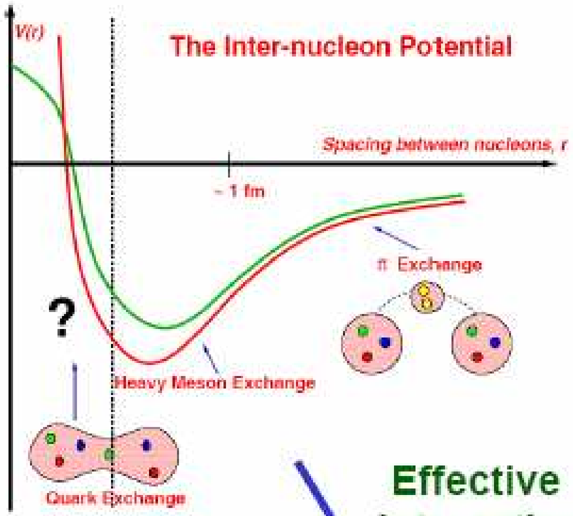
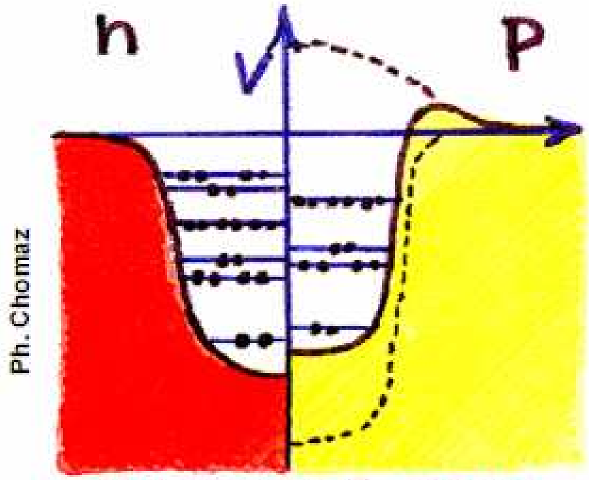
A survey of the relativistic mean field approach

B. D. Serot and J. D. Walecka, The relativistic nuclear many body problem. Adv. Nuc. Phys., 16:1, 1986.

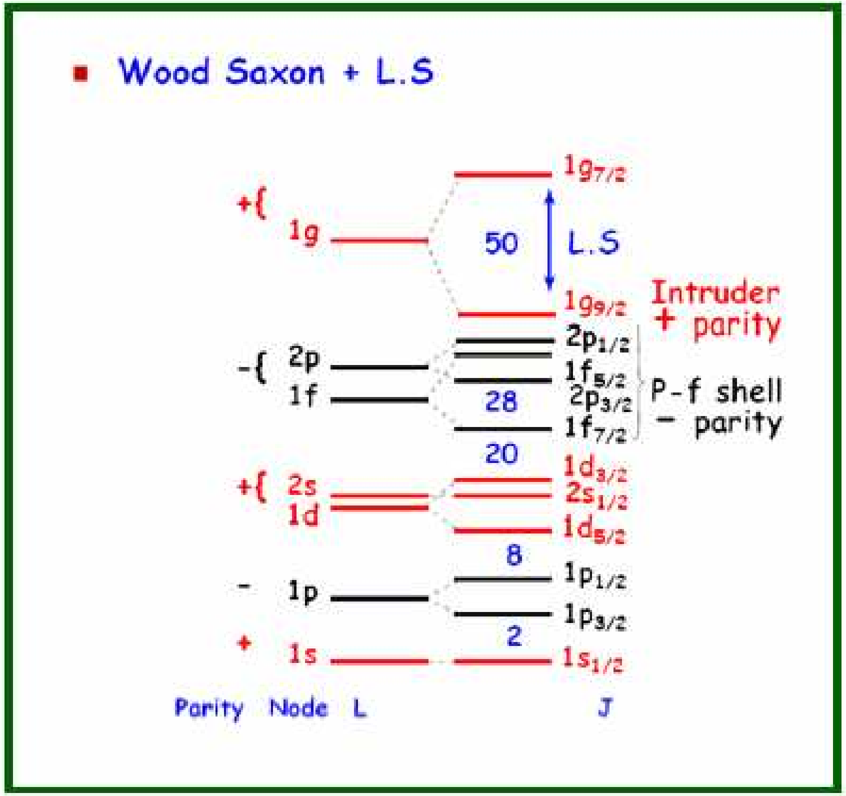
Mean Field Model of Nuclei



Effective interaction



Nucleons in a self-consistent mean-field



- fermion system at low energies
- suppression of collisions by Pauli exclusion
- independent particle motion
- shell structure
- mean field approximation

Non relativistic mean field

•Small potentials (a few tents of MeV), of the order of the binding energy in nuclei

•Separate Central + Spin-Orbit potential

•Importance of Fock (exchange) terms and correlations beyond the mean field

(In non-relativistic models the saturation arises from the interplay between a long range attraction and a short range repulsion, so strong that it **is indispensable to take short range correlations** into account)

Relativistic mean field

•What is the role that relativity plays in nuclear systems?

