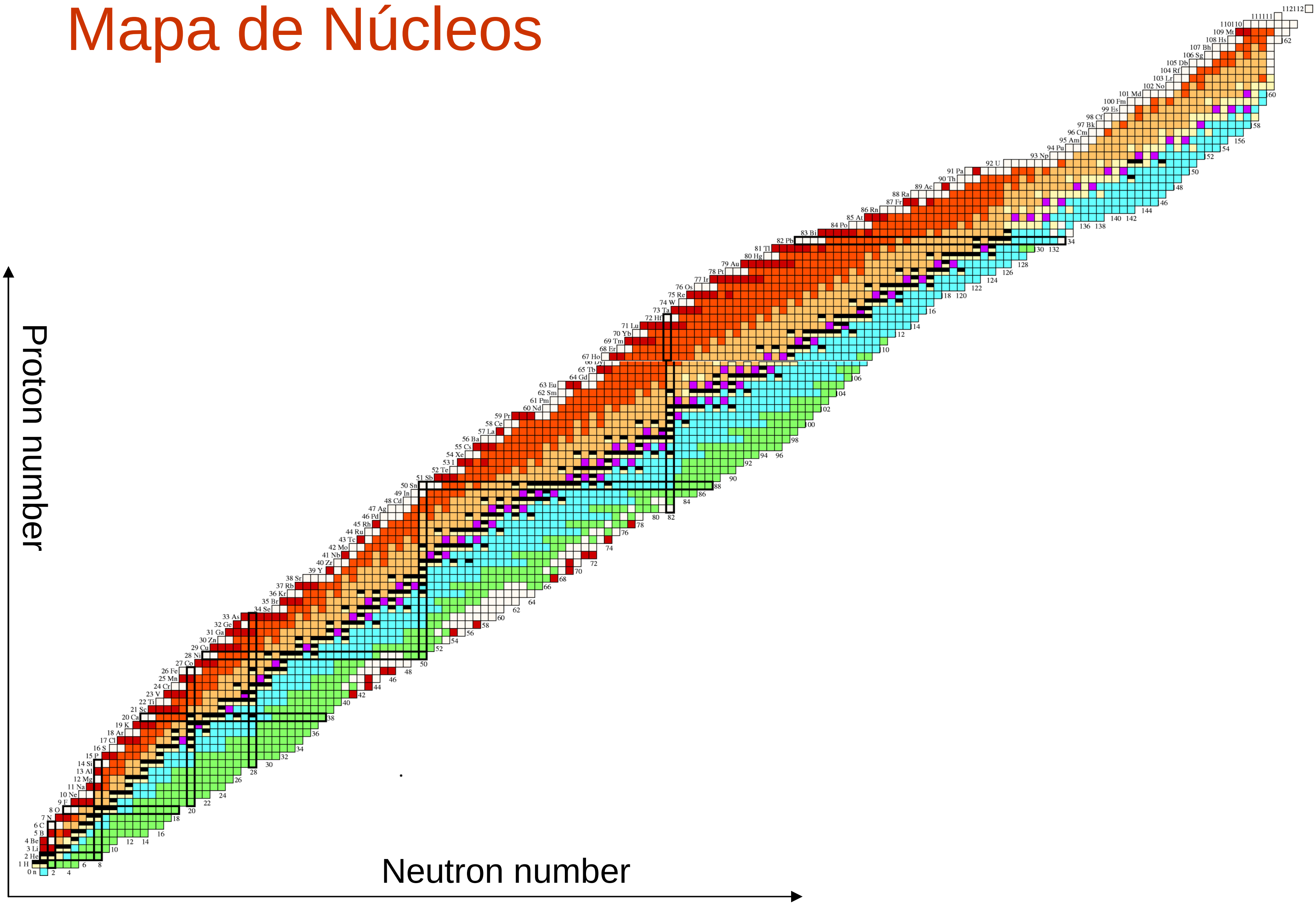
CSIC CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

Eduardo Garrido
Instituto de Estructura de la Materia
CSIC

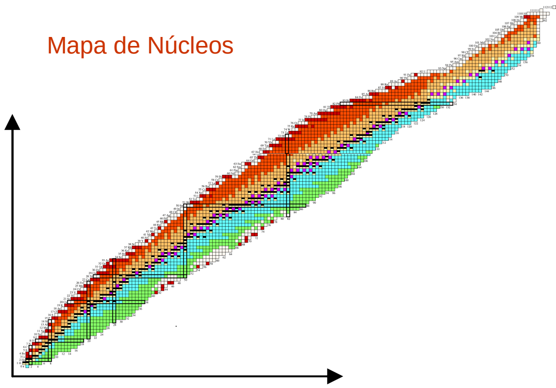
14 de Enero de 2010

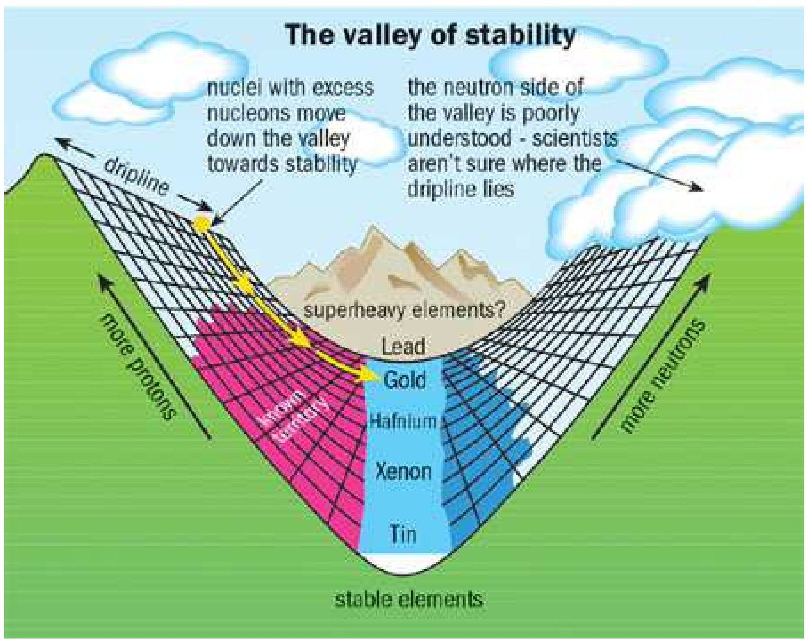
Mapa de Núcleos

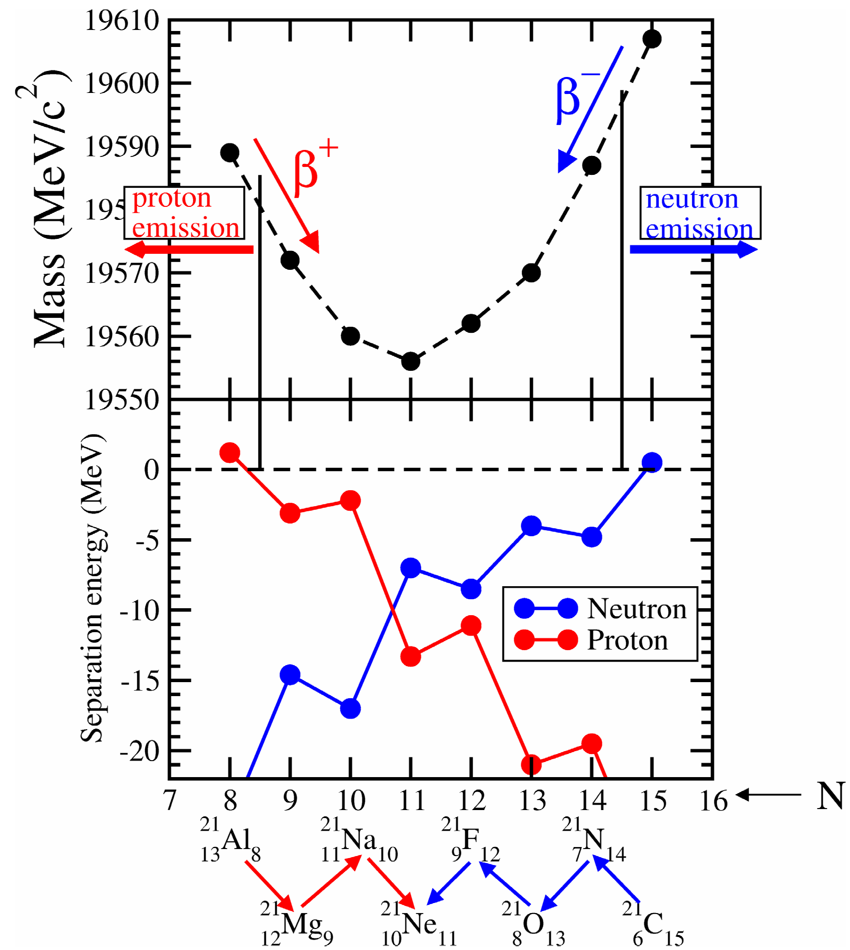


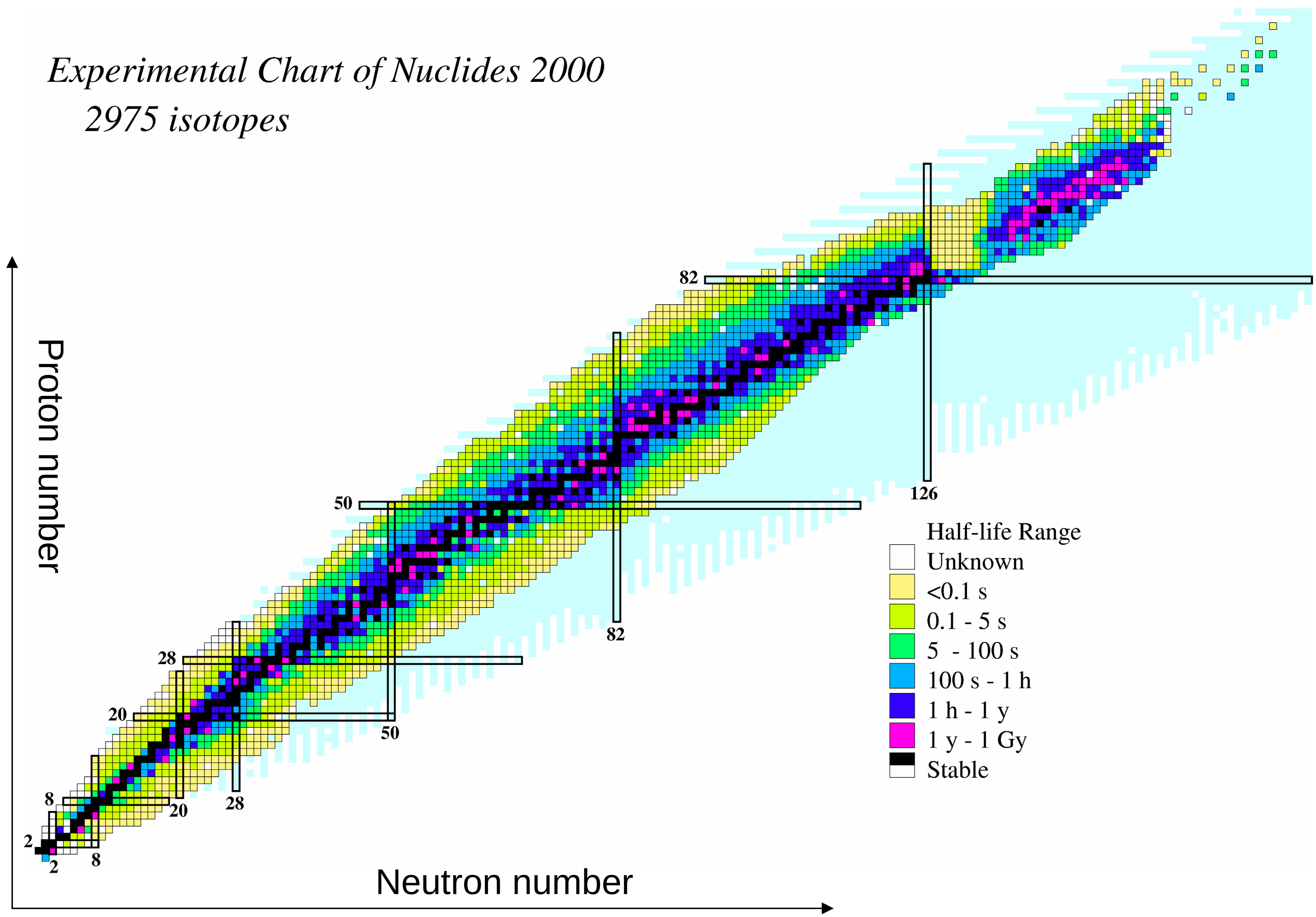
El valle de estabilidad

Mapa de Núcleos



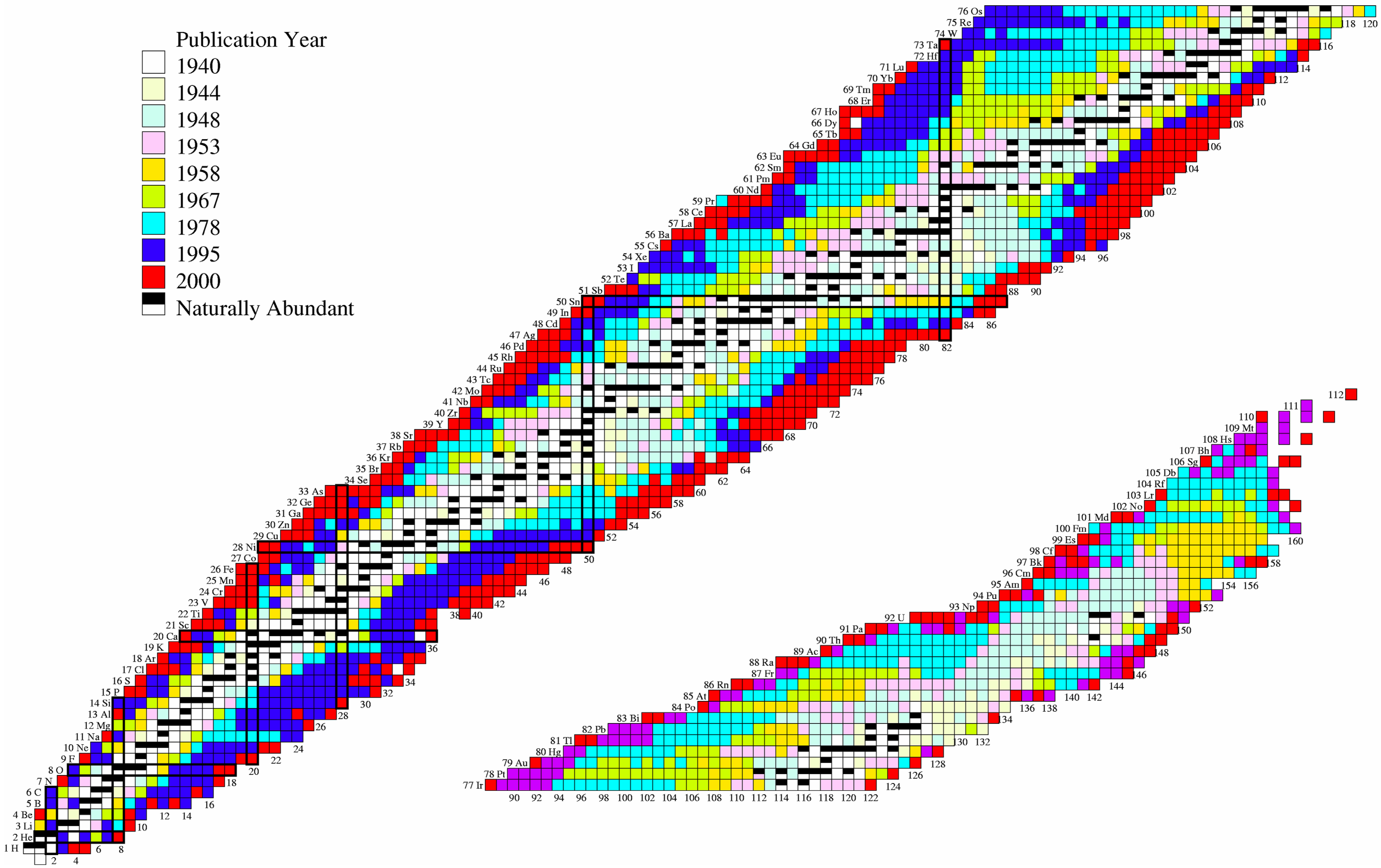




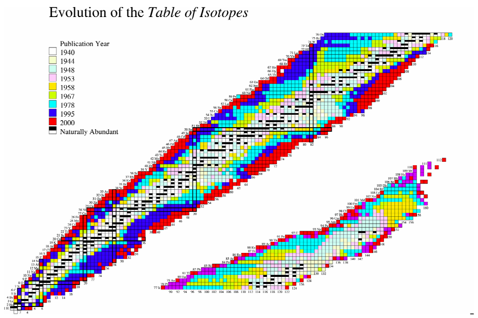


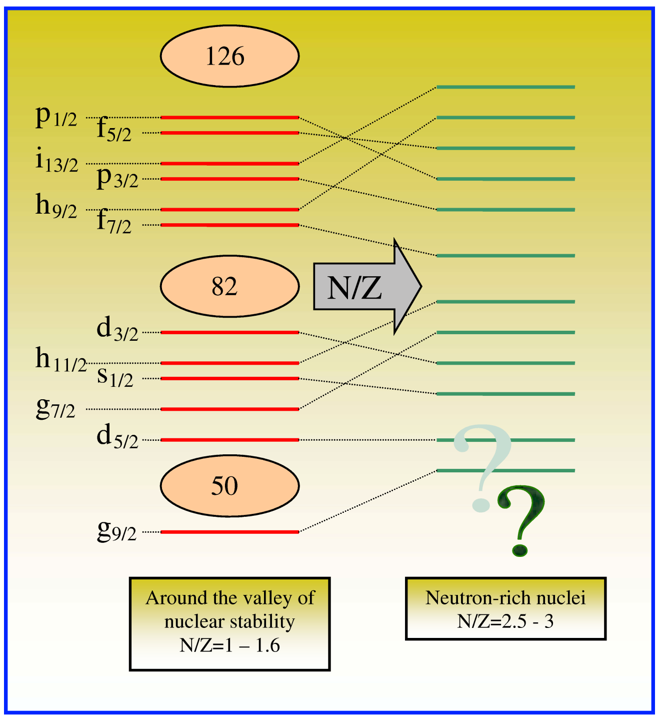
Experimental Chart of Nuclides 2000
2975 isotopes
Proton number
Neutron number
Half-life Range
Unknown
<0.1 s
0.1 - 5 s
5 - 100 s
100 s - 1 h
1 h - 1 y
1 y - 1 Gy
Stable
2
2
8
8
20
20
28
28
50
50
82
82
126

Evolution of the *Table of Isotopes*

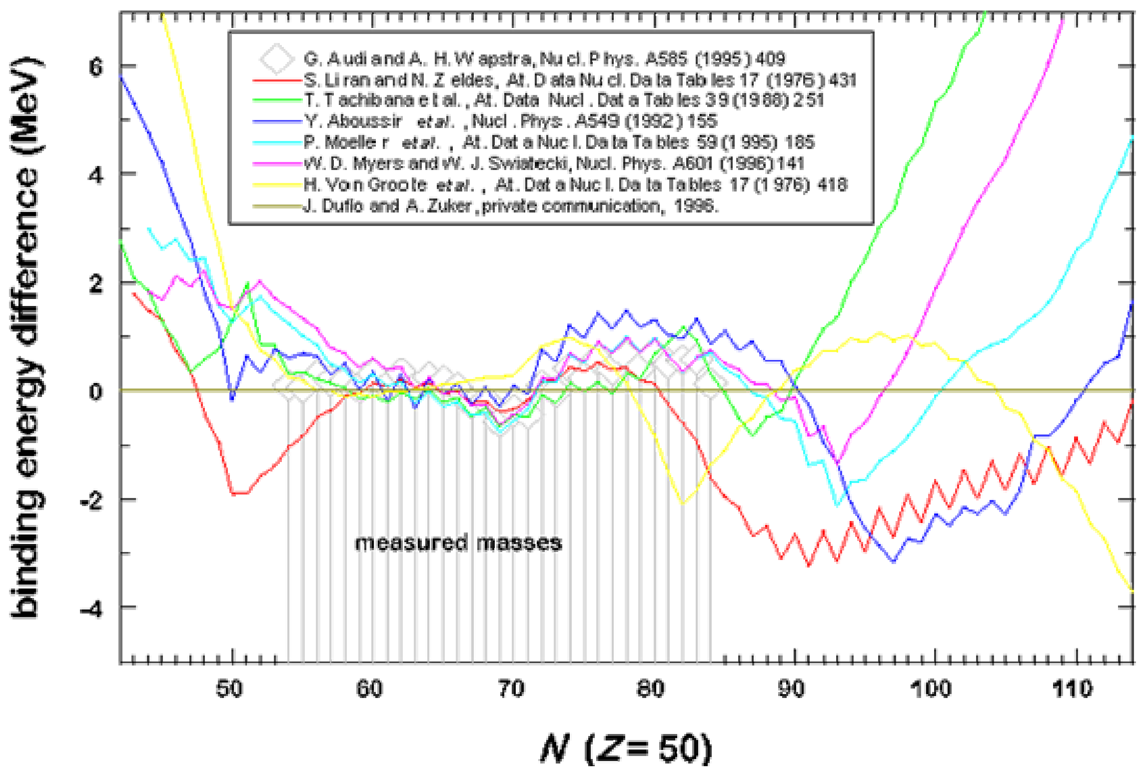


¿¿Nuevos fenómenos en las proximidades de las “driplines”??





¿Validez del modelo de capas?



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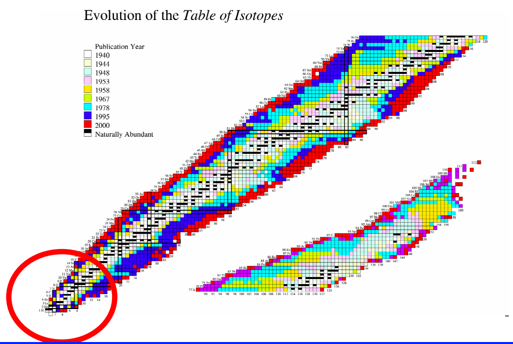

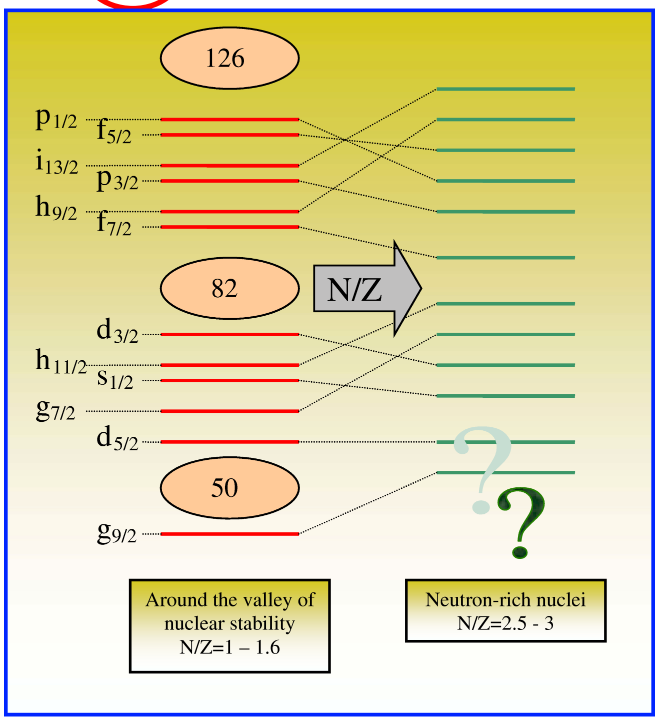

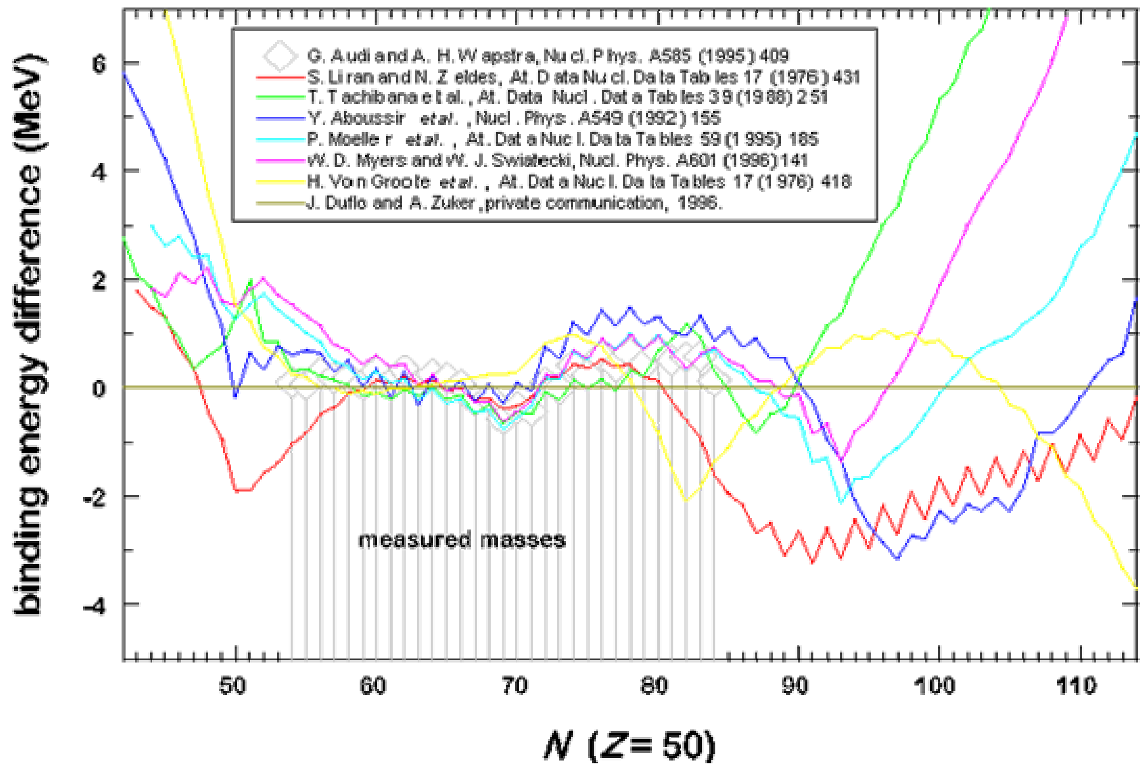

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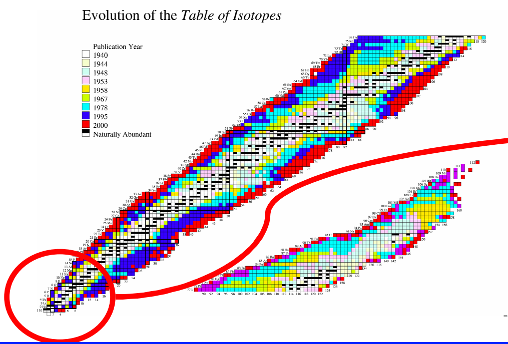
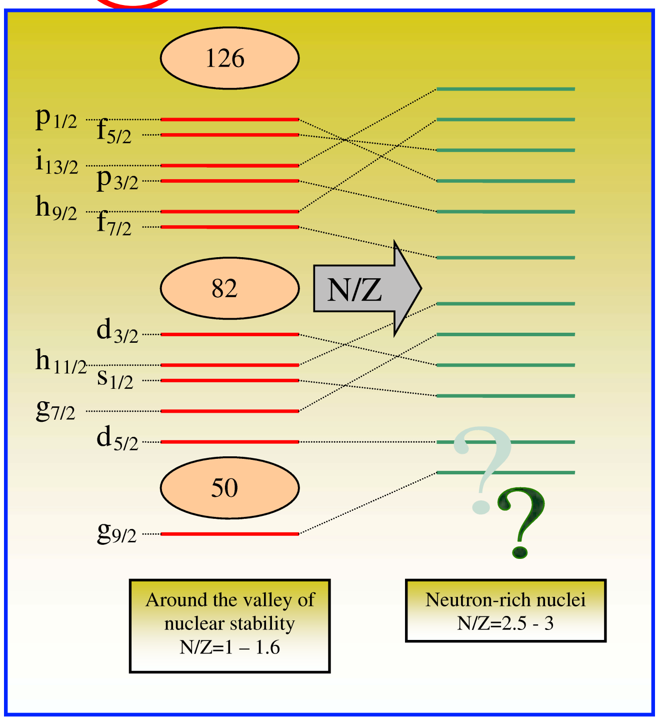


