctuations
- Complementary analysis: Kolmogorov-Smirnov goodness-of-fit tests

Kolmogorov-Smirnov Goodness-of-Fit Test

- To determine whether two datasets differ significantly
- No assumption about the distribution of data (non-parametric and distribution free)
- Based on the maximum distance between cumulative probabilities

Kolmogorov, Giornale dell'Istituto Italiano degli Attuari 4 (1933) 83
Smirnov, Bull. Moscow Univ. 2 (1933) 3; Ann. Math. Stat. 19 (1948) 279
Feller, Ann. Math. Stat. 19 (1948) 177
NAG Libraries, http://www.nag.co.uk

Applied Procedure

Applied Procedure

- We build one Wigner-like and one Poisson-like reference spectra, optimized to study set EXP

Applied Procedure

- We build one Wigner-like and one Poisson-like reference spectra, optimized to study set EXP

  For each sequence $\{s_i\}_X$ we have built two equivalent reference sequences $\{s_i'\}_X$ with the same length (Wigner-like and Poisson-like) as the experimental one

Applied Procedure

- We build one Wigner-like and one Poisson-like reference spectra, optimized to study set EXP

    For each sequence $\{s_i\}_X$ we have built two equivalent reference sequences $\{s_i'\}_X$ with the same length (Wigner-like and Poisson-like) as the experimental one

    We unfold each reference sequence independently and calculate the NNSD for each set

Applied Procedure

- We build one Wigner-like and one Poisson-like reference spectra, optimized to study set EXP
  - For each sequence $\{s_i\}_X$ we have built two equivalent reference sequences $\{s_i'\}_X$ with the same length (Wigner-like and Poisson-like) as the experimental one
  - We unfold each reference sequence independently and calculate the NNSD for each set
  - We run the goodness-of-fit tests

Applied Procedure

- We build one Wigner-like and one Poisson-like reference spectra, optimized to study set EXP
  - For each sequence $\{s_i\}_X$ we have built two equivalent reference sequences $\{s'_i\}_X$ with the same length (Wigner-like and Poisson-like) as the experimental one
  - We unfold each reference sequence independently and calculate the NNSD for each set
  - We run the goodness-of-fit tests
- We proceed in the same way for CI, L1, and L2 sets

Applied Procedure

- We build one Wigner-like and one Poisson-like reference spectra, optimized to study set EXP
  - For each sequence $\{s_i\}_X$ we have built two equivalent reference sequences $\{s_i'\}_X$ with the same length (Wigner-like and Poisson-like) as the experimental one
  - We unfold each reference sequence independently and calculate the NNSD for each set
  - We run the goodness-of-fit tests
- We proceed in the same way for CI, L1, and L2 sets
- Each reference spectra is distorted by the unfolding in the same way as sets EXP, CI, L1, and L2 are