• The ratio of the Fermi momentum over the nucleon mass is about k_F/M = 0.25. Nucleons move with at most about 1/4 of the velocity of light. Only moderate corrections from relativistic kinematics are expected

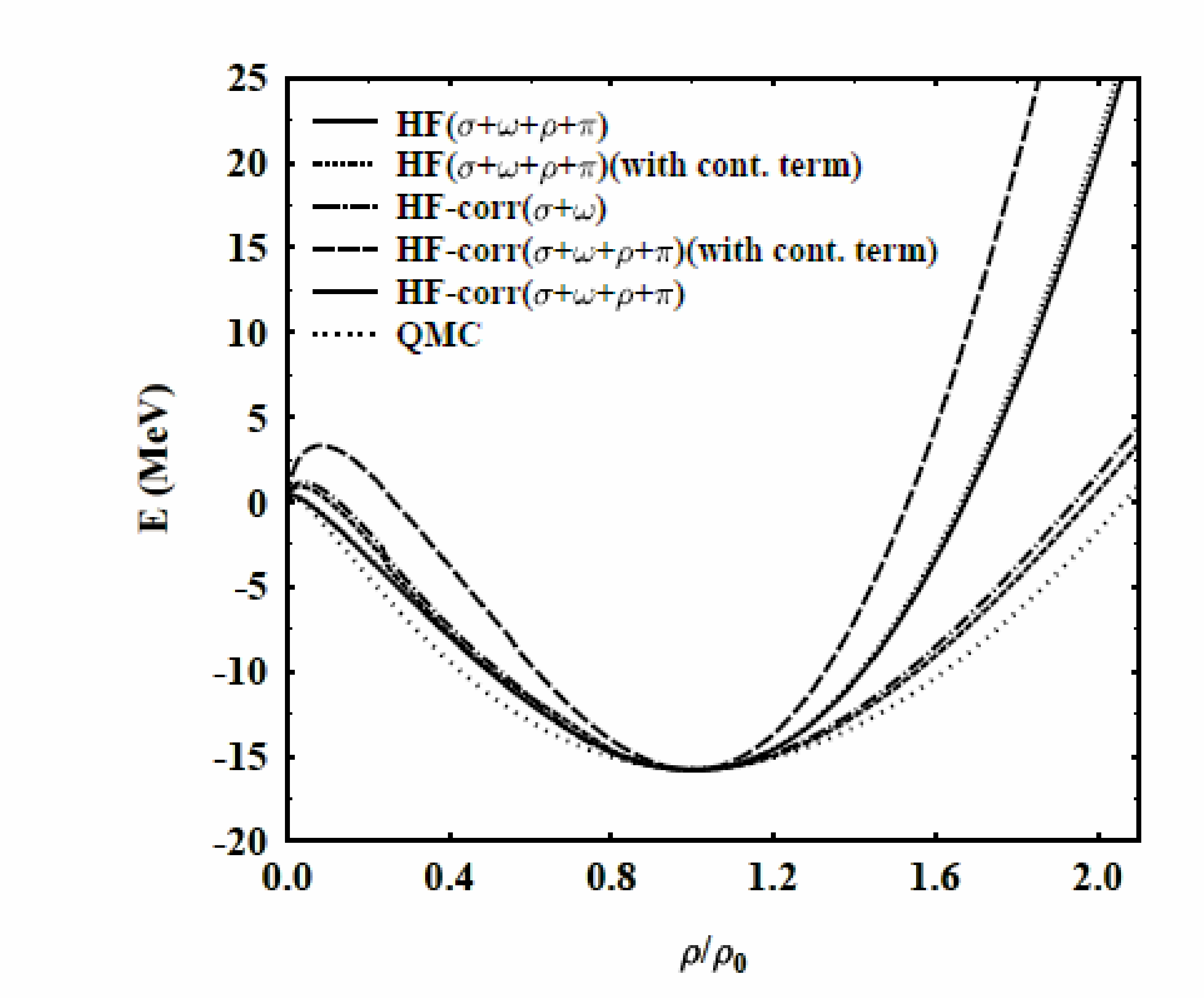
•Strong potentials (a few hundreths of MeV's). Small binding energy is just the 'tip' of the iceberg

•Spin-Orbit potential implicit in the relativistic formalism

•However, there exists a fundamental difference between relativistic and non-relativistic dynamics: a genuine feature of relativistic nuclear dynamics is the appearance of large scalar and vector mean fields, each of a magnitude of several hundreds MeV. The scalar field S is attractive and the vector field V is repulsive

• In relativistic mean field (RMF) theory, both sign and size of the fields are enforced by the nuclear saturation mechanism

- In relativistic mean field models, the parameters are phenomenologically fitted to the saturation properties of nuclear matter
- Short range correlations are not needed to get the right saturation properties
- In this approach short range correlation effects may be accounted for, to some extent, by the model already at mean field level
- Formally, the scalar and vector potentials are usually implemented via (scalar and vector) meson exchanges



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Formally, one can build a field theory. It can seem very convincing...