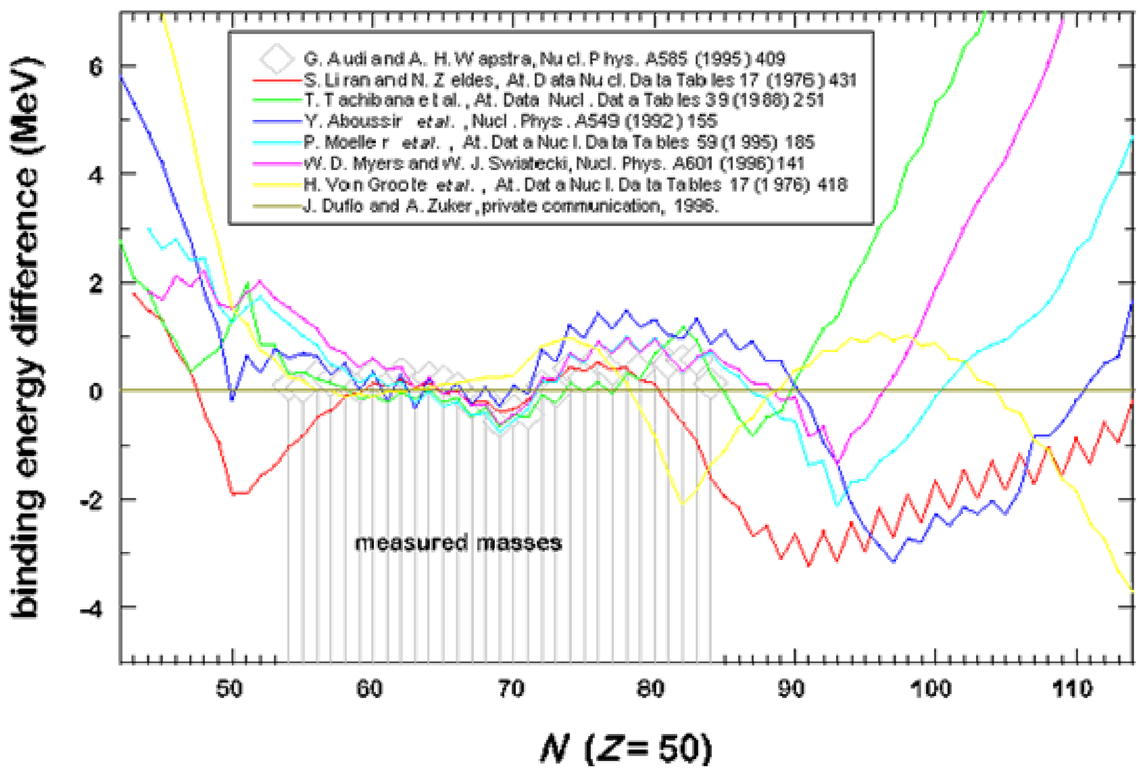
Núcleos ligeros en las “driplines”

Técnicas diferentes

Estructuras exóticas

Núcleos con halo (de neutrones)





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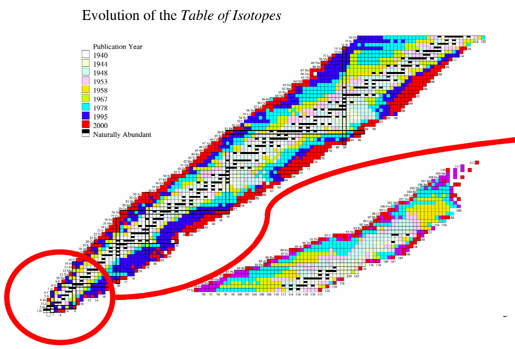


Núcleos ligeros en las “driplines”

Técnicas diferentes

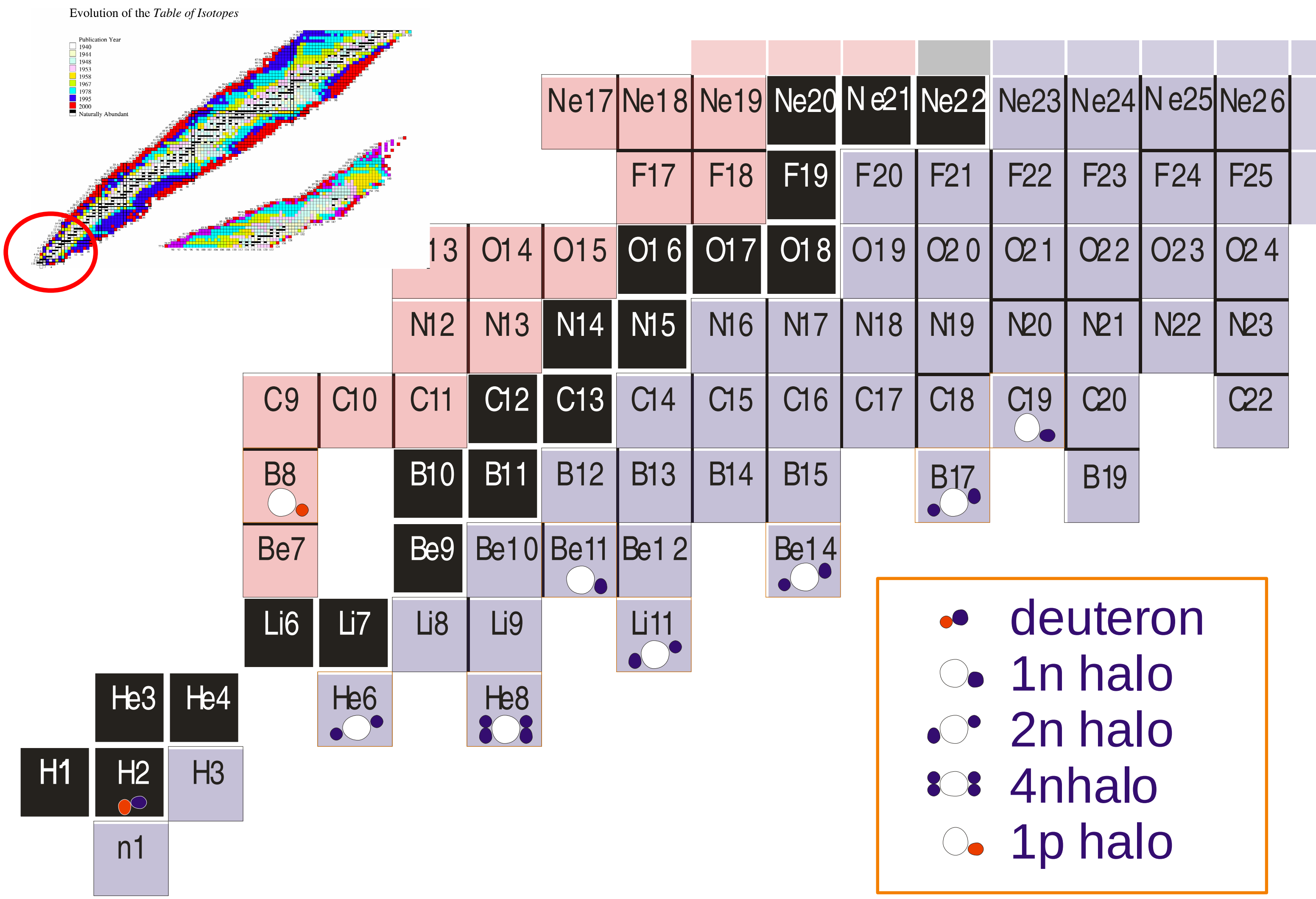
Estructuras exóticas

Núcleos con halo (de neutrones)

Sistemas de tres cuerpos en Física Nuclear

- ✓ Peculiaridades de lo núcleos ligeros en las “*driplines*”
- ✓ Energías y tamaños
- ✓ Evidencias experimentales
- ✓ Reacciones de fragmentación: “*Sudden approximation*”

Núcleos Ligeros en las “driplines”: Núcleos con halo



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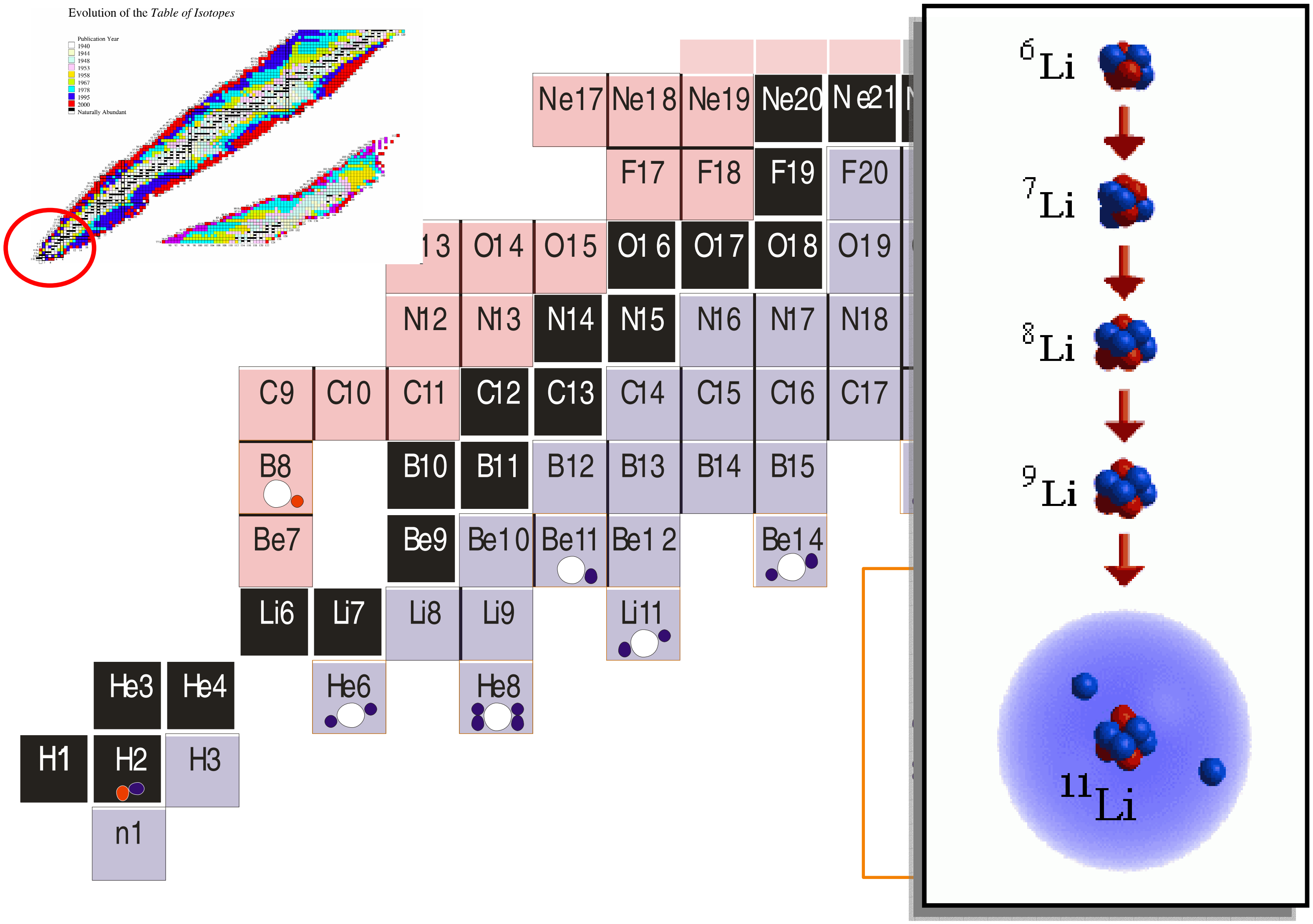

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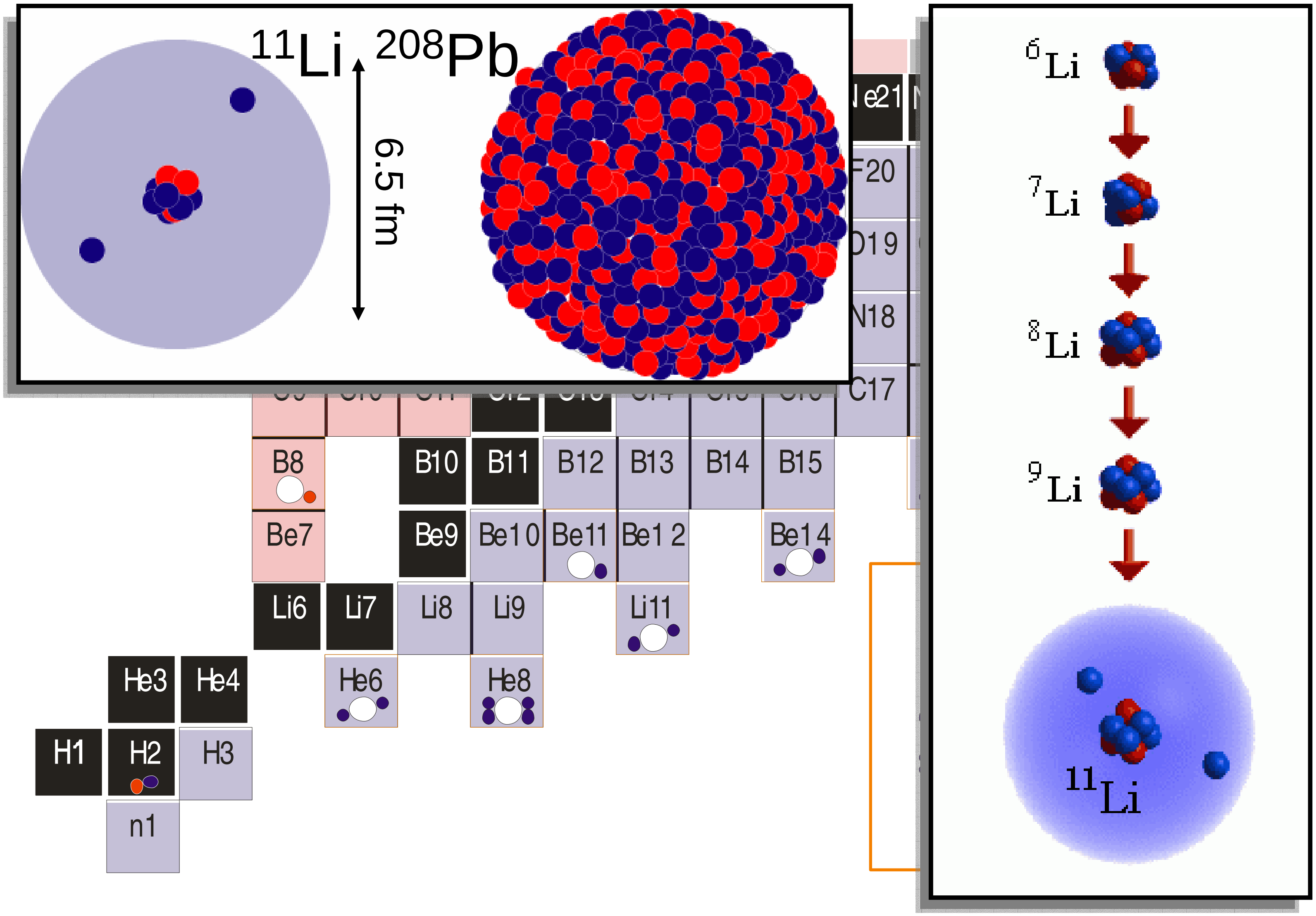
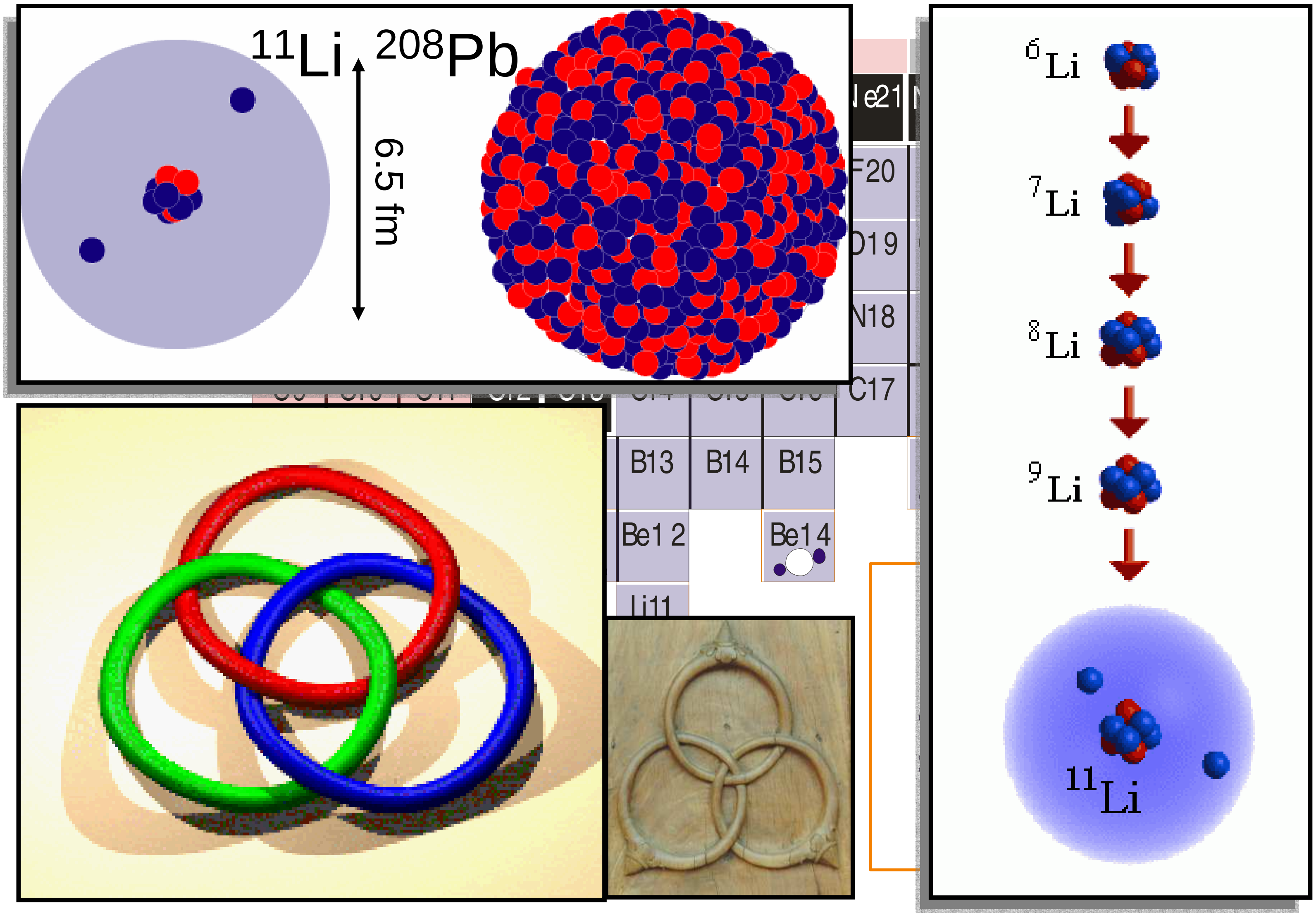


Núcleos Ligeros en las “driplines”: Núcleos de Borromeo



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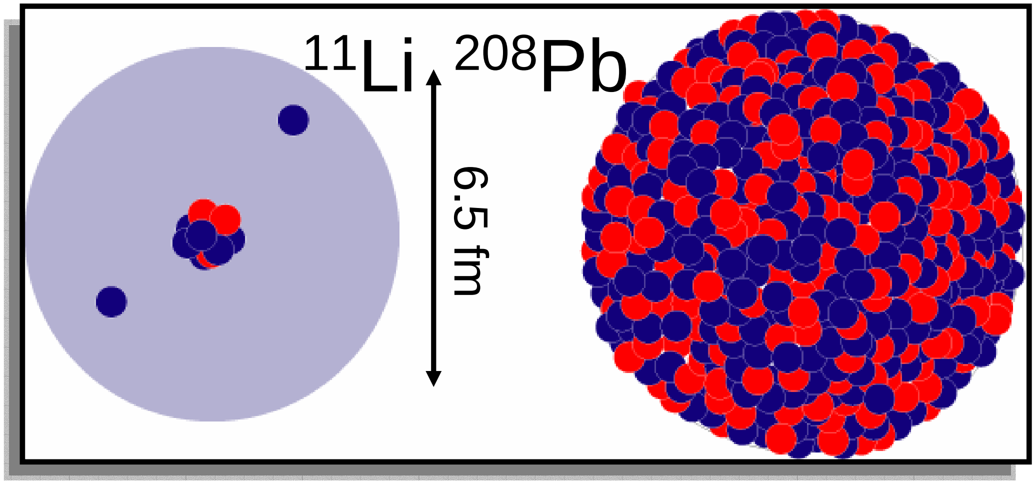
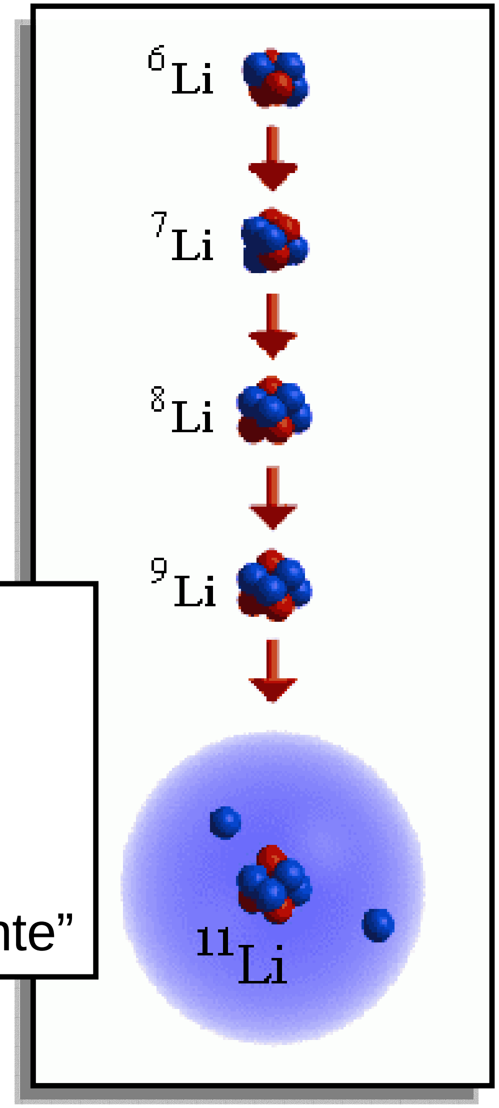
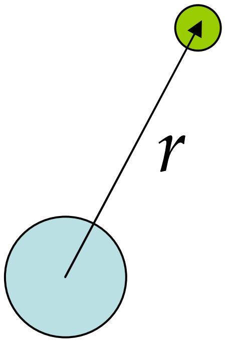