Kolmogorov-Smirnov Test

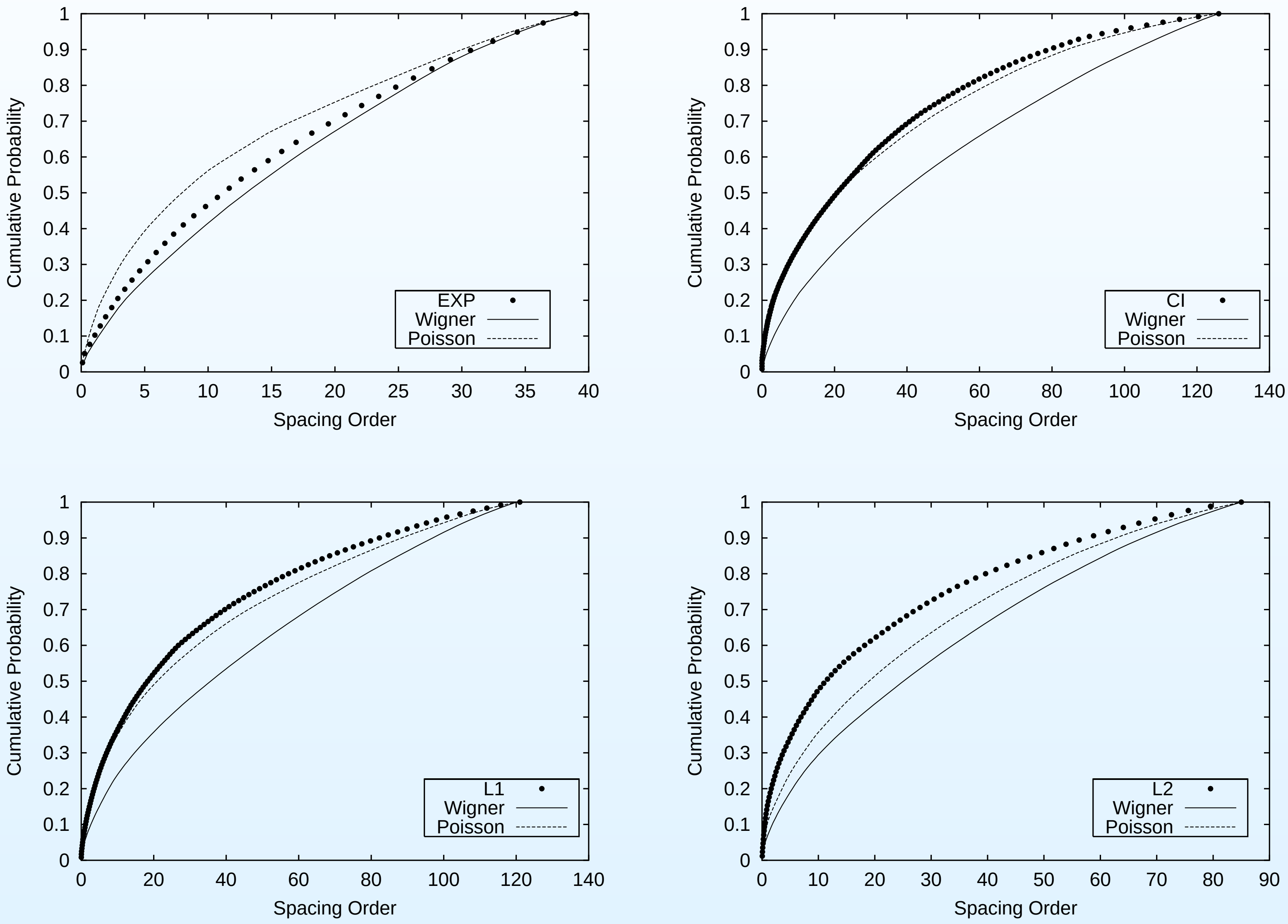
Probability to obtain, under the null hypothesis, a value of the Kolmogorov-Smirnov test statistic as the one observed