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\left(\gamma_\mu\left(\mathrm{i}\partial^\mu - g_\omega\omega^\mu - g_\rho\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu\right) - \left(M + g_\sigma\varphi\right)\right)\psi \\ &+ \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m_\sigma^2\varphi^2 - \frac{1}{3}g_2\varphi^3 - \frac{1}{4}g_3\varphi^4 \\ &- \frac{1}{4}\omega_{\mu\nu}\cdot\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2 \\ &- \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu\end{aligned}$$

$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial_\mu\phi}\right) - \frac{\partial\mathcal{L}}{\partial\phi} = 0.$$

$$\left(\Box + m_\omega^2\right)\omega^\mu = g_\omega\bar{\psi}\gamma^\mu\psi - c_3\left(\omega_\mu\omega^\mu\right)\omega^\mu$$

$$\left(\Box + m_\sigma^2\right)\varphi = -g_s\bar{\psi}\psi - g_2\varphi^2 - g_3\varphi^3$$

$$\gamma_\mu\left(\mathrm{i}\partial^\mu + g_\omega\omega^\mu + g_\rho\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu + \left(M + g_s\varphi\right)\right)\psi = 0.$$

Field theories are difficult to solve. But nuclear systems are dense ones, one can neglect fluctuations and use a semiclassical approximation: Substitute *source-current* terms by their expectation values:

$$
\begin{aligned}
\bar{\psi}\psi &\rightarrow \langle \bar{\psi}\psi \rangle \\
\bar{\psi}\gamma^{\mu}\psi &\rightarrow \langle \bar{\psi}\gamma^{\mu}\psi \rangle \\
\bar{\psi}\boldsymbol{\tau}\gamma^{\mu}\psi &\rightarrow \langle \bar{\psi}\boldsymbol{\tau}\gamma^{\mu}\psi \rangle
\end{aligned}
$$

Also substitute *meson fields* by their expectation values:

$$
\begin{aligned}
\varphi &\rightarrow \langle \varphi \rangle \\
\omega^{\mu} &\rightarrow \langle \omega^{\mu} \rangle \\
\boldsymbol{\rho}^{\mu} &\rightarrow \langle \boldsymbol{\rho}^{\mu} \rangle .
\end{aligned}
$$

$$
\begin{aligned}
\varphi_0 \equiv \langle \varphi \rangle &= -\frac{g_s}{m_\sigma^2} \left\langle \bar{\psi}\psi \right\rangle - \frac{1}{m_\sigma^2} \left(g_2 \varphi_0^2 + g_3 \varphi_0^3 \right) \\
\omega_0 \equiv \left\langle \omega^0 \right\rangle &= \frac{g_\omega}{m_\omega^2} \left\langle \bar{\psi}\gamma^0\psi \right\rangle - \frac{c_3}{m_\omega^2} \omega_0^3 \\
\rho_0 \equiv \left\langle \rho_3^0 \right\rangle &= \frac{g_\rho}{m_\rho^2} \left\langle \bar{\psi}\tau_3\gamma^0\psi \right\rangle .
\end{aligned}
$$

In the mean field style, one expands the multi-particle state into a product of single-particle states ψ_α. In the simplest approach (Hartree), the product is not antisymmetrized

$$
\begin{aligned}
\left\langle \bar{\psi}\psi \right\rangle &= \sum_\alpha a_\alpha \bar{\psi}_\alpha \psi_\alpha \\
\left\langle \bar{\psi}\gamma^0\psi \right\rangle &= \sum_\alpha a_\alpha \bar{\psi}_\alpha \gamma^0 \psi_\alpha \\
\left\langle \bar{\psi}\tau_3\gamma^0\psi \right\rangle &= \sum_\alpha a_\alpha \bar{\psi}_\alpha \tau_3 \gamma^0 \psi_\alpha .
\end{aligned}
$$

And the result is a Dirac equation for each single-nucleon state ψ_α

$$\left(-\mathrm{i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}+\beta\left(M+g_s\varphi_0\right)+g_\omega\omega_0+g_\rho\tau_3\rho_0\right)\psi_\alpha=E_\alpha\psi_\alpha.$$