Núcleos con halo:

- ✓Sistemas que “clusterizan”
- ✓Sistemas poco ligados
- ✓Espacialmente muy extensos
- ✓Se extienden a la zona prohibida “clásicamente”

Un caso sencillo: Sistemas de dos cuerpos



Si la interacción es central $\Longrightarrow$ $\Psi_{\ell m}(\vec{r}) = \frac{u_\ell(r)}{r} Y_{\ell m}(\Omega)$

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + V(r) - E + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}\right] u_\ell(r) = 0$$

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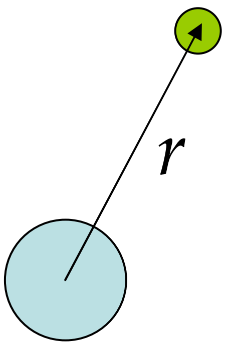

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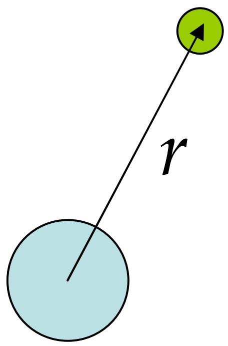

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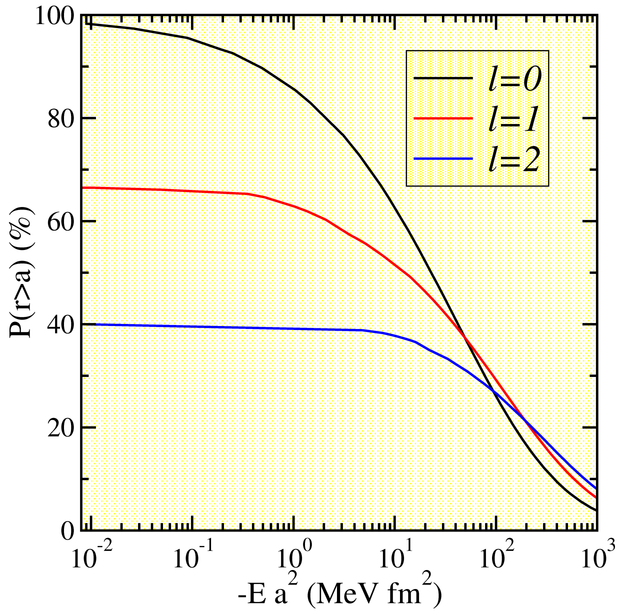

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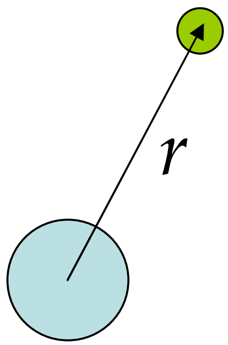

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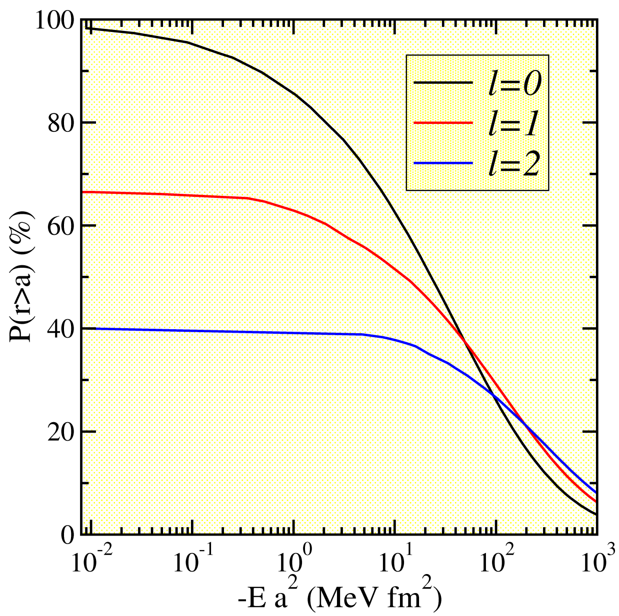

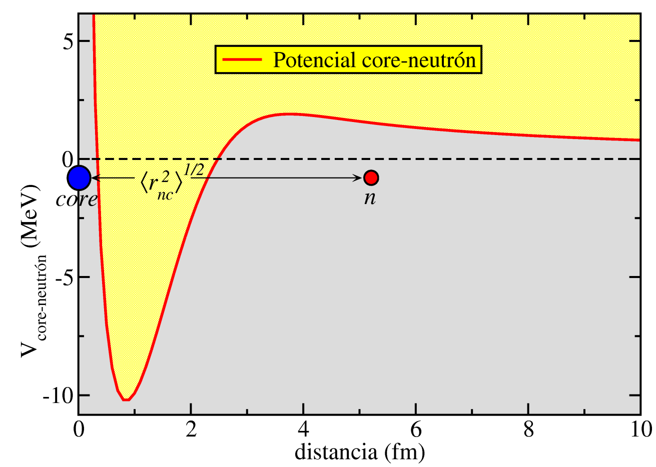

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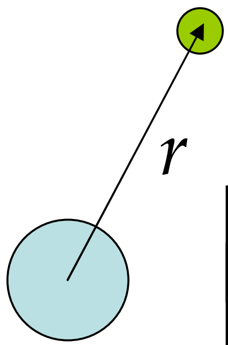

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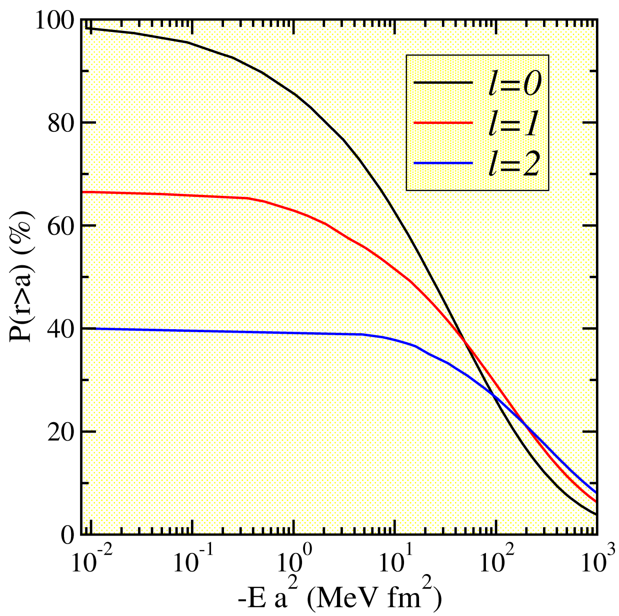

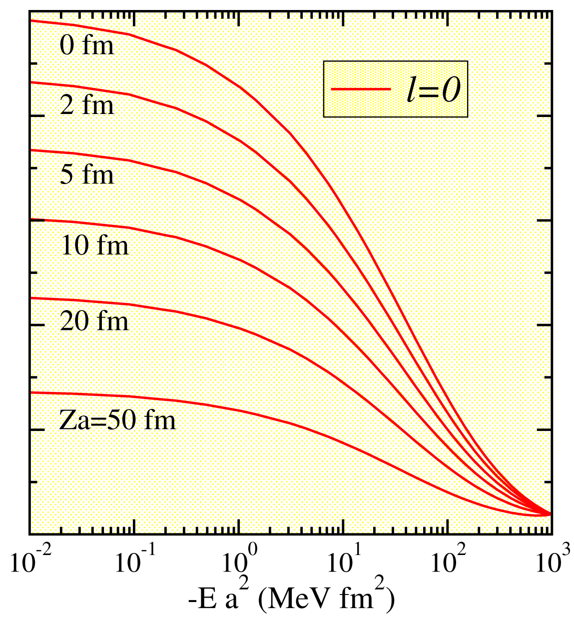

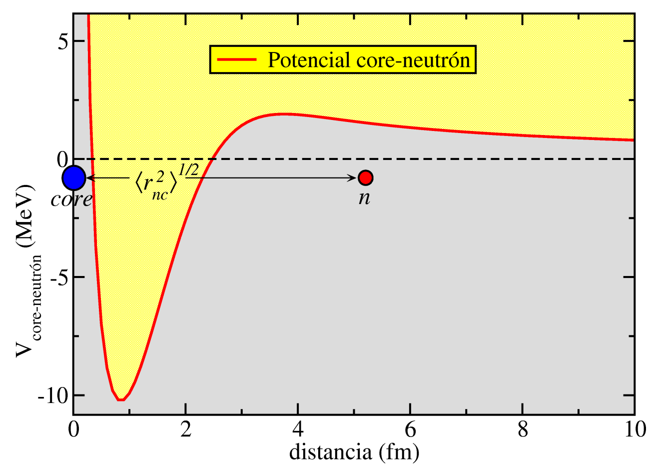

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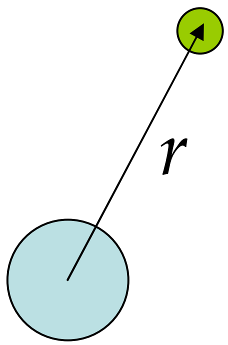

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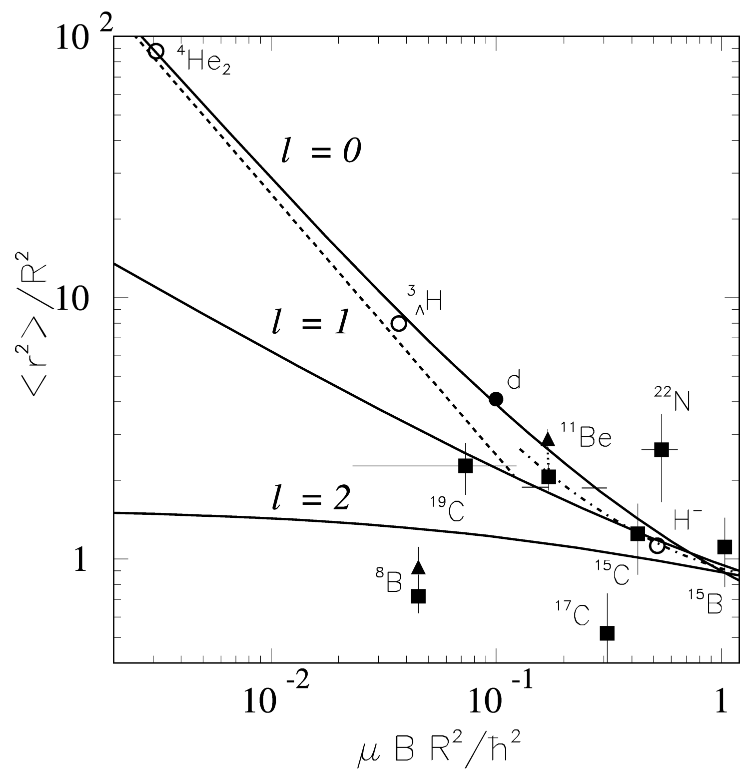

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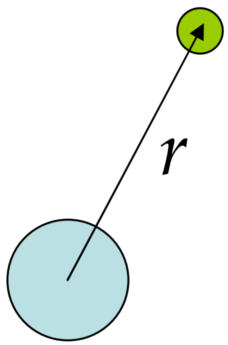

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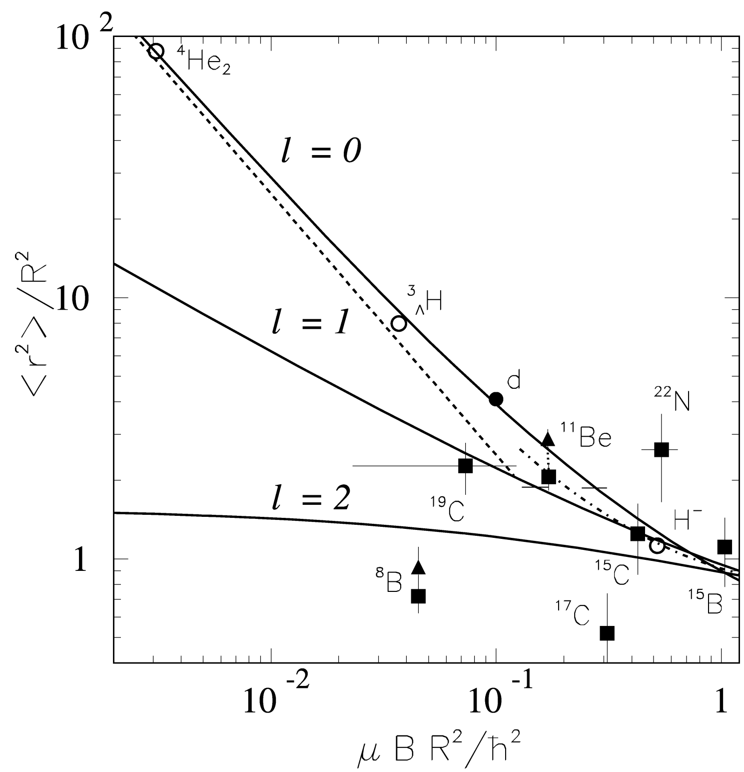
$$\text{Si } \ell = 0 \Rightarrow \langle r^2 \rangle = \frac{\hbar^2}{4\mu B}$$



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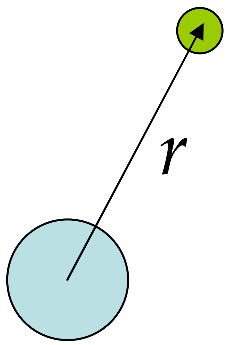

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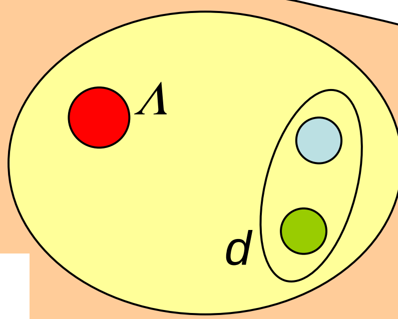
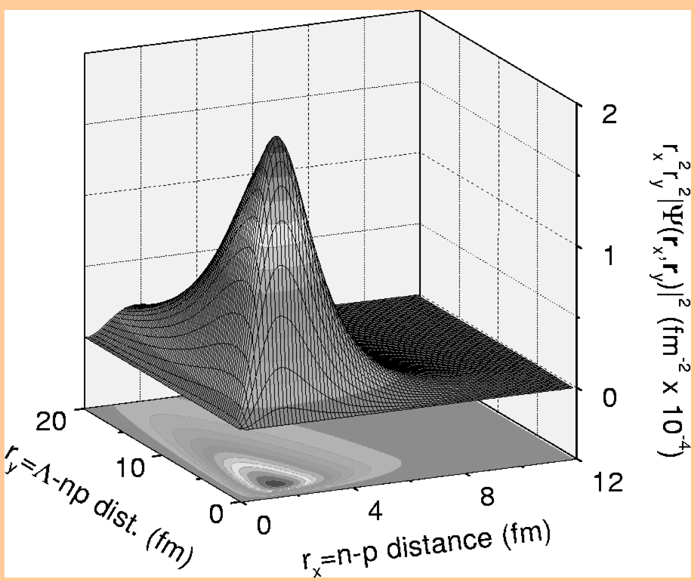
Hypertriton= *n* + *p* + *Λ*

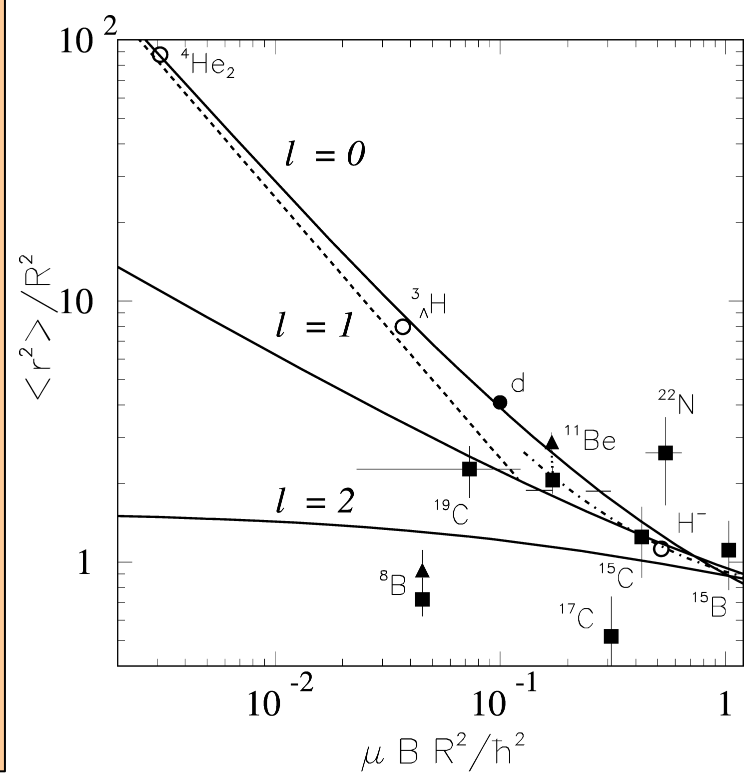




$$\left\langle r^2_{n-p} \right\rangle^{1/2} = 1.97 \text{ fm}$$

$$\left\langle r^2_{\Lambda-d} \right\rangle^{1/2} = 11.7 \text{ fm}$$

$$S_\Lambda = 0.13 \pm 0.05 \text{ MeV}$$



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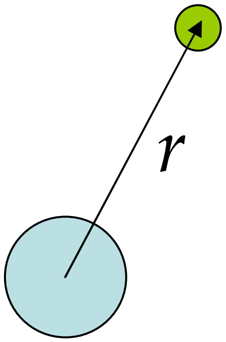

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¿Sistemas de N cuerpos?

Coordenadas $\Rightarrow$ $(\rho, \alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_{3N-4})$

$$m\rho^2 = \sum_{i<k} \frac{m_i m_k}{M} (\vec{r}_i - \vec{r}_k)^2$$

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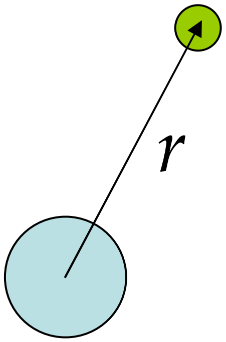

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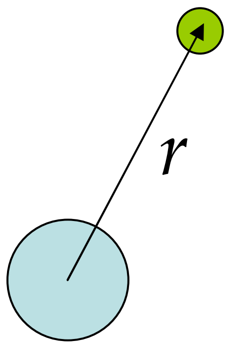
Barrera $\Rightarrow \frac{\hbar^2}{2m}\frac{(3N-4)(3N-6)}{4\rho^2}$

¡¡ Incluso si solamente hay ondas s !!

Un caso sencillo: Sistemas de dos cuerpos



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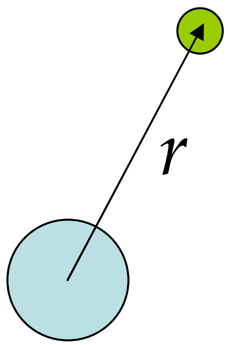

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$\ell^* < 2 \Rightarrow N < \frac{10}{3} \Rightarrow$ No más de 3 partículas!!

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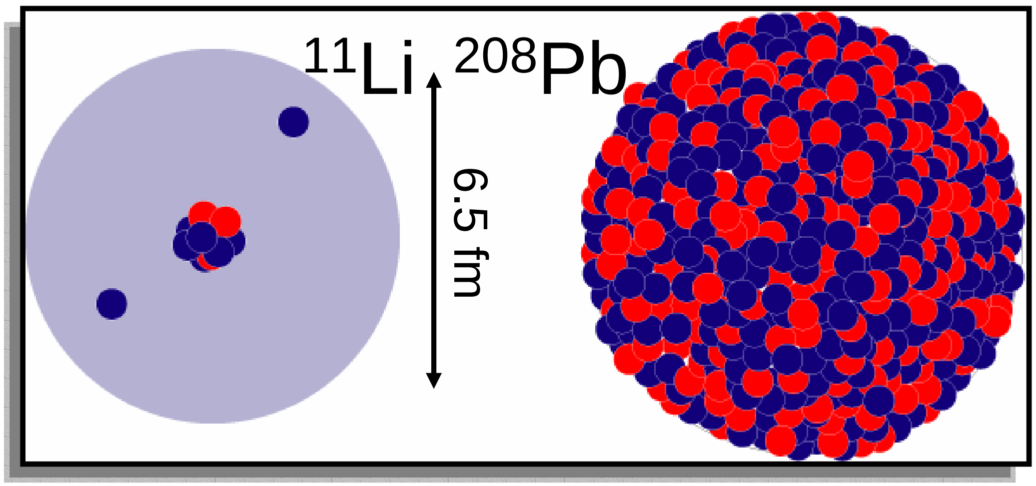
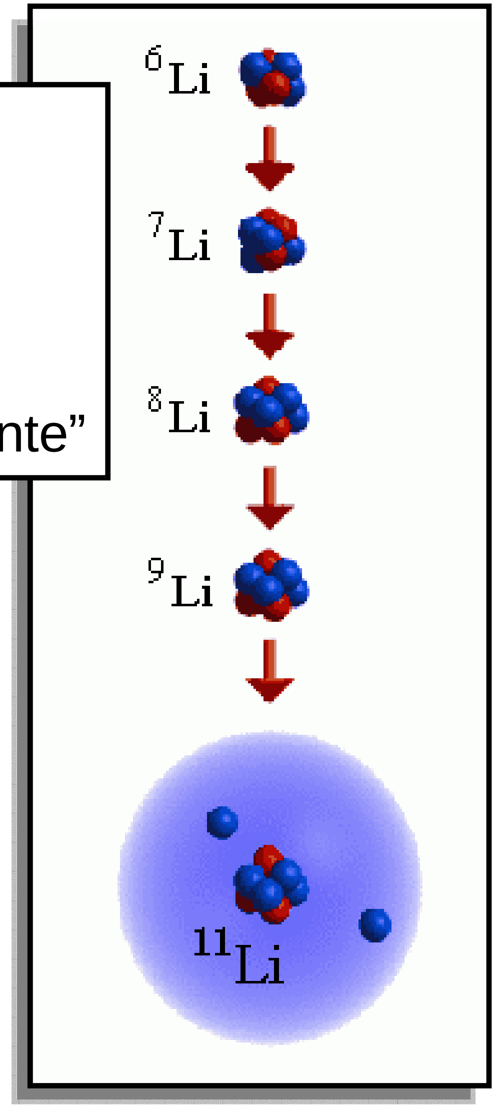
✓Sistemas poco ligados

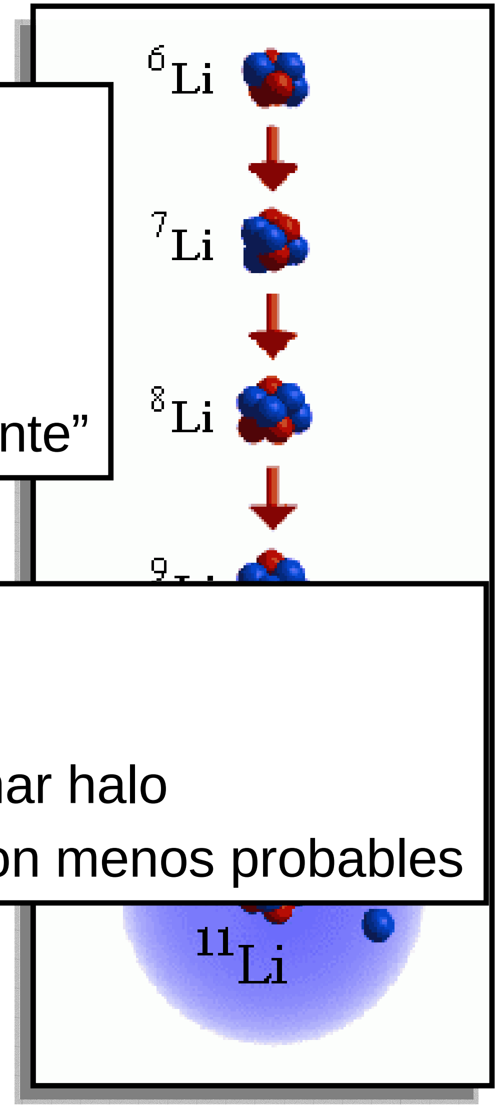
✓Espacialmente muy extensos

✓Se extienden a la zona prohibida "clásicamente"





Núcleos Ligeros en las "driplines": Núcleos de Borromeo

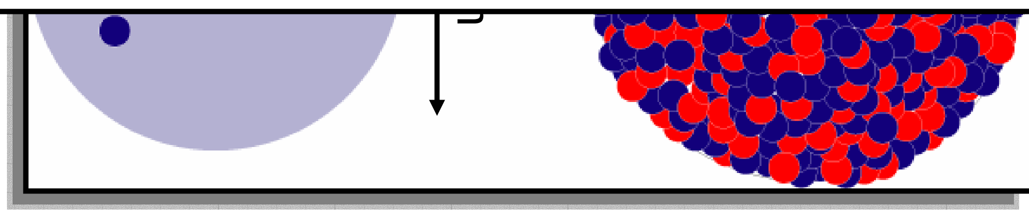


Núcleos con halo:

✓Sistemas que "clusterizan"
✓Sistemas poco ligados
✓Espacialmente muy extensos
✓Se extienden a la zona prohibida "clásicamente"

Además:

✓*ℓ = 0,1* favorece la formación del halo
✓Los protones tienen menos tendencia a formar halo
✓Halos en sistemas de más de 3 partículas son menos probables



Evidencias experimentales: Radios

Radios

$$\sigma_I = \pi(R_T + R_P)^2$$

Proy.	σ_I (mb)	R_P (fm)
^{4}He		1.41 ± 0.03
^{6}He		2.18 ± 0.02
^{8}He		2.48 ± 0.03
^{6}Li	688 ± 10	2.09 ± 0.02
^{7}Li	736 ± 6	2.23 ± 0.02
^{8}Li	768 ± 9	2.36 ± 0.02
^{9}Li	796 ± 6	2.41 ± 0.02
^{11}Li	1040 ± 60	3.14 ± 0.16
^{7}Be	738 ± 9	2.22 ± 0.02
^{9}Be	806 ± 9	2.45 ± 0.01
^{10}Be	813 ± 10	2.46 ± 0.03
^{11}Be		2.73 ± 0.05
^{14}Be	1109 ± 69	3.16 ± 0.38

I. Tanihata, NPA 522 (1991) 275

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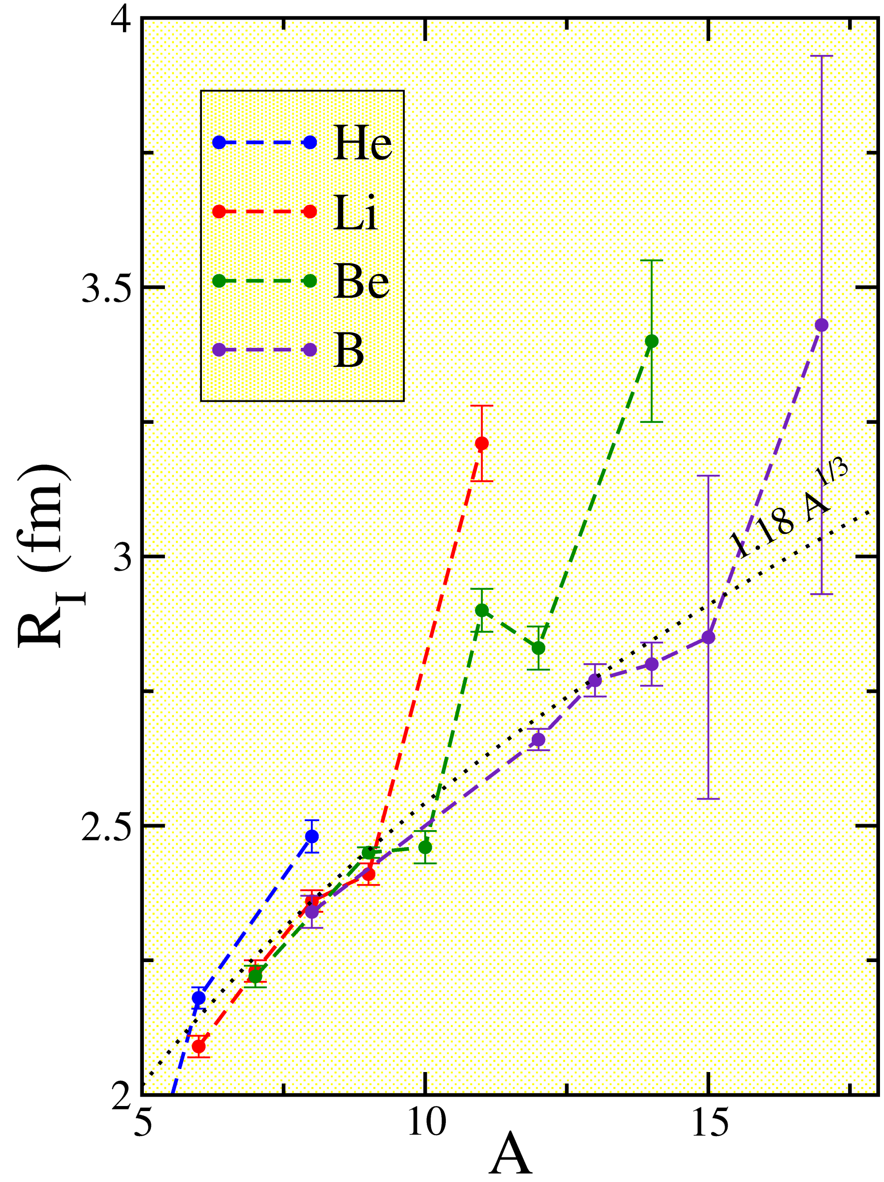
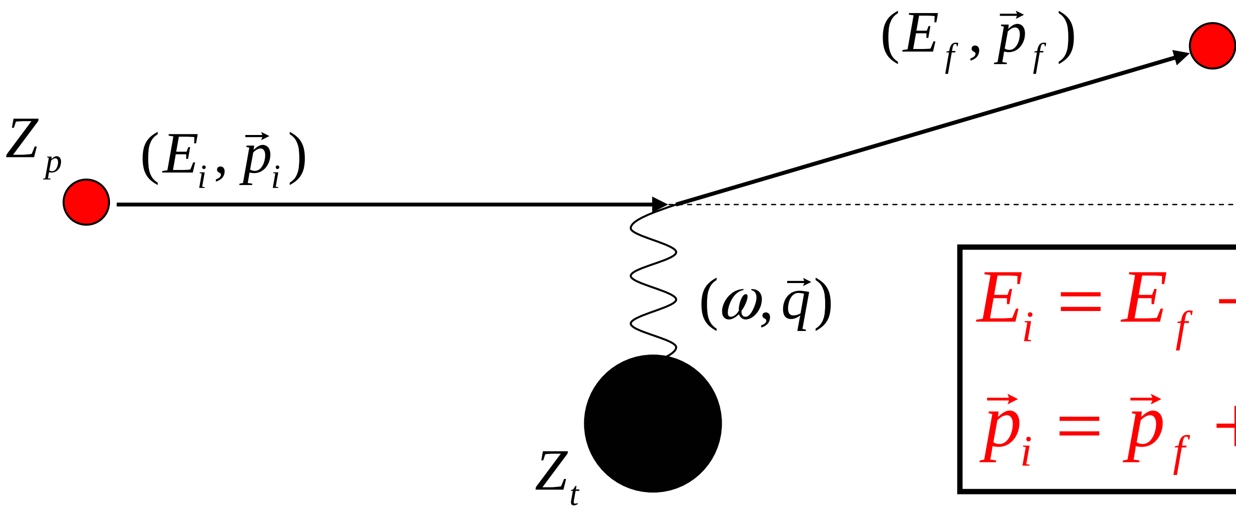


I. Tanihata, NPA 522 (1991) 275

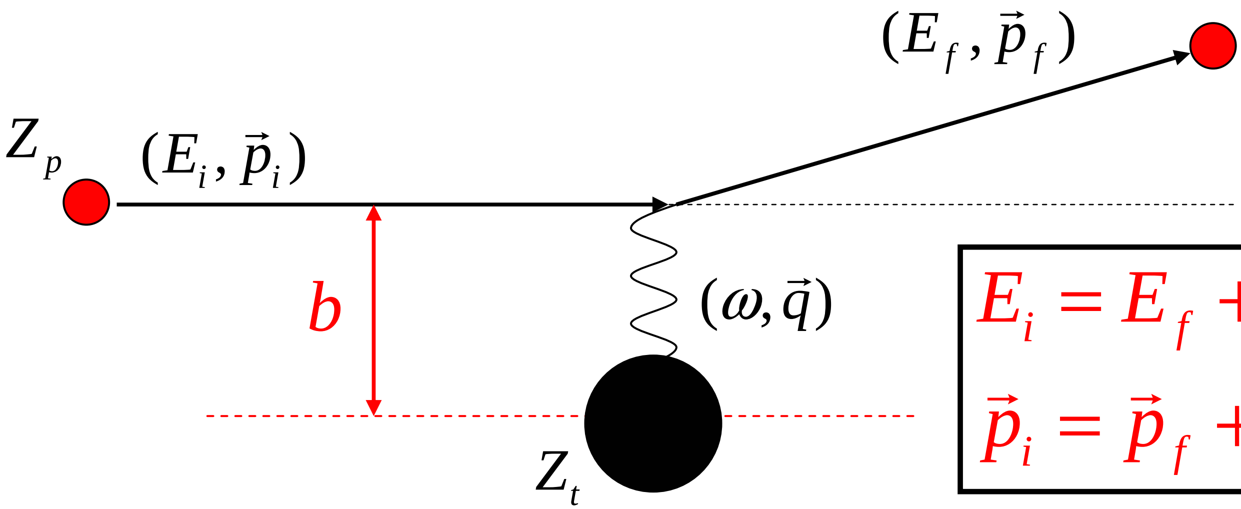
Evidencias experimentales: Disociación Coulombiana



$$E_i = E_f + \omega$$

$$\vec{p}_i = \vec{p}_f + \vec{q}$$

Evidencias experimentales: Disociación Coulombiana

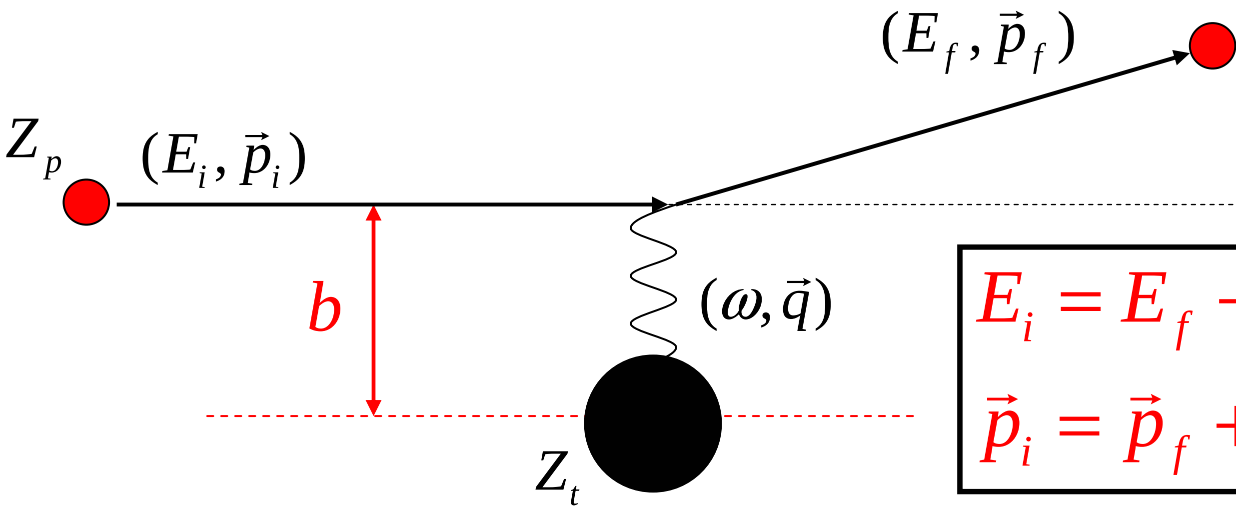


$$E_i = E_f + \omega$$

$$\vec{p}_i = \vec{p}_f + \vec{q}$$

$$b = \frac{Z_p Z_t e^2}{q} \frac{p_i}{T_i}$$

Evidencias experimentales: Disociación Coulombiana



$$E_i = E_f + \omega$$

$$\vec{p}_i = \vec{p}_f + \vec{q}$$

$$b = \frac{Z_p Z_t e^2}{q} \frac{p_i}{T_i}$$

Si q es suficientemente pequeño ($q < q_{\max}$)

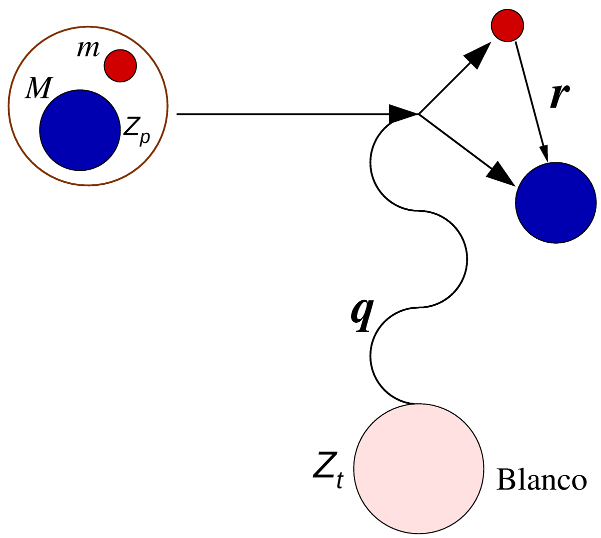
entonces $b > R_p + R_t$

$\Downarrow$

Sólo interacción Coulombiana

$$q_{\max} = \frac{Z_p Z_t e^2}{R_p + R_t} \frac{p_i}{T_i}$$

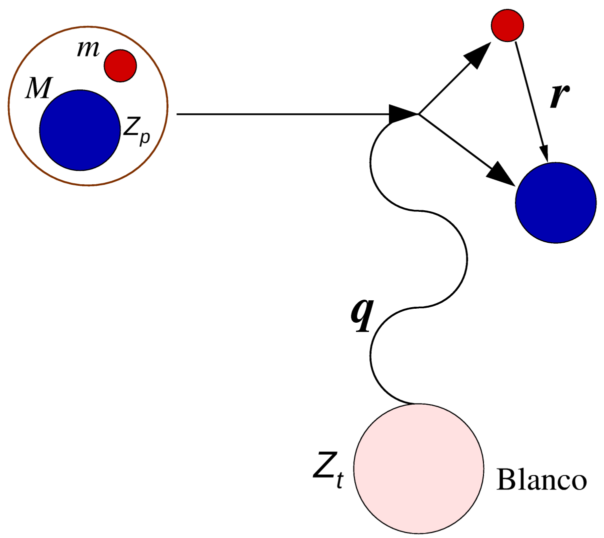
Evidencias experimentales: Disociación Coulombiana



Evidencias experimentales: Disociación Coulombiana

$$\vec{p}_r = \mu\left(\frac{\vec{p}_M}{M} - \frac{\vec{p}_m}{m}\right)$$

$$\vec{p}\,'_r = \mu\left(\frac{\vec{p}\,'_M}{M} - \frac{\vec{p}\,'_m}{m}\right)$$



Evidencias experimentales: Disociación Coulombiana

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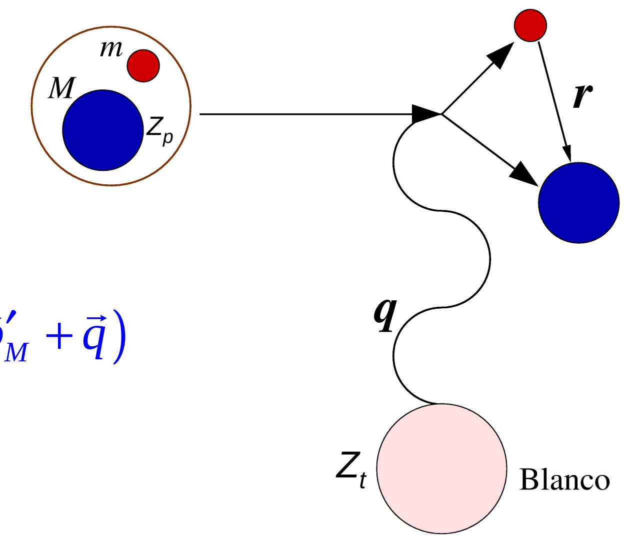
$$\vec{p}_r' = \mu\left(\frac{\vec{p}_M'}{M} - \frac{\vec{p}_m'}{m}\right) = \mu\left(\frac{\vec{p}_M - \vec{q}}{M} - \frac{\vec{p}_m}{m}\right)$$

$$\left(\vec{p}_M = \vec{p}_M' + \vec{q}\right)$$



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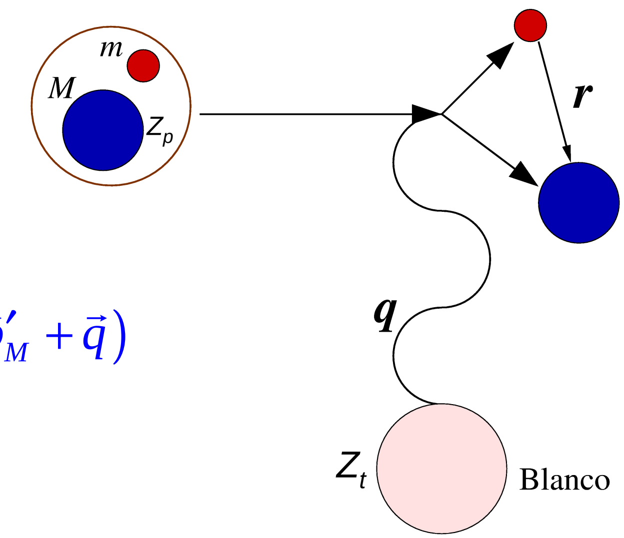
$$\left(\vec{p}_M = \vec{p}_M' + \vec{q}\right)$$

$$\Delta\vec{p}_r = \vec{p}_r - \vec{p}_r' = \frac{m}{m+M}\vec{q}$$



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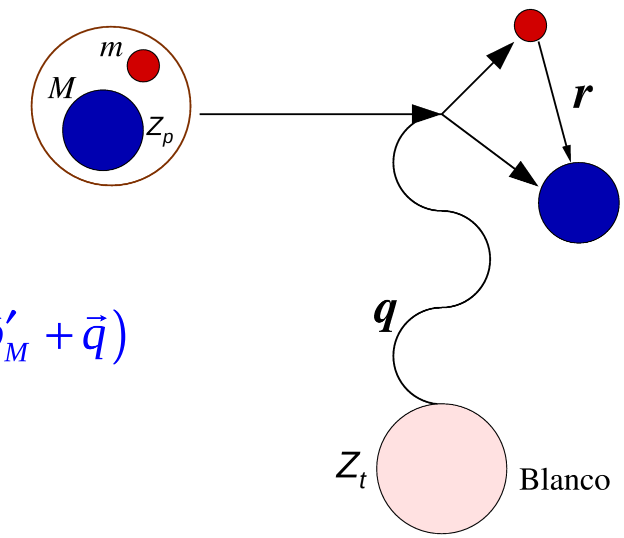

$$\text{En un proceso elástico} \Rightarrow \Psi_{\text{final}}(\vec{r}) = e^{i\Delta\vec{p}_r\cdot\vec{r}}\,\Psi(\vec{r})$$

$$P_{\text{elas}} = \left|\left\langle\Psi(\vec{r})\right|e^{i\Delta\vec{p}_r\cdot\vec{r}}\left|\Psi(\vec{r})\right\rangle\right|^2$$

Evidencias experimentales: Disociación Coulombiana

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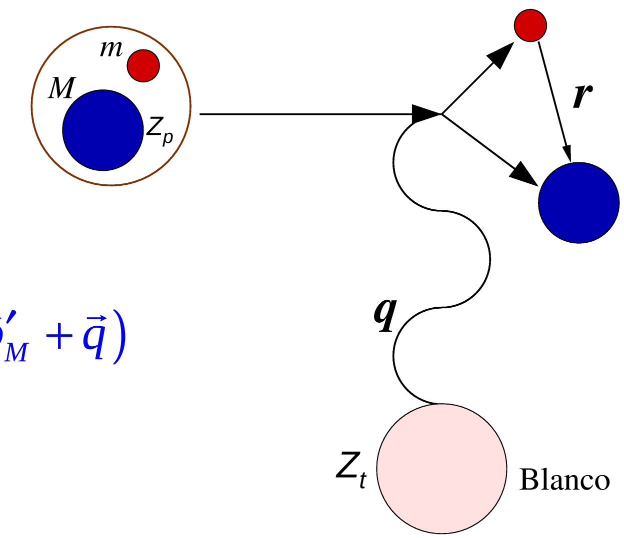

$$\Delta\vec{p}_r \cdot \vec{r} \ll 1 \Rightarrow e^{i\Delta\vec{p}_r \cdot \vec{r}} \approx 1 + i\Delta\vec{p}_r \cdot \vec{r} - \frac{1}{2}\left(\Delta\vec{p}_r \cdot \vec{r}\right)^2 + \cdots$$

$$P_{\text{elas}} = \left|\left\langle \Psi(\vec{r}) \right| e^{i\Delta\vec{p}_r \cdot \vec{r}} \left| \Psi(\vec{r}) \right\rangle\right|^2$$

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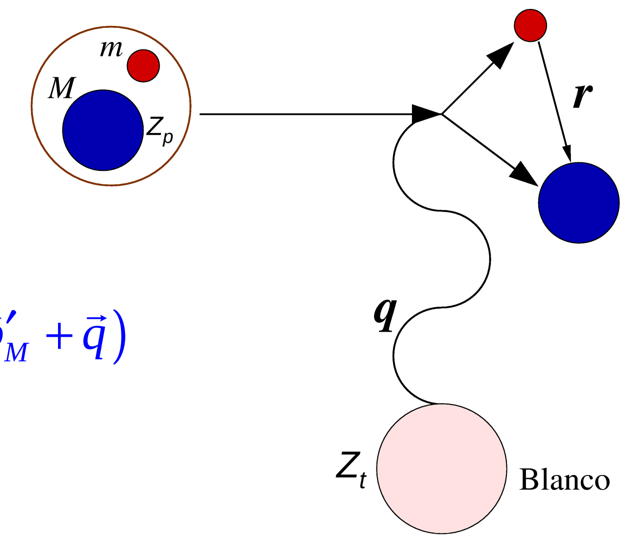

$$\Delta\vec{p}_r = \vec{p}_r - \vec{p}\,'_r = \frac{m}{m+M}\vec{q}$$

$$P_{\text{elas}} \approx 1 - (\Delta p_r)^2 \langle r^2 \rangle = 1 - \frac{m^2}{(m+M)^2} q^2 \langle r^2 \rangle$$

$$\Delta\vec{p}_r \cdot \vec{r} \ll 1 \Rightarrow e^{i\Delta\vec{p}_r\cdot\vec{r}} \approx 1 + i\Delta\vec{p}_r \cdot \vec{r} - \frac{1}{2}\left(\Delta\vec{p}_r \cdot \vec{r}\right)^2 + \cdots$$

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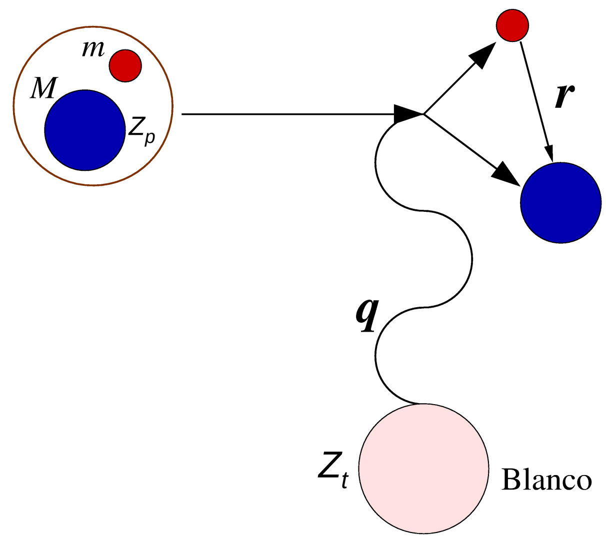
Evidencias experimentales: Disociación Coulombiana

Si sólo un estado ligado: $P_{\text{dis}} = 1 - P_{\text{elas}}$



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Evidencias experimentales: Disociación Coulombiana

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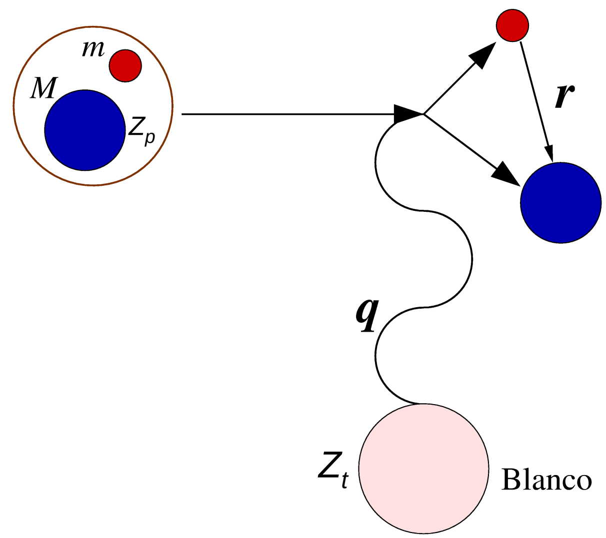
$$\frac{d\sigma_d}{dq} = \frac{8\pi \left(Z_p Z_t e^2\right)^2}{v^2} \frac{1}{q^3} \frac{m^2}{(m+M)^2} q^2 \langle r^2 \rangle$$

Sección eficaz de Rutherford



Evidencias experimentales: Disociación Coulombiana

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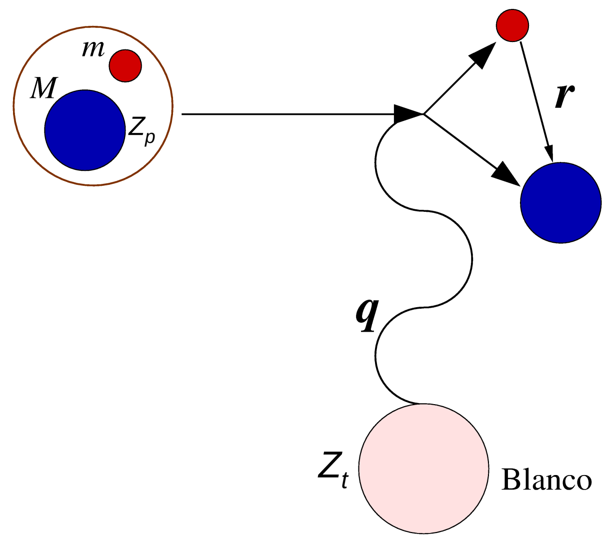