Spectrum	EXP	CI	L1	L2
Poisson	0.51	0.49	0.25	0.53
Wigner	0.80	0.18	0.05	0.01

Null hypothesis: Both distributions display equal spectral fluctuations

We study 500 realizations to have enough statistics

Cumulative Probabilities: The K-S Test in more Detail

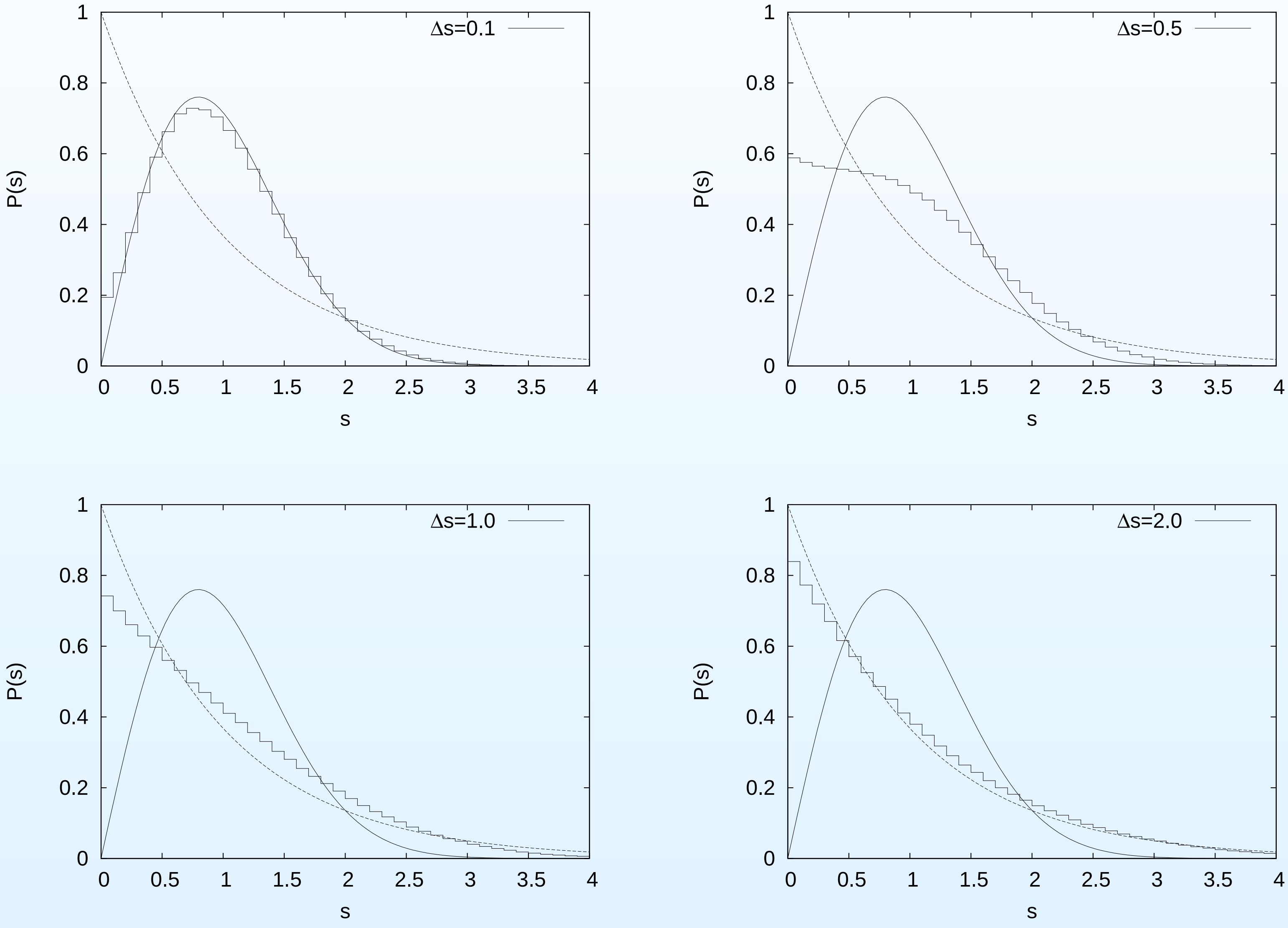


Deviations from Wigner Surmise

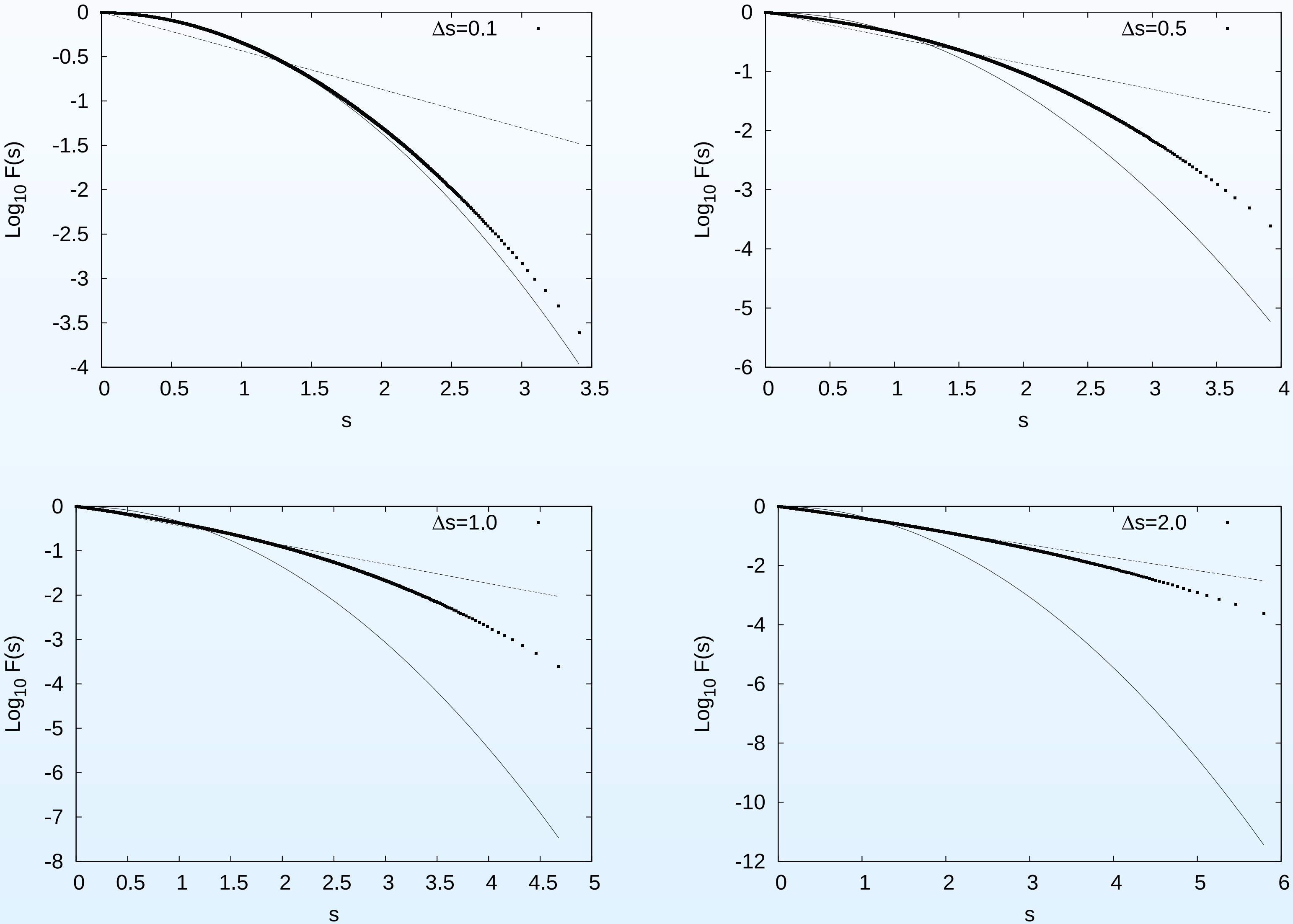
Due to

- Uncertainties in the masses (error bars)
  $\longrightarrow$ Errors mean random noise, which brings the NNSD closer to a Poisson distribution
- Existence of missing states
  $\longrightarrow$ Missing resonances

Error Bars (Toy Model Simulation): NNSD



Error Bars (Toy Model Simulation): Accumulated NNSD



Missing Levels in a Spectrum

Statistical tools allow to identify the existence of missing states

Bohigas, Pato, PLB 595 (2004) 25; PRE 74 (2006) 036212

Molina, Retamosa, Muñoz, Relaño, Faleiro, PLB 644 (2007) 25

Missing levels cause the spectral fluctuations of a spectrum with Wigner distribution look more like a Poisson distribution

We can use this property to identify missing levels in a spectrum

Estimation of Missing Levels

If we assume that the *real* distribution is 100% Wigner we can speculate on the maximum amount of missing states

- We remove levels randomly from a Wigner distribution until we get values for the K-S test closer to what is observed
- Very rough estimation: <20% of missing levels

Conclusions (I)

Conclusions (I)

- The statistical techniques developed to study the chaotic character of quantum systems have evolved into powerful and reliable techniques that can provide new insight in hadron physics

Conclusions (I)

- The statistical techniques developed to study the chaotic character of quantum systems have evolved into powerful and reliable techniques that can provide new insight in hadron physics
- From the spectral fluctuations of the *experimental* baryon spectrum one can conclude the importance of correlations in the underlying physics