And many different versions of the lagrangians have been cooked

	Wa [19]	HS [8]	NL1 [15]	NL-SH [22]	TM1 [23]	TM2 [23]
M (MeV)	938.0	939.0	938.0	939.0	938.0	938.0
m_σ (MeV)	550.00	492.0	492.250	526.059	511.198	526.443
m_ω (MeV)	783.00	783.0	795.359	783.0	783.0	783.0
m_ρ (MeV)	763.00	770.0	763.0	763.0	770.0	770.0
g_s	9.58289	10.47	10.1377	10.444	10.0289	11.4694
g_ω	11.683586	13.80	13.2846	12.945	12.6139	14.6377
g_ρ	0.0	8.07	4.9757	4.383	4.6322	4.6783
g_2 (fm^{-1})	0.0	0.0	-12.1724	-6.9099	-7.2325	-4.4440
g_3	0.0	0.0	-36.2646	-15.8337	0.6183	4.6076
c_3	0.0	0.0	0.0	0.0	71.3075	84.5318
E/A (MeV)	-15.70	-15.70	-16.40	-16.32	-18.56	-14.22
ρ_B (fm^{-1})	-0.193	0.148	0.152	0.146	0.146	0.111
M^*/M	0.55	0.54	0.573	0.597	0.66	0.618

Relativistic mean field RMF

•Use Dirac equation with local potentials, obtained with a lagrangian fitted to reproduce saturation properties of nuclear matter, and/or radii and mass of selected nuclei. Or use any phenomenological S-V potentials of Woods-Saxon kind

•RMF does saturate, even if no Fock terms are introduced (Dirac Hartree) and without considering correlations

•Generally speaking, introducing Fock terms or correlations shifts the saturation point, but a change of the parameters of the model puts the saturation point wherever we want it

•This is at serious variance with the nonrelativistic case. Are there any observables sensitive to these differences?

- At nuclear saturation density 0.16 fm^{-3}, the empirical fields deduced from fits to finite nuclei or nuclear matter are of the order of 300 to 500 MeV
- The single particle potential in which the nucleons move originates from the cancellation of the two contributions yielding around −50 MeV which makes it difficult to observe relativistic effects in nuclear systems
- There exist, however, several features in nuclear structure which can naturally be explained within Dirac phenomenology while models based on non-relativistic dynamics have difficulties:
- Best established is the large spin-orbit splitting in finite nuclei
- Also the so-called pseudo-spin symmetry, observed more than thirty years ago in single particle levels of spherical nuclei, can naturally be understood within RMF theory as a consequence of the coupling to the lower components of the Dirac equation
- QCD sum rules also suggest attractive scalar and repulsive vector self-energies which are astonishingly close to the empirical values derived from RMF fits to the nuclear chart
- Also relativistic many-body calculations [6, 7, 8] yield scalar/vector fields of the same sign and magnitude as obtained from RMF theory or, alternatively, from QCD sum rules

These facts suggest that preconditions for the existence of large fields in matter or, alternatively, the density dependence of the QCD condensates, must already be inherent in the vacuum nucleon-nucleon (NN) interaction

If this is true, then a simple, phenomenological RMF would be a very effective way of emulating the underlying microscopic relativistic strongly interacting theory

$$(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta(M+U_s)-E+U_\omega)\psi(\mathbf{x})=0.$$

Local potentials?

$$\psi=\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix}.$$

$$\left[\frac{1}{2m}\vec{\mathrm{p}}^{\,2}+U_e(r;E)+U_{so}(r)\frac{\vec{\sigma}\cdot\vec{\mathrm{L}}}{r}\right]\psi_>(\vec{\mathrm{r}})=\frac{1}{2m}(E^2-m^2)\psi_>(\vec{\mathrm{r}}),$$

$$U_{so}(r)\approx(2m)^{-2}\frac{d}{dr}[U_0(r)-U_s(r)],$$

$$U_e(r;E)\approx U_s(r)+Em^{-1}U_0(r)+(2m)^{-1}[U_s^{\,2}(r)-U_0^{\,2}(r)].$$

Relativistic mean field RMF (II)

•By chosing the parameters of the lagrangians to reproduce the saturation point at the mean field approximation, the effects of correlations on the saturation curve has been taken into account, at least partly

•The relativistic mean field, being able of incorporating at the same time repulsive and attractive effects, via vector and scalar potentials, should be more succesful in emulating correlations

• Nonlocalities (dependences of the potentials or the effective mass on the density, the energy or the position) as well as other effects introduced by correlations or even Fock terms, will be recovered from the relativistic formalisms when performing the nonrelativistic reduction, even if the relativistic equations and potentials are local

•Of course, RMF also incorporates relativistic effects!!