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Sección eficaz de disociación Coulombiana

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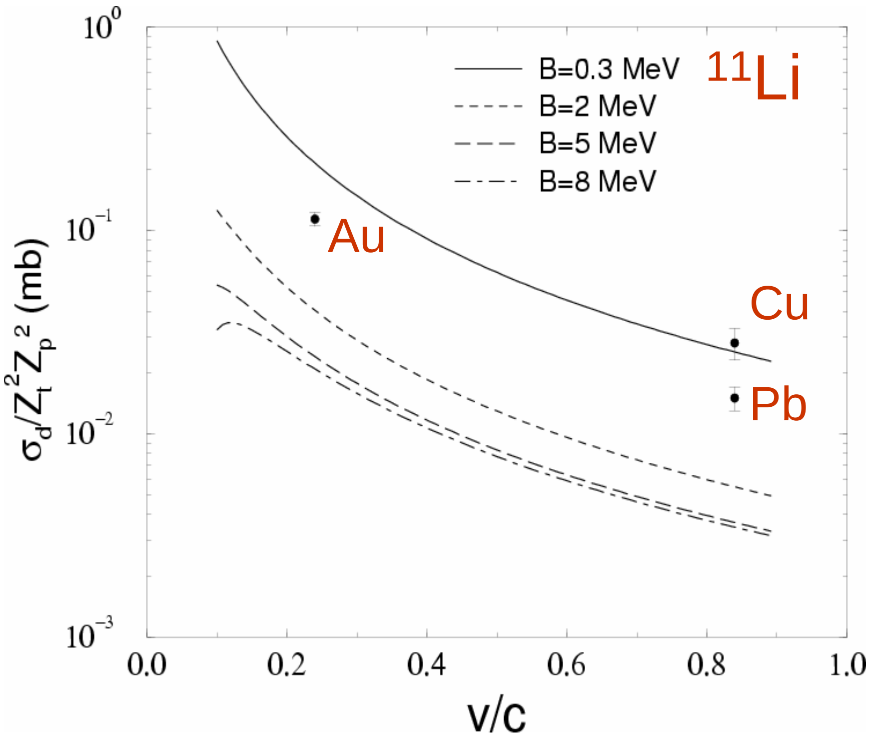
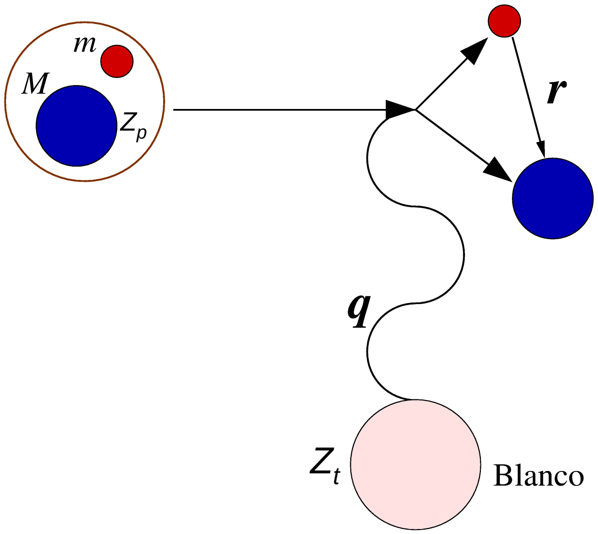
Evidencias experimentales: Disociación Coulombiana





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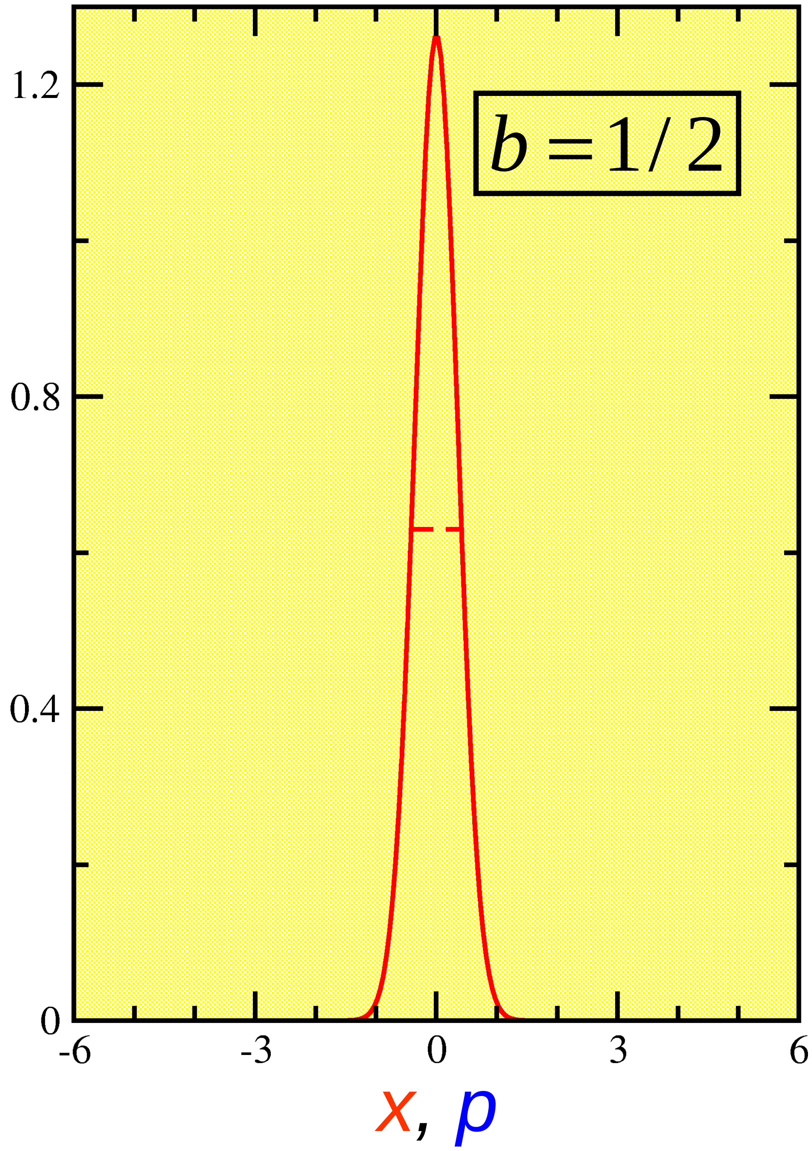
$$q_{\min} < q < q_{\max}$$

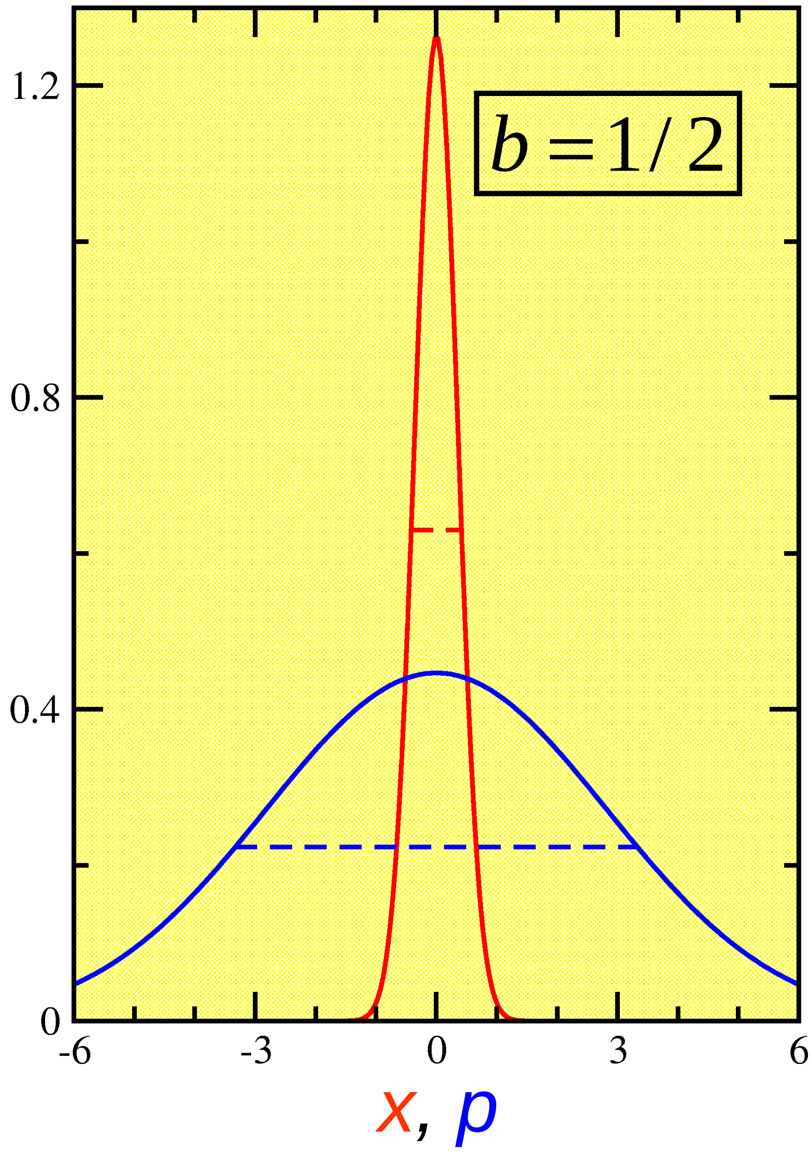
Evidencias experimentales: Distribuciones de momentos



$$\Gamma_x = 2b\sqrt{\ln 2}$$

$$\Psi(x) = \left(\frac{2}{\pi}\frac{1}{b^2}\right)^{\frac{1}{4}} \exp\left(-\frac{x^2}{b^2}\right)$$

Evidencias experimentales: Distribuciones de momentos



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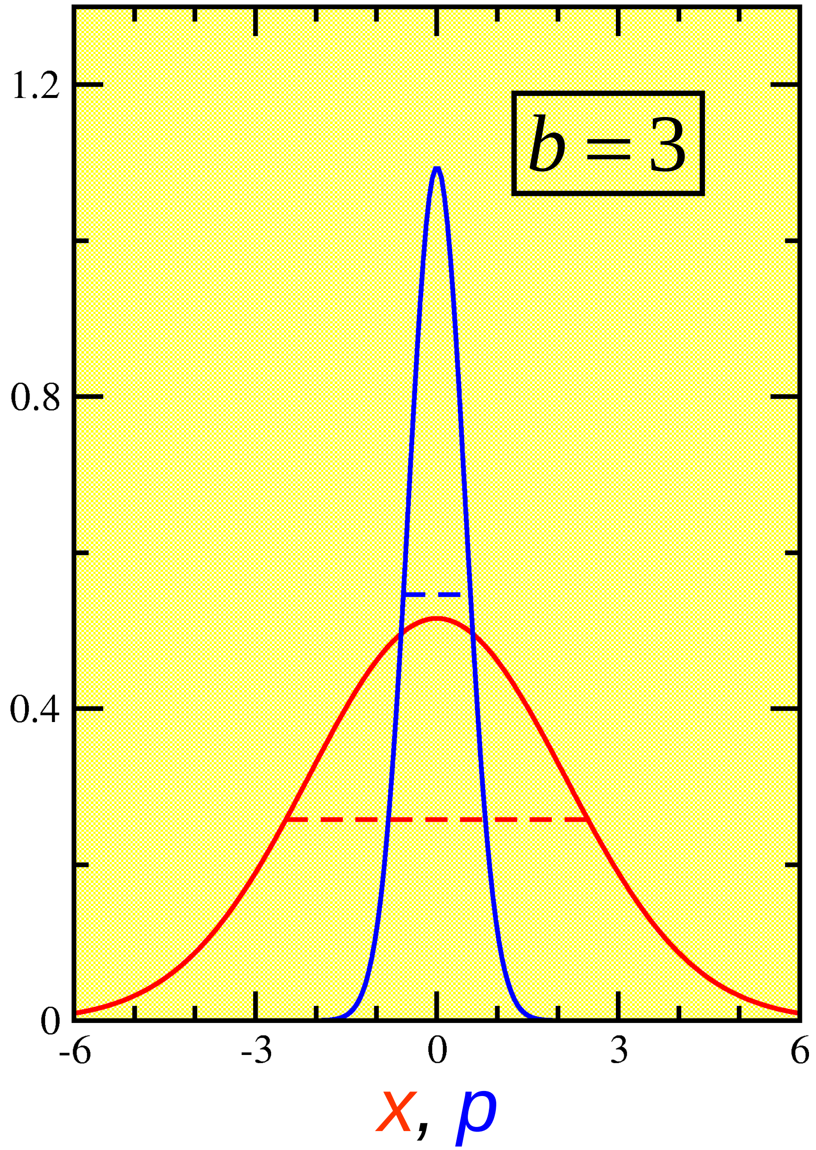

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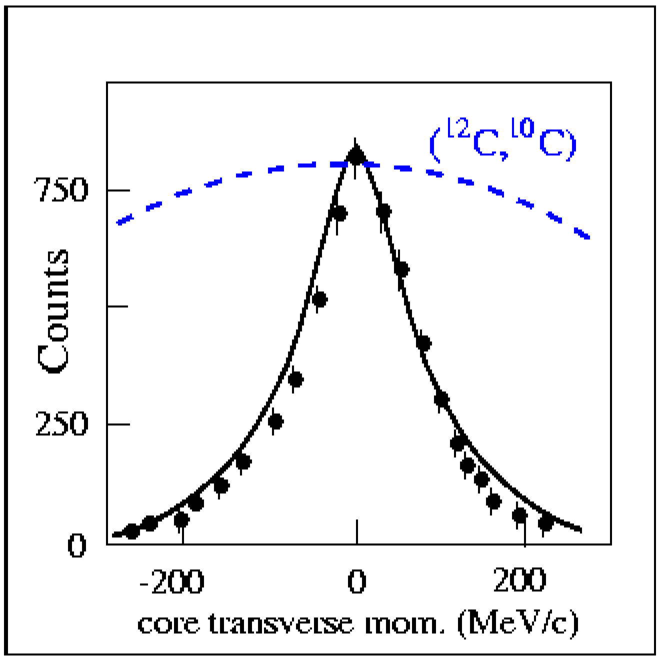
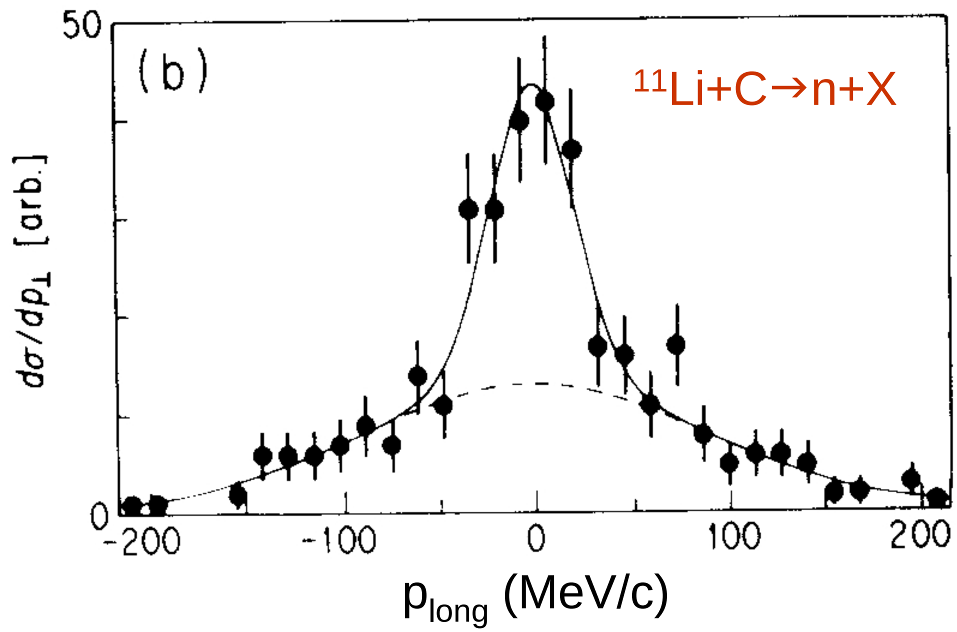
Evidencias experimentales: Distribuciones de momentos

$^{6}He + T \longrightarrow \alpha + n + n + T$

@ 400 MeV/u





Reacciones de fragmentación

“SUDDEN APPROXIMATION”

Si la energía del proyectil es suficientemente alta

sólo un constituyente interacciona con el blanco, siendo arrancado del mismo

Reacciones de fragmentación en blancos ligeros (No Coulomb)

“SUDDEN APPROXIMATION”

Si la energía del proyectil es suficientemente alta

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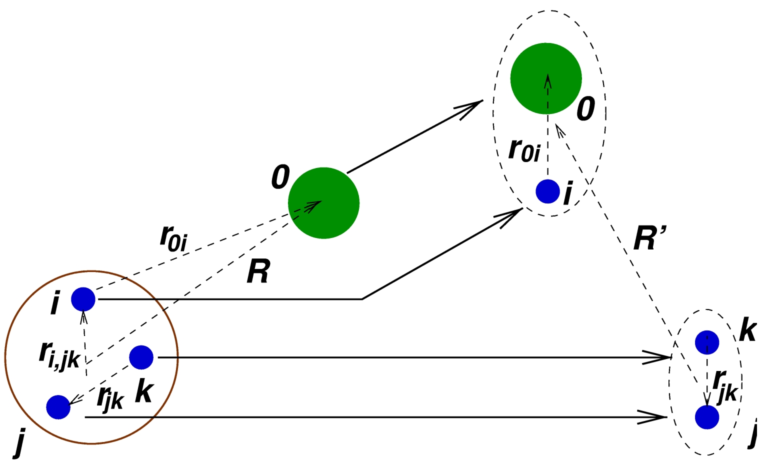

Reacciones de fragmentación en blancos ligeros (No Coulomb)

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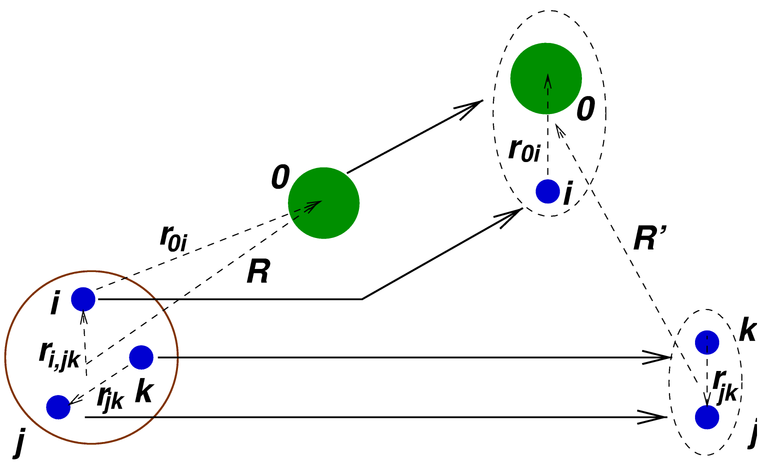

$$T^{(i)} = \left\langle \Phi^{(0i)}_{p'_{0i}\sigma'_i} \Phi^{(jk)}_{p'_{jk} s_{jk} \sigma_{jk}} e^{i\vec{P}'\cdot\vec{R}'} \middle| V_{0i} \middle| \Psi^{JM} e^{i\vec{P}\cdot\vec{R}} \right\rangle$$

$$T^{(i)} = \sum_{\sigma_i} T^{(0i)}_{\sigma_i \sigma_i} \left\langle \Phi^{(jk)}_{p'_{jk} s_{jk} \sigma_{jk}} e^{i\vec{p}_{i,jk}\cdot\vec{r}_{i,jk}} \chi_{s_i \sigma_i} \middle| \Psi^{JM} \right\rangle$$

Reacciones de fragmentación en blancos ligeros (No Coulomb)

"SUDDEN APPROXIMATION"

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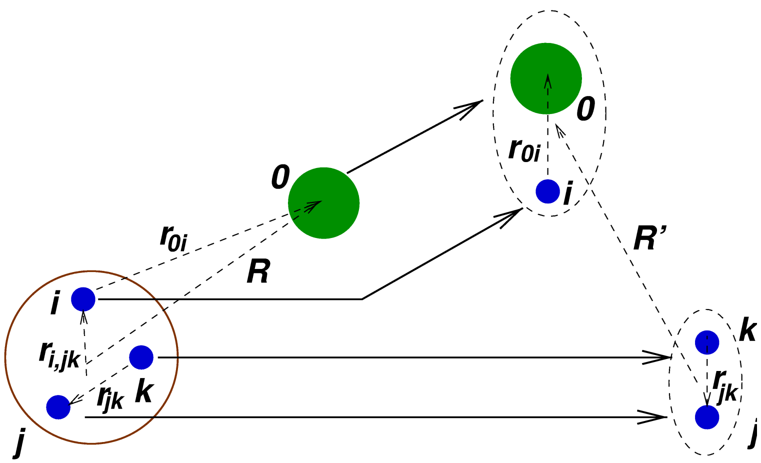

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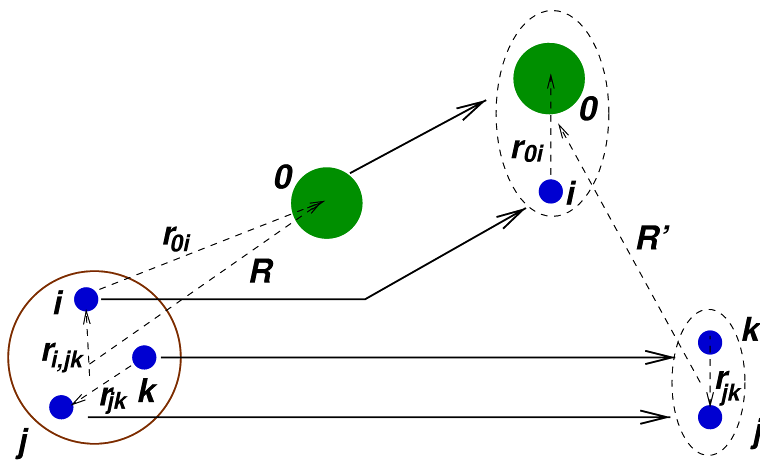
$$d\sigma^{(i)} = \frac{2\pi}{v} \left| T^{(i)} \right|^2 \delta(E'_{0i} - E_{0i}) \frac{d\vec{p}'_{0i}}{(2\pi)^3} \frac{d\vec{p}'_{jk}}{(2\pi)^3} \frac{d\vec{P}'}{(2\pi)^3}$$

Regla de oro de Fermi

Reacciones de fragmentación en blancos ligeros (No Coulomb)

$$\frac{d^6\sigma^{(0i)}_{(\text{elas,abs})}}{d\vec{p}'_{i,jk}\,d\vec{p}'_{jk}} = \sigma^{(0i)}_{(\text{elas,abs})}\,\frac{1}{2J+1}\sum\left|\left\langle \Phi^{(jk)}_{p'_{jk}s_{jk}\sigma_{jk}}\,e^{i\vec{p}_{i,jk}\cdot\vec{r}_{i,jk}}\,\chi_{s_i\sigma_i}\right|\Psi^{JM}\right\rangle\right|^2$$



$$T^{(i)} = \left\langle \Phi^{(0i)}_{p'_{0i}\sigma'_i}\,\Phi^{(jk)}_{p'_{jk}s_{jk}\sigma_{jk}}\,e^{i\vec{P}'\cdot\vec{R}'}\right|V_{0i}\left|\Psi^{JM}\,e^{i\vec{P}\cdot\vec{R}}\right\rangle$$

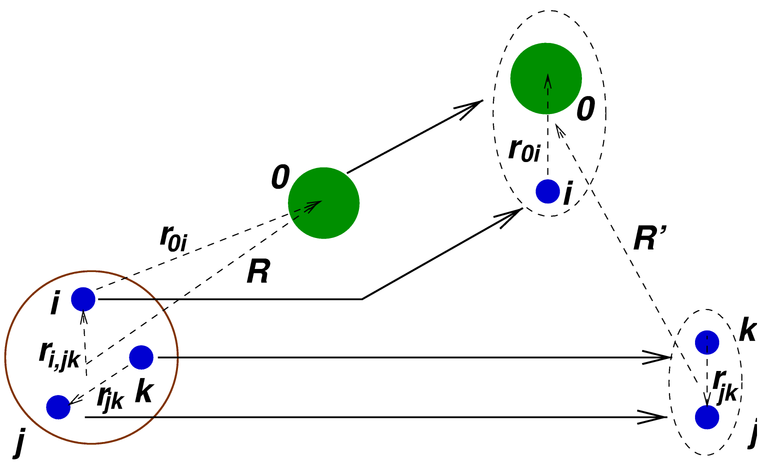
$$T^{(i)} = \sum_{\sigma_i} T^{(0i)}_{\sigma_i\sigma_i}\left\langle \Phi^{(jk)}_{p'_{jk}s_{jk}\sigma_{jk}}\,e^{i\vec{p}_{i,jk}\cdot\vec{r}_{i,jk}}\,\chi_{s_i\sigma_i}\right|\Psi^{JM}\right\rangle$$

$$d\sigma^{(i)} = \frac{2\pi}{v}\left|T^{(i)}\right|^2\,\delta(E'_{0i}-E_{0i})\,\frac{d\vec{p}'_{0i}}{(2\pi)^3}\,\frac{d\vec{p}'_{jk}}{(2\pi)^3}\,\frac{d\vec{P}'}{(2\pi)^3}$$