Conclusions (I)

- The statistical techniques developed to study the chaotic character of quantum systems have evolved into powerful and reliable techniques that can provide new insight in hadron physics
- From the spectral fluctuations of the *experimental* baryon spectrum one can conclude the importance of correlations in the underlying physics
- From the analysis of *theoretical* spectra from constituent quark models, one can conclude that, as presently built, they do <u>not</u> describe the basic statistical properties of the low-lying baryon spectrum and they need to include more correlations

Conclusions (II)

- This result is model-independent

Conclusions (II)

- This result is model-independent
- There is room for missing resonances at the 20% level

Conclusions (II)

- This result is model-independent
- There is room for missing resonances at the 20% level
- Predictions of missing states derived from constituent quark models are not reliable

Conclusions (II)

- This result is model-independent
- There is room for missing resonances at the 20% level
- Predictions of missing states derived from constituent quark models are not reliable

F-R, Relaño, PRL 98 (2007) 062001

What's Next?

- Extend this analysis to other models: Lattice QCD and Effective QCD-inspired models
- Improve quark models to account for fluctuations properties (re-thinking the interaction?)
- Test the universality of fluctuations
- Further experimental research on baryon states
- Study the spectral properties of other quantities beyond energy levels (helicity amplitudes, decay widths), both from baryon models as well as from experimental data

Acknowledgments

- Co-author: Armando Relaño (IEM-CSIC, Spain)
- Thanks to: F.J. Llanes-Estrada, E. Moya de Guerra, J. Retamosa, J.M. Udías (UCM, Spain), R.A. Molina (IEM-CSIC, Spain), and T. van Cauteren (U. Ghent, Belgium)

Work partly funded by Ministerio de Educación y Ciencia,

Universidad Complutense de Madrid and CSIC (Spain)