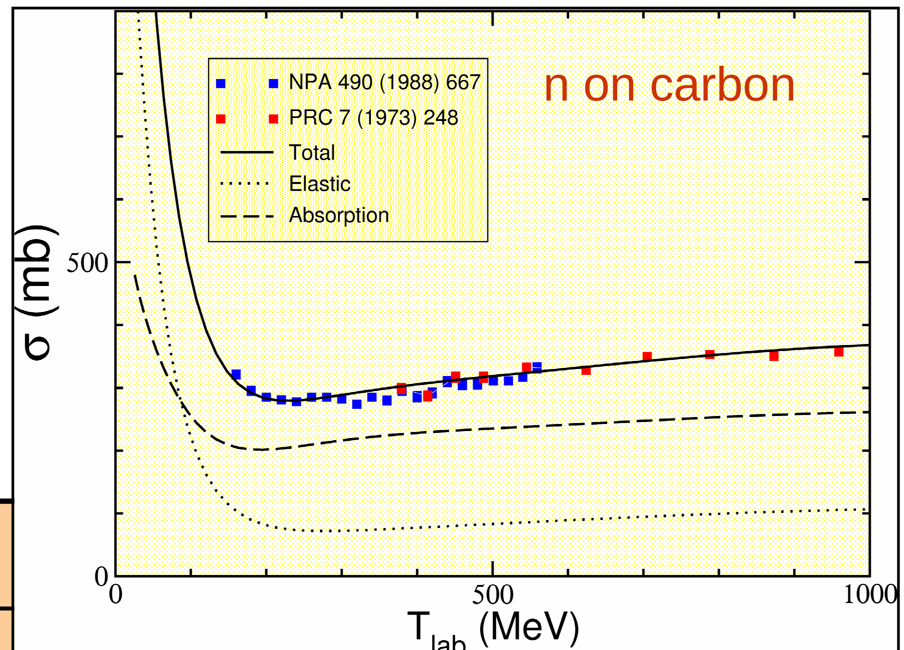
Regla de oro de Fermi

Reacciones de fragmentación en blancos ligeros (No Coulomb)

$$\frac{d^6\sigma^{(0i)}_{(\text{elas,abs})}}{d\vec{p}'_{i,jk}\,d\vec{p}'_{jk}} = \sigma^{(0i)}_{(\text{elas,abs})}\,\frac{1}{2J+1}\sum\left|\left\langle \Phi^{(jk)}_{p'_{jk}s_{jk}\sigma_{jk}}\, e^{i\vec{p}_{i,jk}\cdot\vec{r}_{i,jk}}\,\chi_{s_i\sigma_i}\,\middle|\,\Psi^{JM}\right\rangle\right|^2$$



$$d\sigma^{(i)} = \frac{2\pi}{v}\left|T^{(i)}\right|^2 \delta(E'_{0i} - \ldots \quad (2\pi)\ \ (2\pi)\ \ (2\pi)$$



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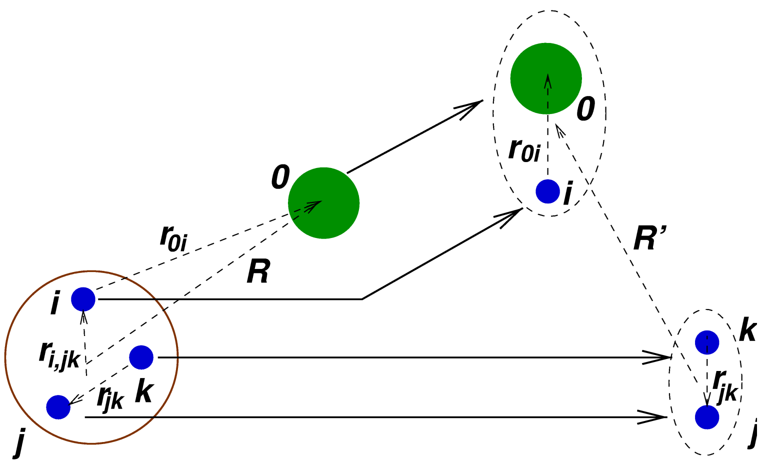
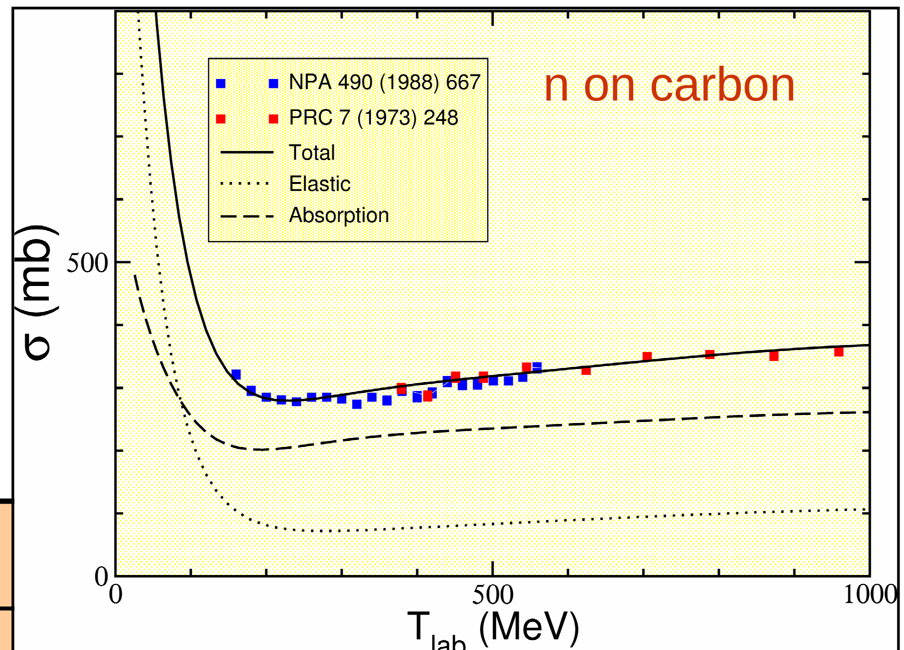
Reacciones de fragmentación en blancos ligeros (No Coulomb)

$$\frac{d^6\sigma^{(0i)}}{d\vec{p}'_{i,jk}\, d\vec{p}'_{jk}} \propto \sum \left| \left\langle \Phi^{(jk)}_{p'_{jk} s_{jk} \sigma_{jk}} \, e^{i\vec{p}_{i,jk}\cdot\vec{r}_{i,jk}} \chi_{s_i \sigma_i} \right| \Psi^{JM} \right\rangle \Big|^2$$

Pure sudden approximation





$$d\sigma^{(i)} = \frac{2\pi}{v} \left| T^{(i)} \right|^2 \delta(E'_{0i} - \ldots \quad (2\pi)^3 \quad (2\pi)^3 \quad (2\pi)^3$$

Regla de oro de Fermi

Reacciones de fragmentación en blancos ligeros (No Coulomb)

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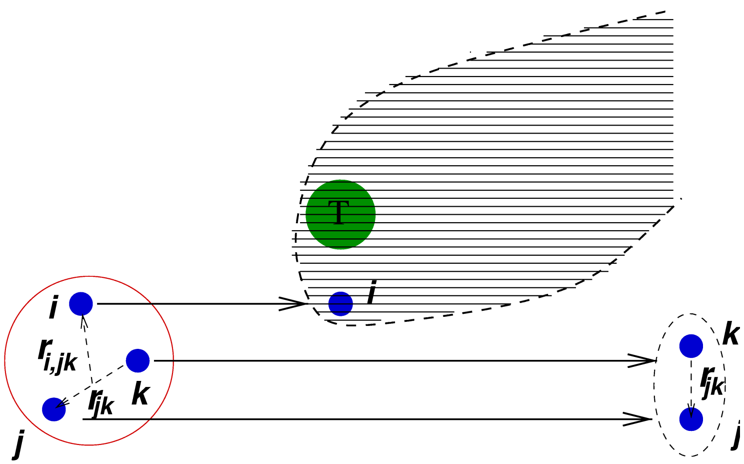

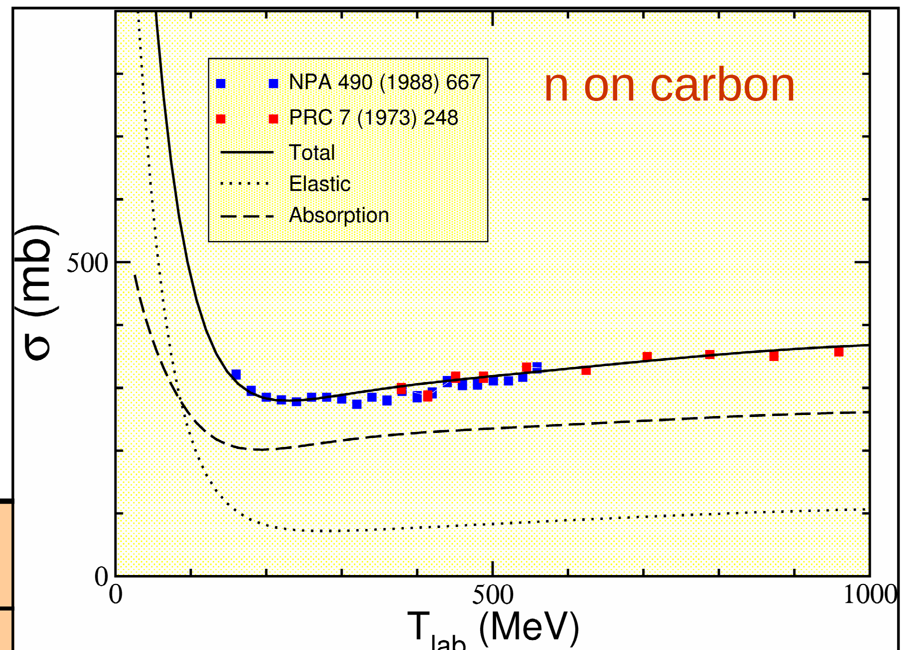

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Regla de oro de Fermi

Reacciones de fragmentación en blancos ligeros (No Coulomb)

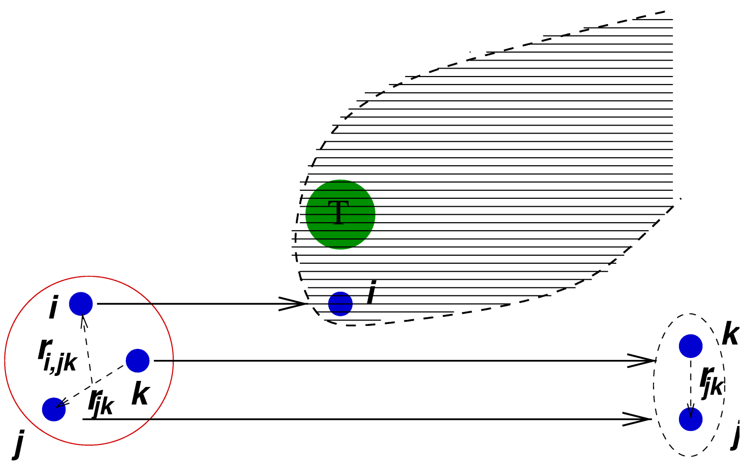
$$\frac{d^6\sigma^{(0i)}}{d\vec{p}'_{i,jk}\, d\vec{p}'_{jk}} \propto \sum \left| \left\langle \Phi^{(jk)}_{p'_{jk} s_{jk} \sigma_{jk}}\, e^{i\vec{p}_{i,jk}\cdot\vec{r}_{i,jk}}\, \chi_{s_i\sigma_i} \right| \Psi^{JM} \right\rangle \right|^2$$

Pure sudden approximation

Si no FSI

$$\Phi^{(jk)}_{p'_{jk} s_{jk} \sigma_{jk}} = e^{i\vec{p}_{jk}\cdot r_{jk}}$$

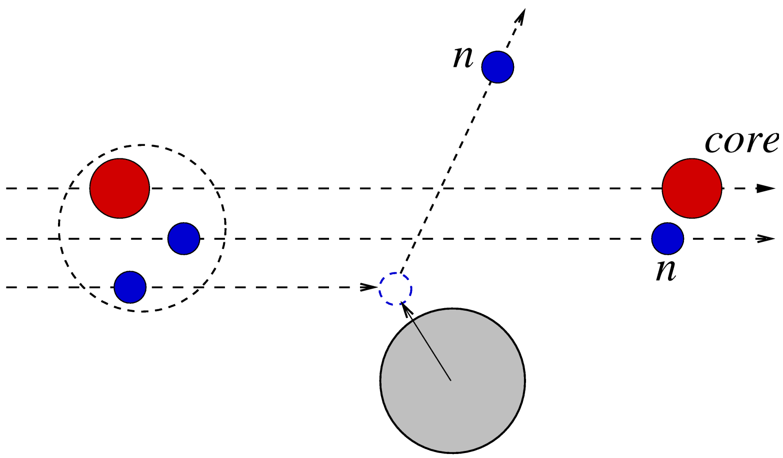
Transformada de Fourier



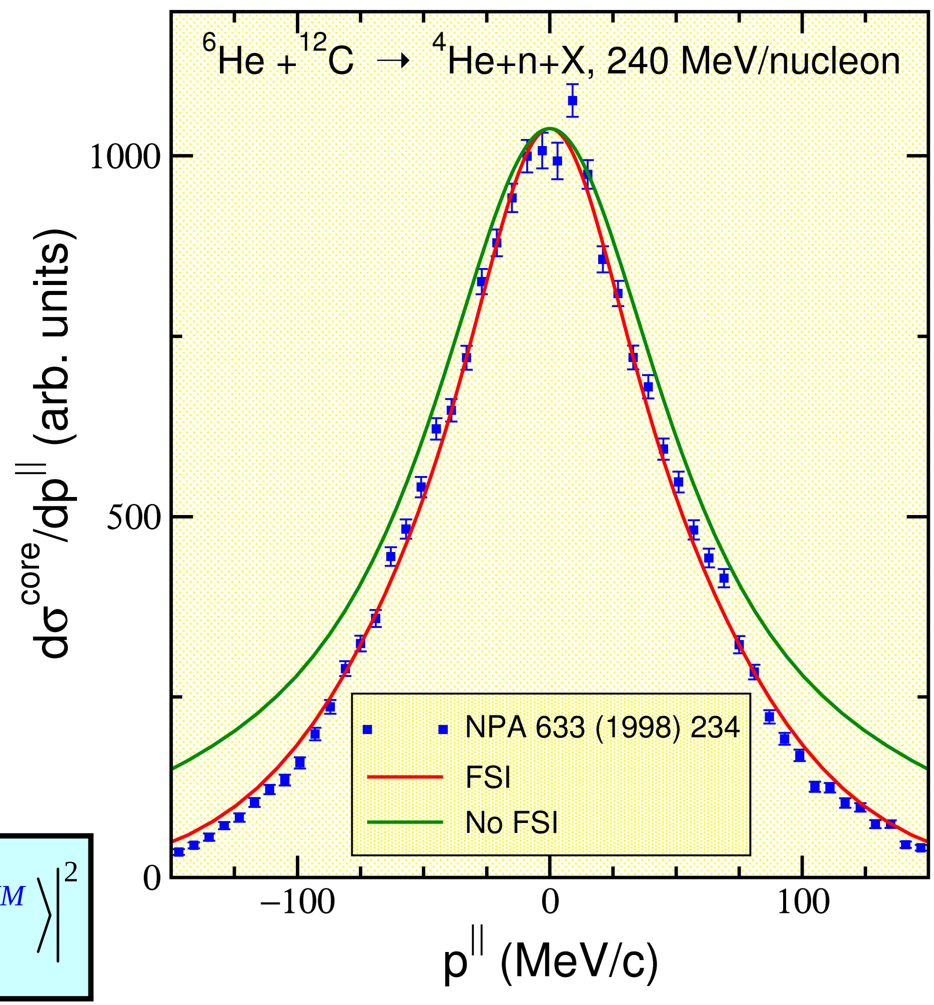
$$d\sigma^{(i)} = \frac{2\pi}{v} \left| T^{(i)} \right|^2 \delta(E'_{0i} - E_{0i}) \frac{d\vec{p}'_{0i}}{(2\pi)^3} \frac{d\vec{p}'_{jk}}{(2\pi)^3} \frac{d\vec{P}'}{(2\pi)^3}$$

Regla de oro de Fermi

El caso del ^{6}He: $n+n+\alpha$

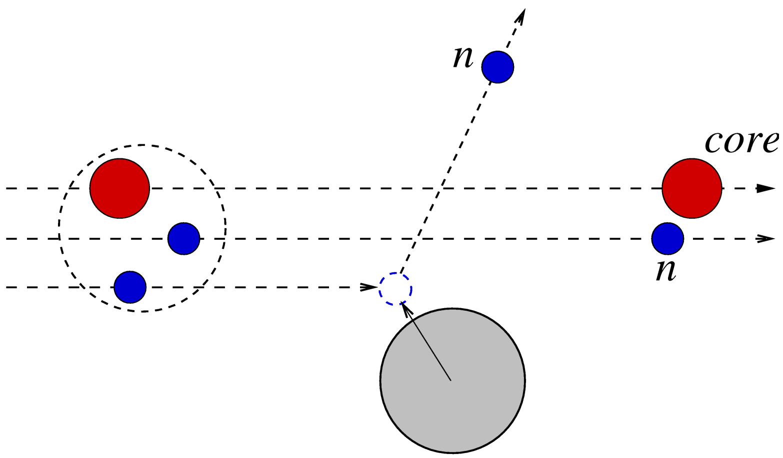


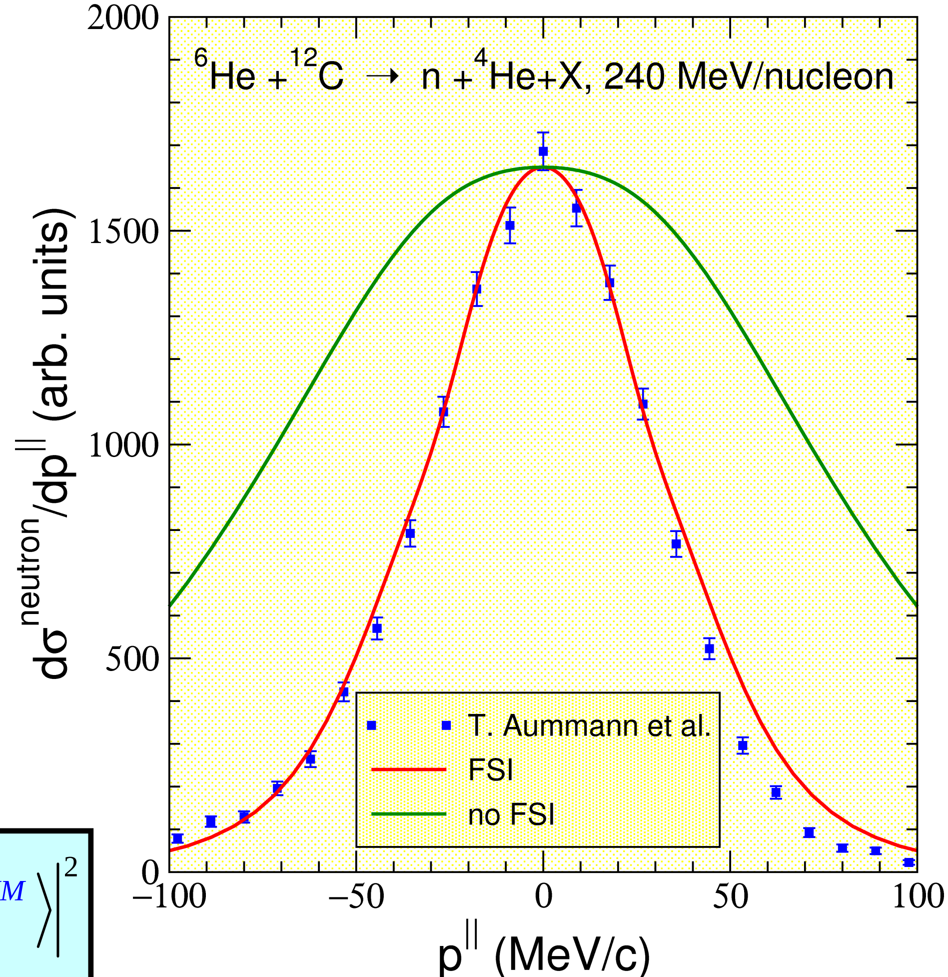
Distribución de momentos del "core"



$$\frac{d^6\sigma^{(0i)}}{d\vec{p}'_{i,jk}\, d\vec{p}'_{jk}} \propto \sum \left| \left\langle \Phi^{(jk)}_{p'_{jk} s_{jk} \sigma_{jk}}\, e^{i\vec{p}_{i,jk}\cdot\vec{r}_{i,jk}}\, \chi_{s_i\sigma_i} \middle| \Psi^{JM} \right\rangle \right|^2$$

El caso del ^{6}He: $n+n+\alpha$

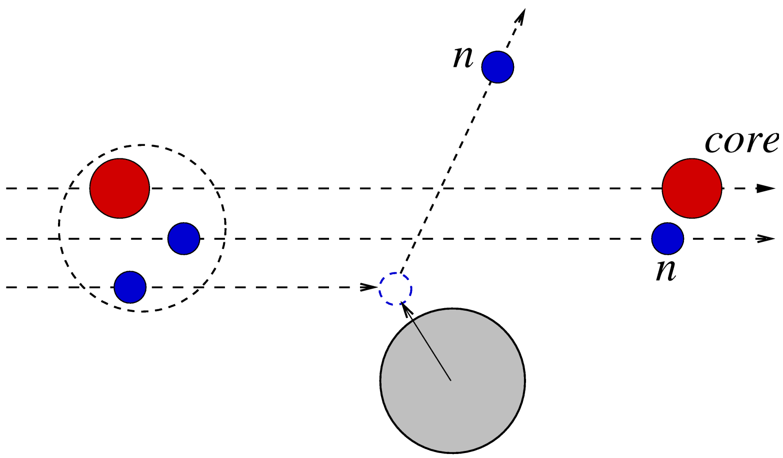




Distribución de momentos del neutrón

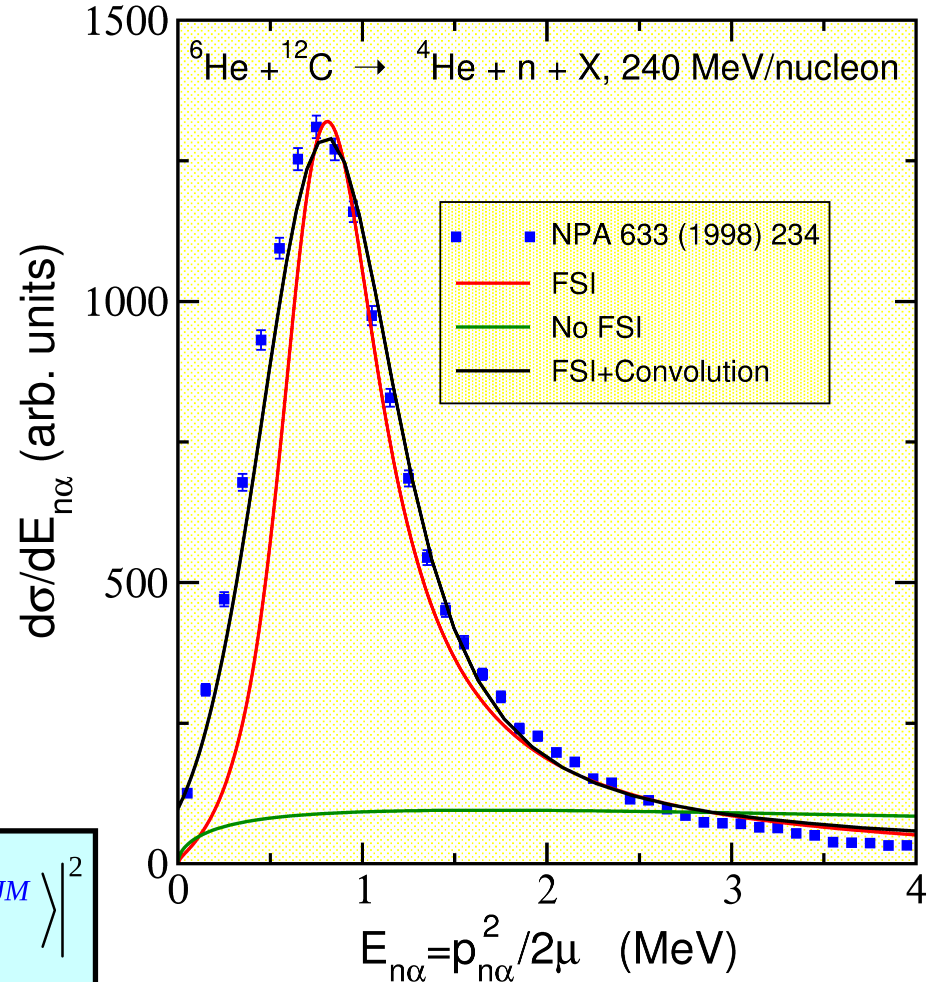
$$\frac{d^6\sigma^{(0i)}}{d\vec{p}'_{i,jk}\,d\vec{p}'_{jk}} \propto \sum \left| \left\langle \Phi^{(jk)}_{p'_{jk}s_{jk}\sigma_{jk}} e^{i\vec{p}_{i,jk}\cdot\vec{r}_{i,jk}} \chi_{s_i\sigma_i} \middle| \Psi^{JM} \right\rangle \right|^2$$

El caso del ^{6}He: $n+n+\alpha$

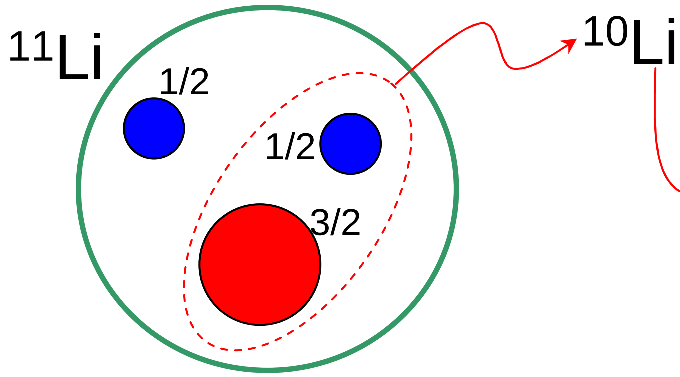


Distribución de momentos del energías en ^{5}He

$$\frac{d^6\sigma^{(0i)}}{d\vec{p}\,'_{i,jk}\,d\vec{p}\,'_{jk}} \propto \sum \left| \left\langle \Phi^{(jk)}_{p'_{jk}s_{jk}\sigma_{jk}} \, e^{i\vec{p}_{i,jk}\cdot\vec{r}_{i,jk}} \, \chi_{s_i\sigma_i} \middle| \Psi^{JM} \right\rangle \right|^2$$

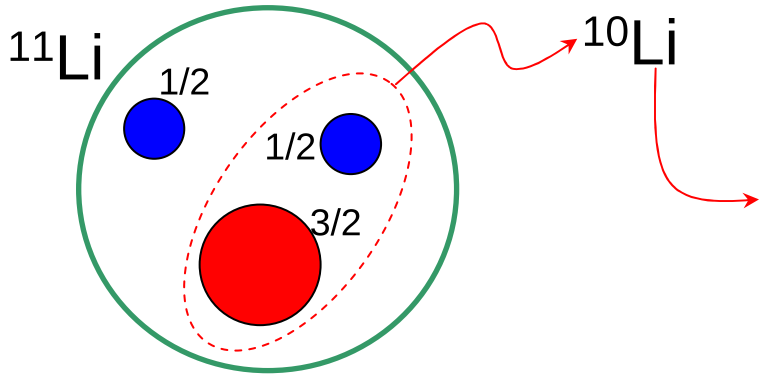


El caso del ^{11}Li: $n+n+{}^{9}Li$



1) E_s < 200 keV
2) E_p ~ 0.5 MeV
3) Contenido en onda *p*??

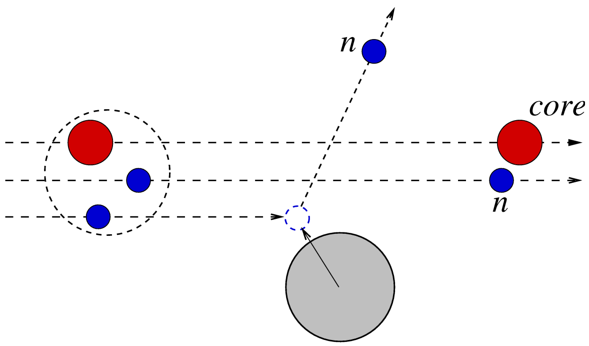
El caso del ^{11}Li: $n+n+^{9}Li$

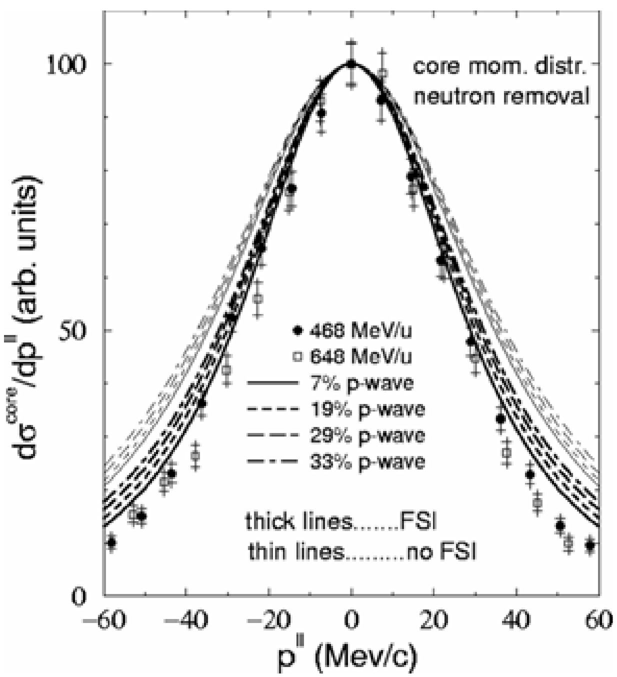


1) $E_s < 200$ keV
2) $E_p \sim 0.5$ MeV
3) Contenido en onda *p*??

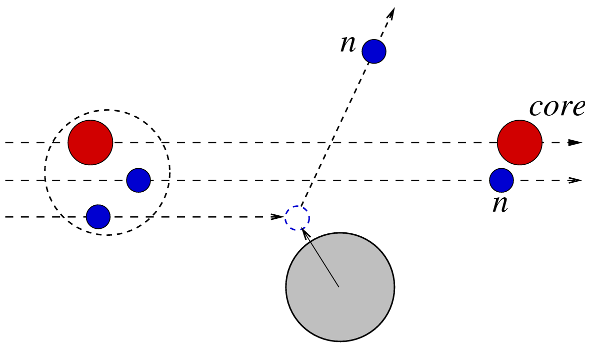
$$V^{(\ell)}_{^9Li-n}(r) = V_c^{(\ell)}(r) + V_{so}^{(\ell)}(r)\vec{\ell}_{^9Li-n} \cdot \vec{s}_n + V_{ss}^{(\ell)}(r)\vec{s}_c \cdot \vec{s}_n$$

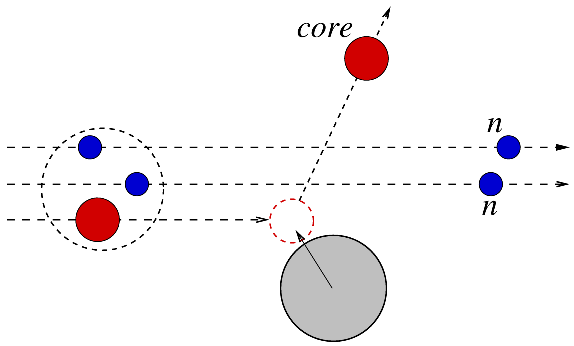
El caso del ^{11}Li: *n+n+^{9}Li*

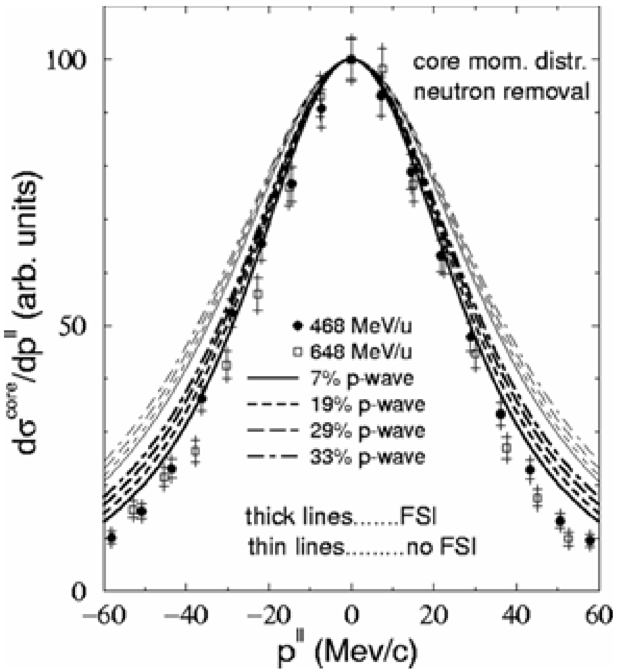


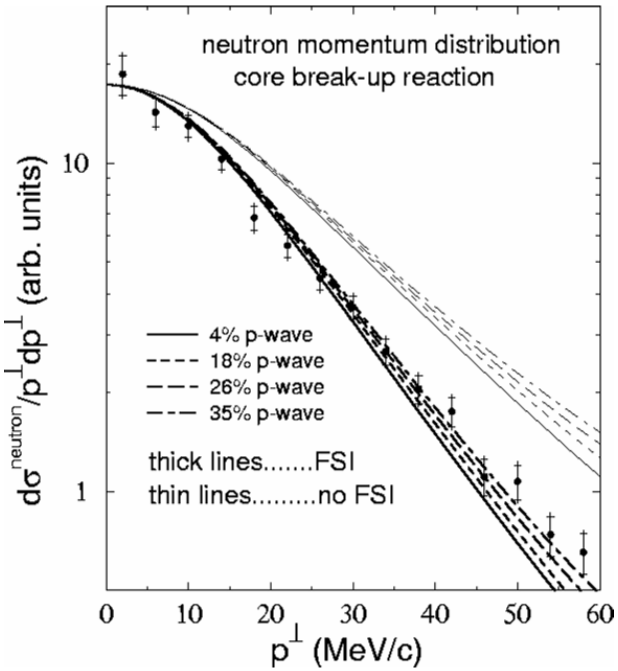


El caso del ^{11}Li: $n+n+^{9}Li$

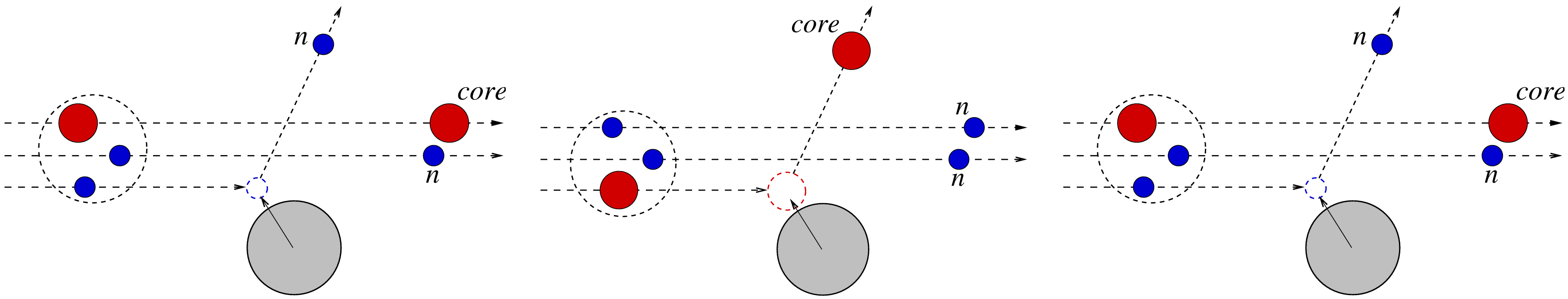


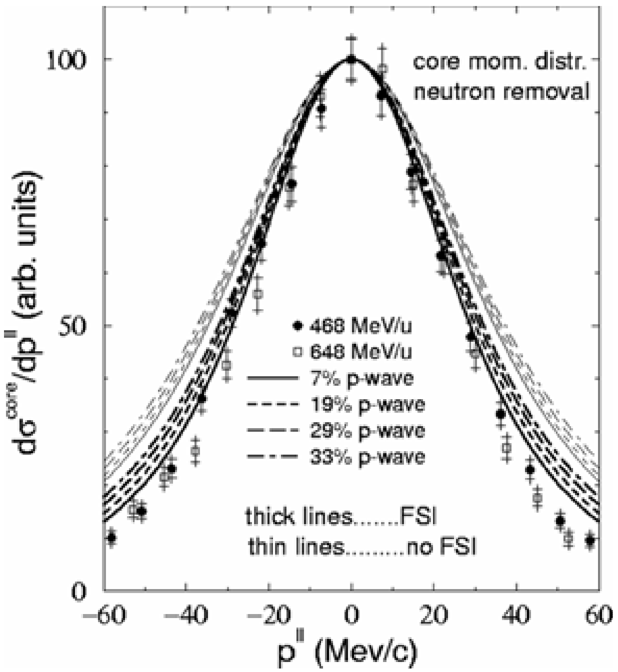


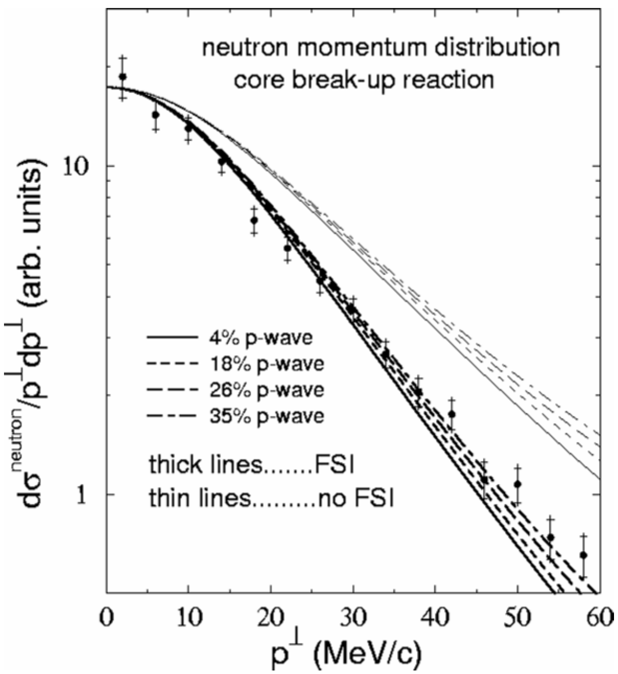


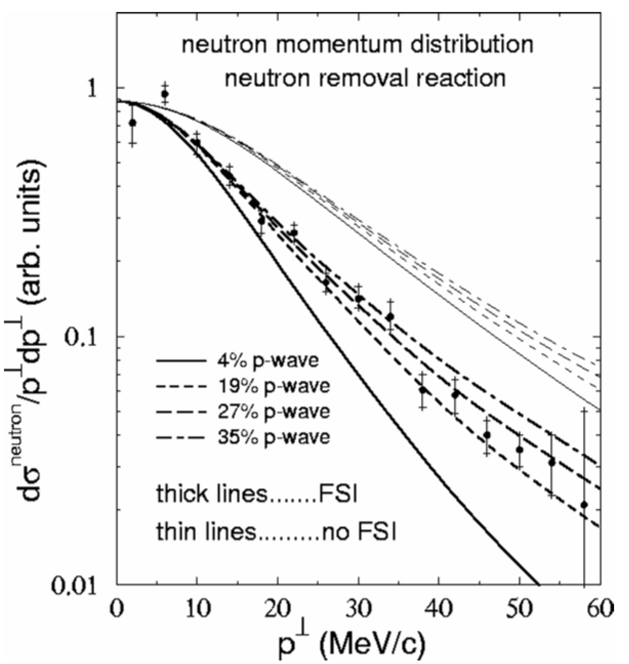


El caso del ^{11}Li: $n+n+^{9}Li$

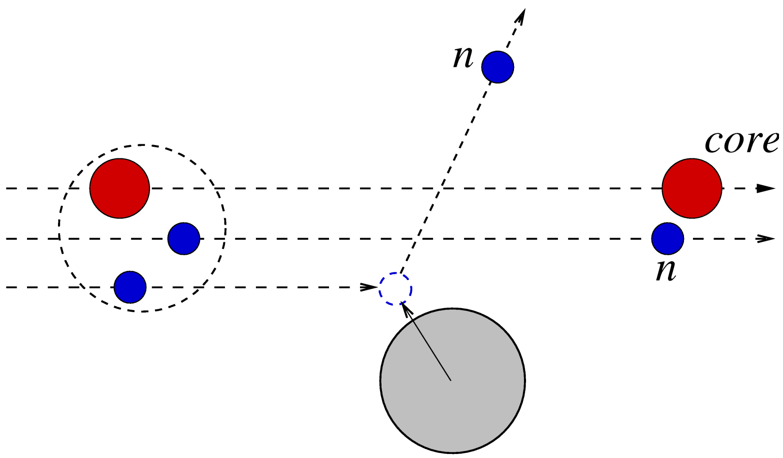




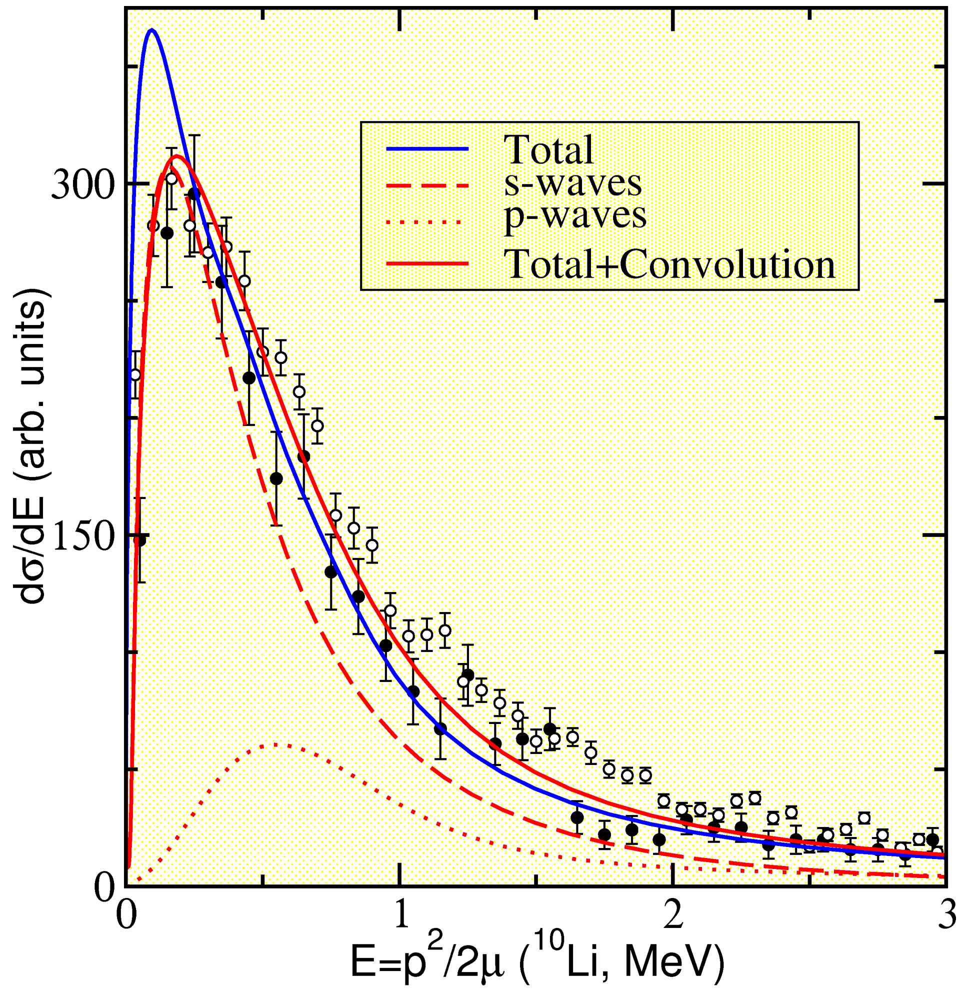




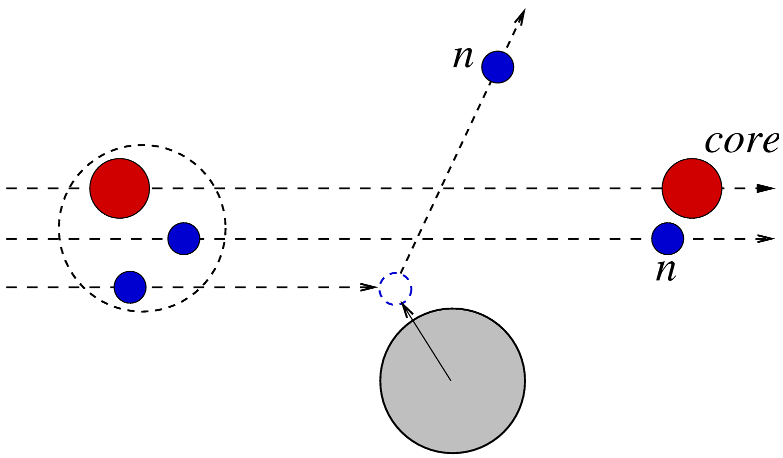
El caso del ^{11}Li: $n+n+^{9}Li$

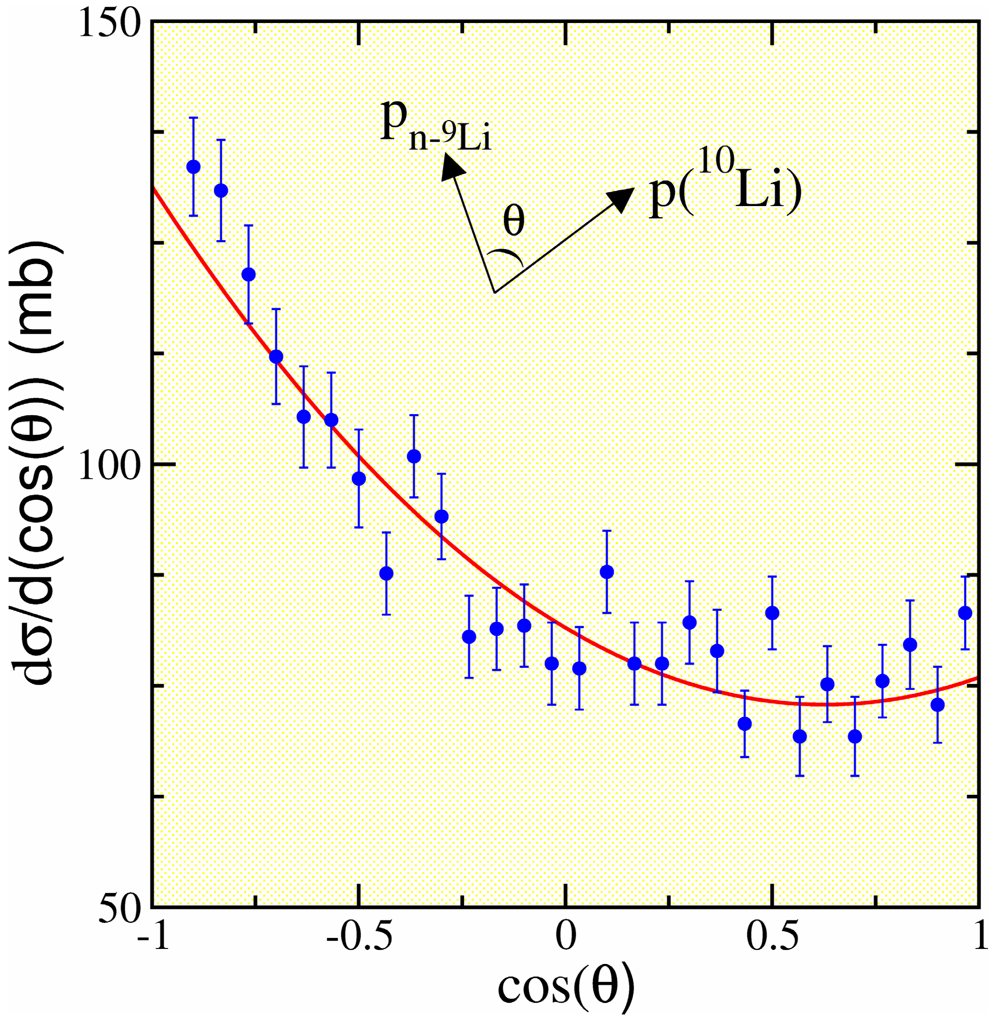


Distribución de momentos del energías en ^{10}Li



El caso del ^{11}Li: $n+n+^9Li$





Distribución angular del ^{10}Li

Ángulo entre el momento relativo ^{9}Li-neutrón y el momento del ^{10}Li

En la “sudden approximation”……

$$\frac{d^6\sigma^{(0i)}}{d\vec{p}\,'_{i,jk}\,d\vec{p}\,'_{jk}} \propto \sum \left| \left\langle \Phi^{(jk)}_{p'_{jk} s_{jk} \sigma_{jk}} e^{i\vec{p}_{i,jk}\cdot\vec{r}_{i,jk}} \chi_{s_i \sigma_i} \middle| \Psi^{JM} \right\rangle \right|^2$$

- ✓¿Dependencia en la energía del haz?
- ✓¿Dependencia en el blanco empleado?
- ✓¿Distribuciones longitudinales y transversales?
- ✓¿Valores absolutos de las secciones eficaces?

En la “sudden approximation”……

$$\frac{d^6\sigma^{(0i)}}{d\vec{p}'_{i,jk}\,d\vec{p}'_{jk}} \propto \sum \left| \left\langle \Phi^{(jk)}_{p'_{jk}s_{jk}\sigma_{jk}}\, e^{i\vec{p}_{i,jk}\cdot\vec{r}_{i,jk}}\, \chi_{s_i\sigma_i} \middle| \Psi^{JM} \right\rangle \right|^2$$

- ✓¿Dependencia en la energía del haz?
- ✓¿Dependencia en el blanco empleado?
- ✓¿Distribuciones longitudinales y transversales?
- ✓¿Valores absolutos de las secciones eficaces?

$$\frac{d^6\sigma^{(0i)}_{(\mathrm{elas,abs})}}{d\vec{p}'_{i,jk}\,d\vec{p}'_{jk}} = \sigma^{(0i)}_{(\mathrm{elas,abs})}\,\frac{1}{2J+1}\sum \left| \left\langle \Phi^{(jk)}_{p'_{jk}s_{jk}\sigma_{jk}}\, e^{i\vec{p}_{i,jk}\cdot\vec{r}_{i,jk}}\, \chi_{s_i\sigma_i} \middle| \Psi^{JM} \right\rangle \right|^2$$

Resumiendo….

- ✓Los núcleos ligeros próximos a la “dripline” de neutrones pueden presentar una estructura de halo: ^{6}He, ^{11}Li, ^{11}Be, ^{19}C….

- ✓Además de poder describirse como sistemas de pocos cuerpos los neutrones del halo residen preferentemente en la zona clásicamente prohibida.

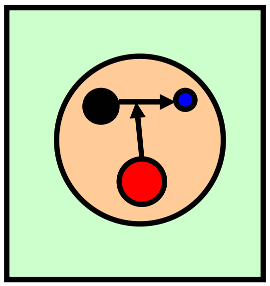
- ✓Experimentalmente se observa que estos núcleos presentan un comportamiento anómalo: Radios de interacción, disociación Coulombiana, distribuciones de momentos…

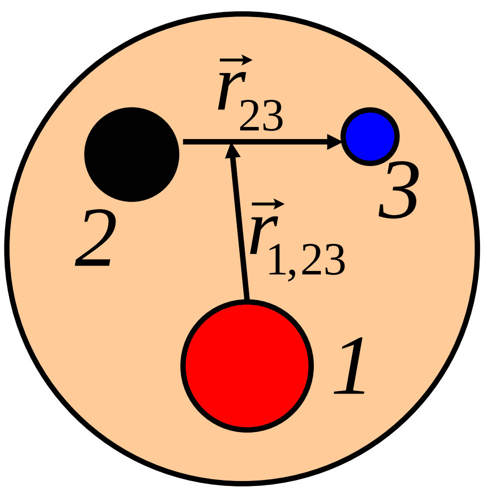
- ✓La descripción de estos núcleos como sistemas de pocos cuerpos permite reproducir datos experimentales, no sólo de su estructura, sino también de procesos de fragmentación.

Función de onda para un sistema de tres cuerpos

Función de onda para un sistema de tres cuerpos

Coordenadas de Jacobi:





Coordenadas de Jacobi

$$\vec{x} = \sqrt{\frac{\mu_{23}}{m}}\vec{r}_{23}$$

$$\vec{y} = \sqrt{\frac{\mu_{1,23}}{m}}\vec{r}_{1,23}$$

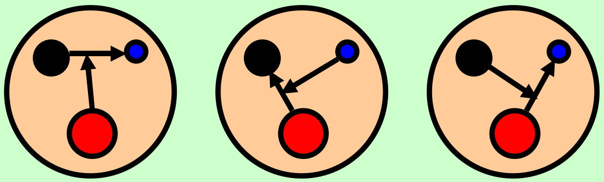
Hyperspheric coordinates

$$\rho^2 = x^2 + y^2$$

$$\alpha = \arctan(x/y),\ \Omega_x,\ \Omega_y$$

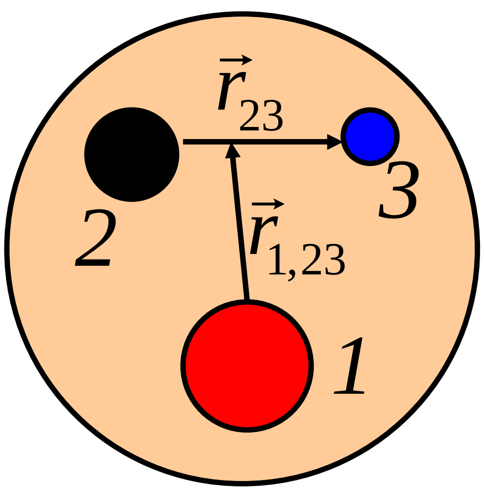
Función de onda para un sistema de tres cuerpos

Coordenadas de Jacobi:



✓¿¿Qué sistema de Jacobi??

▪Un subsistema de dos cuerpos está "privilegiado"



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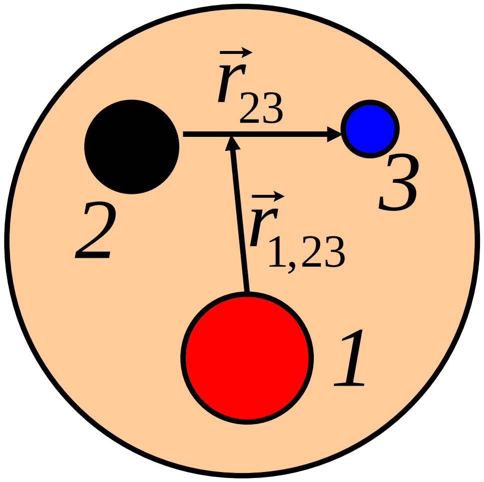
Función de onda para un sistema de tres cuerpos

Coordenadas de Jacobi:

¿Sistemas de N cuerpos?

$$\text{Coordenadas} \Rightarrow (\rho, \alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_{3N-4}) \qquad m\rho^2 = \sum_{i<k} \frac{m_i m_k}{M} (\vec{r}_i - \vec{r}_k)^2$$



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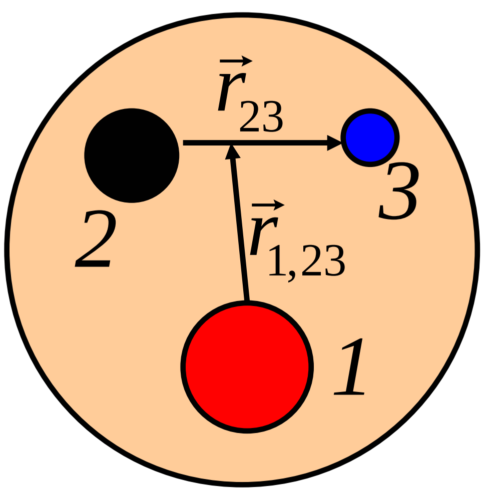
Función de onda para un sistema de tres cuerpos

Two-body ⇨ $$\left[\frac{p_r^2}{2\mu}+V(r)\right]\Psi(\vec{r})=E\Psi(\vec{r})$$



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Función de onda para un sistema de tres cuerpos

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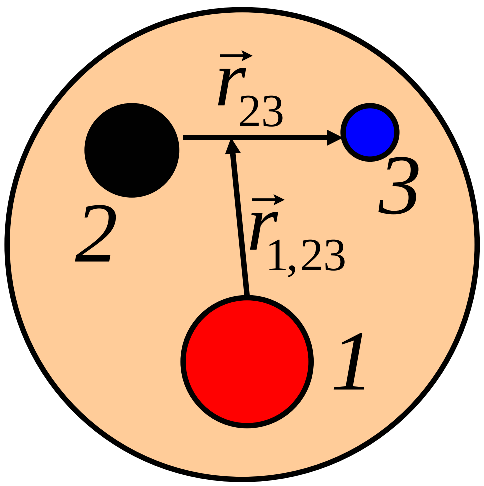
Three-body ⇨ $\left[\frac{p_{23}^2}{2\mu_{23}}+\frac{p_{1,23}^2}{2\mu_{1,23}}+V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right]\Psi(\vec{x},\vec{y})=E\Psi(\vec{x},\vec{y})$



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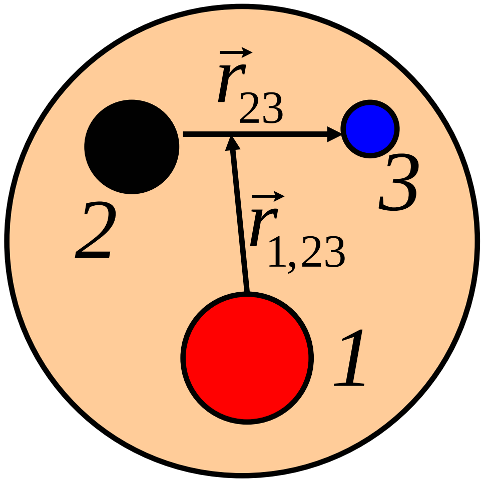
Función de onda para un sistema de tres cuerpos

Two-body ⇨ $$\frac{p_r^2}{2\mu} = -\frac{\hbar^2}{2\mu}\left[\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{\hat{L}^2(\Omega)}{r^2}\right]$$

Three-body ⇨ $$\left[\frac{p_{23}^2}{2\mu_{23}} + \frac{p_{1,23}^2}{2\mu_{1,23}} + V_{12}(r_{12}) + V_{13}(r_{13}) + V_{23}(r_{23})\right]\Psi(\vec{x},\vec{y}) = E\Psi(\vec{x},\vec{y})$$



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Función de onda para un sistema de tres cuerpos

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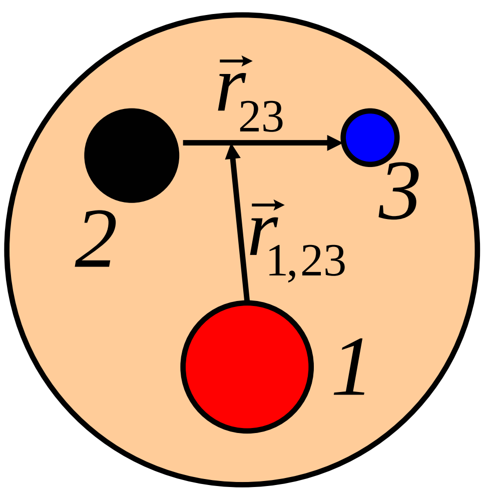
Three-body ⇨ $$\frac{p_{23}^2}{2\mu_{23}} + \frac{p_{1,23}^2}{2\mu_{1,23}} = -\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial \rho^2} + \frac{5}{\rho}\frac{\partial}{\partial \rho} - \frac{\hat{\Lambda}^2(\alpha,\Omega_x,\Omega_y)}{\rho^2}\right]$$



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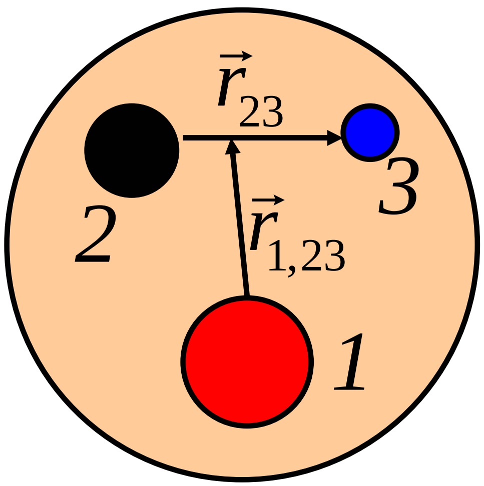
Función de onda para un sistema de tres cuerpos

Two-body ⇨ $\hat{L}^2 Y_{\ell m}(\Omega) = \ell(\ell+1) Y_{\ell m}(\Omega)$

Three-body ⇨ $\dfrac{p_{23}^2}{2\mu_{23}} + \dfrac{p_{1,23}^2}{2\mu_{1,23}} = -\dfrac{\hbar^2}{2m}\left[\dfrac{\partial^2}{\partial\rho^2} + \dfrac{5}{\rho}\dfrac{\partial}{\partial\rho} - \dfrac{\hat{\Lambda}^2(\alpha,\Omega_x,\Omega_y)}{\rho^2}\right]$



Coordenadas de Jacobi

$$\vec{x} = \sqrt{\frac{\mu_{23}}{m}}\,\vec{r}_{23}$$

$$\vec{y} = \sqrt{\frac{\mu_{1,23}}{m}}\,\vec{r}_{1,23}$$

Hyperspheric coordinates

$$\rho^2 = x^2 + y^2$$

$$\alpha = \arctan(x/y),\ \Omega_x,\ \Omega_y$$

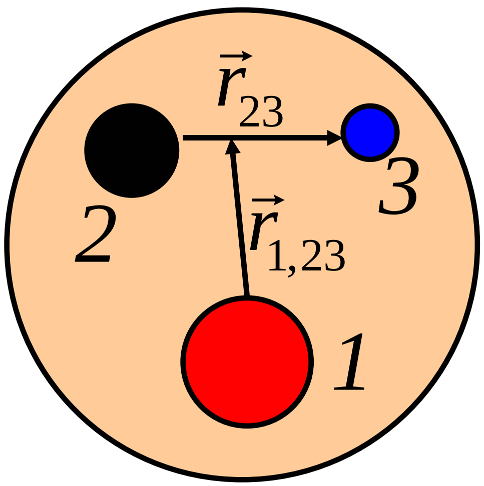
Función de onda para un sistema de tres cuerpos

Two-body ⇨ $\hat{L}^2 Y_{\ell m}(\Omega) = \ell(\ell+1) Y_{\ell m}(\Omega)$

Three-body ⇨ $\hat{\Lambda}^2 \mathcal{Y}^{KLM}_{\ell_x \ell_y} = K(K+4)\mathcal{Y}^{KLM}_{\ell_x \ell_y}$; $K = 2n + \ell_x + \ell_y$

hiperarmónicos esféricos

hipermomento



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Función de onda para un sistema de tres cuerpos

Two-body ⇨ $Y_{\ell m}(\Omega) = (-1)^m \sqrt{\frac{2\ell+1}{4\pi}} \sqrt{\frac{(\ell-m)!}{(\ell+m)!}} P_\ell^m(\cos\theta) e^{im\varphi}$

Three-body ⇨ $\hat{\Lambda}^2 \mathcal{Y}_{\ell_x \ell_y}^{KLM} = K(K+4) \mathcal{Y}_{\ell_x \ell_y}^{KLM}$; $K = 2n + \ell_x + \ell_y$

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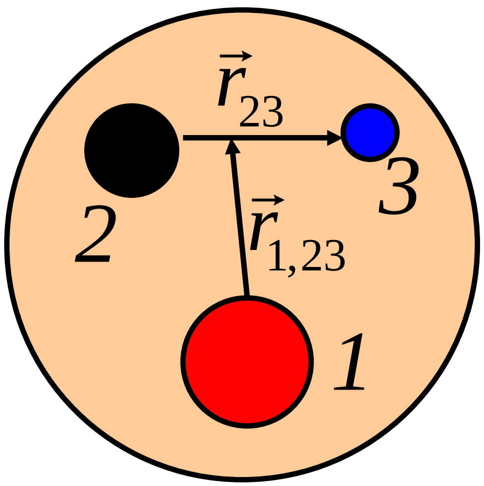

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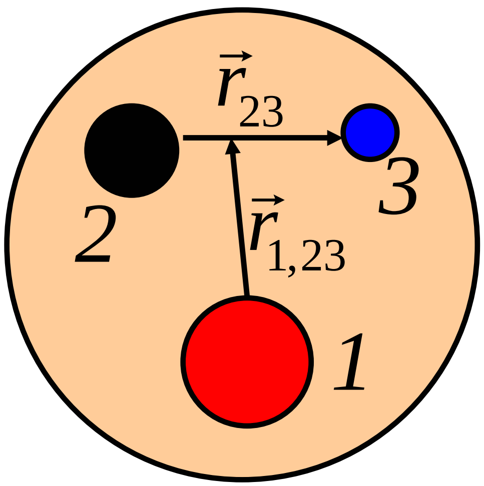
Three-body ⇨ $\mathcal{Y}_{\ell_x \ell_y}^{KLM} = N_K^{\ell_x \ell_y} (\sin\alpha)^{\ell_x} (\cos\alpha)^{\ell_y} P_n^{(\ell_x+\frac{1}{2},\ell_y+\frac{1}{2})}(\cos 2\alpha) \left[Y_{\ell_x} \otimes Y_{\ell_y} \right]^{LM}$



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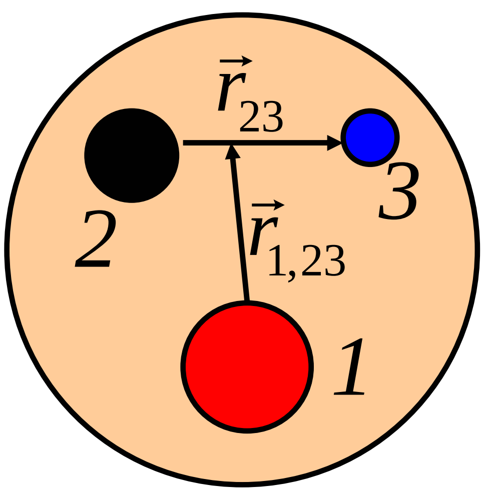
Función de onda para un sistema de tres cuerpos

Two-body ⇨ $\Psi^{\ell m}(\vec{r}) = \frac{u_\ell(r)}{r} Y_{\ell m}(\Omega)$

Three-body ⇨ $\mathcal{Y}^{KLM}_{\ell_x \ell_y} = N_K^{\ell_x \ell_y} (\sin\alpha)^{\ell_x} (\cos\alpha)^{\ell_y} P_n^{(\ell_x+\frac{1}{2},\ell_y+\frac{1}{2})}(\cos 2\alpha)\left[Y_{\ell_x} \otimes Y_{\ell_y}\right]^{LM}$



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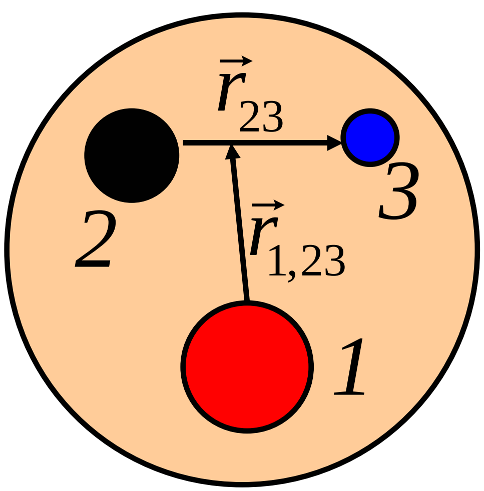
Three-body ⇨ $\Psi^{LM}(\vec{x},\vec{y}) = \sum_{K\ell_x\ell_y} \frac{\chi^{L}_{K\ell_x\ell_y}(\rho)}{\rho^{5/2}} \mathcal{Y}^{KLM}_{\ell_x\ell_y}(\alpha,\Omega_x,\Omega_y)$



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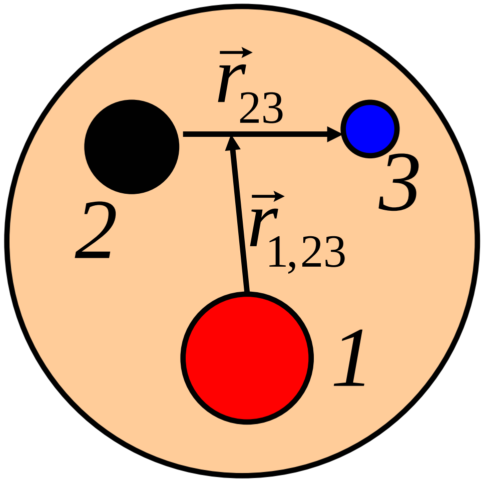
Función de onda para un sistema de tres cuerpos

Two-body ⇨ $$\left[-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}-\frac{\hat{L}^2}{r^2}\right)+V(r)\right]\frac{u_\ell(r)}{r}Y_{\ell m}(\Omega)=E\frac{u_\ell(r)}{r}Y_{\ell m}(\Omega)$$

Three-body ⇨ $$\Psi^{LM}(\vec{x},\vec{y})=\sum_{K\ell_x\ell_y}\frac{\chi^L_{K\ell_x\ell_y}(\rho)}{\rho^{5/2}}\mathcal{Y}^{KLM}_{\ell_x\ell_y}(\alpha,\Omega_x,\Omega_y)$$



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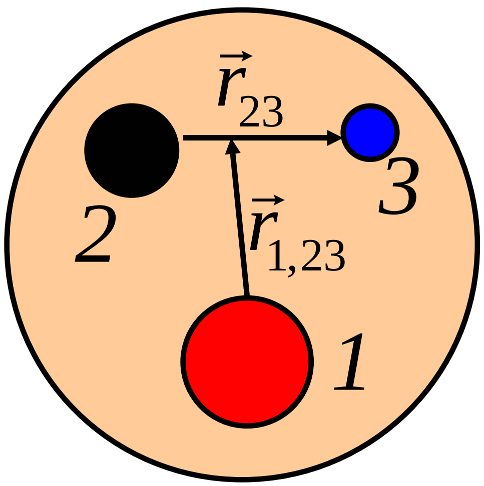
Función de onda para un sistema de tres cuerpos

Two-body ⇨ $$\left[-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}-\frac{\hat{L}^2}{r^2}\right)+V(r)\right]\frac{u_\ell(r)}{r}Y_{\ell m}(\Omega)=E\,\frac{u_\ell(r)}{r}Y_{\ell m}(\Omega)$$

Three-body ⇨ $$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}-\frac{\hat{\Lambda}^2}{\rho^2}\right)+V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right]\sum_{K\ell_x\ell_y}\frac{\chi^L_{K\ell_x\ell_y}(\rho)}{\rho^{5/2}}\mathcal{Y}^{KLM}_{\ell_x\ell_y}=E\sum_{K\ell_x\ell_y}\frac{\chi^L_{K\ell_x\ell_y}(\rho)}{\rho^{5/2}}\mathcal{Y}^{KLM}_{\ell_x\ell_y}$$



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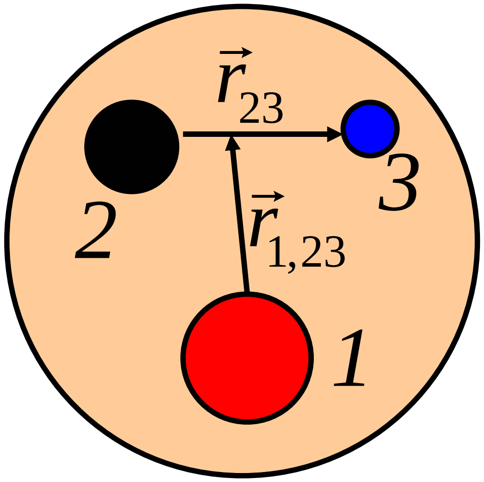
Función de onda para un sistema de tres cuerpos

Two-body ⇨ $$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+(V(r)-E)+\frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}\right]u_\ell(r)=0$$

Three-body ⇨ $$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}-\frac{\hat{\Lambda}^2}{\rho^2}\right)+V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right]\sum_{K\ell_x\ell_y}\frac{\chi^L_{K\ell_x\ell_y}(\rho)}{\rho^{5/2}}\mathcal{Y}^{KLM}_{\ell_x\ell_y}=E\sum_{K\ell_x\ell_y}\frac{\chi^L_{K\ell_x\ell_y}(\rho)}{\rho^{5/2}}\mathcal{Y}^{KLM}_{\ell_x\ell_y}$$



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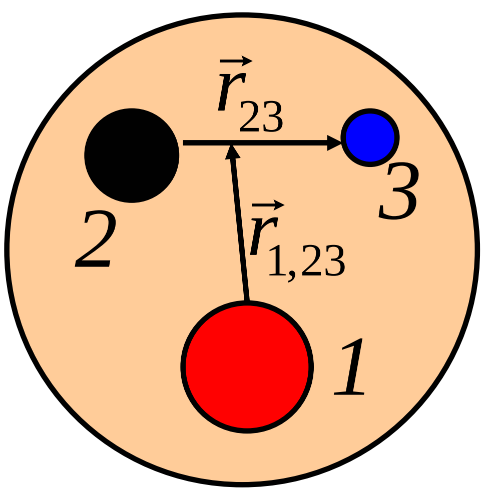
Three-body ⇨ $$\left[-\frac{\hbar^2}{2m}\frac{d^2}{d\rho^2}-E+\frac{\hbar^2}{2m}\frac{(K+\frac{3}{2})(K+\frac{5}{2})}{\rho^2}\right]\chi^L_{K\ell_x\ell_y}(\rho)+\sum_{K'\ell'_x\ell'_y}V_{K\ell_x\ell_y,K'\ell'_x\ell'_y}(\rho)\chi^L_{K'\ell'_x\ell'_y}(\rho)=0$$



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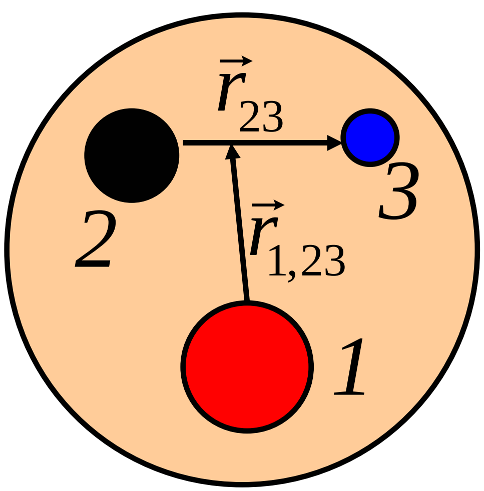
Función de onda para un sistema de tres cuerpos

Two-body ⇨ $$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+(V(r)-E)+\frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}\right]u_\ell(r)=0$$

$$\left\langle \mathcal{Y}^{KLM}_{\ell_x\ell_y}(\alpha_{23},\Omega_{23},\Omega_{1,23})\middle| V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23}) \middle| \mathcal{Y}^{K'LM}_{\ell'_x\ell'_y}(\alpha_{23},\Omega_{23},\Omega_{1,23})\right\rangle$$

Three-body ⇨ $$\left[-\frac{\hbar^2}{2m}\frac{d^2}{d\rho^2}-E+\frac{\hbar^2}{2m}\frac{(K+\frac{3}{2})(K+\frac{5}{2})}{\rho^2}\right]\chi^L_{K\ell_x\ell_y}(\rho)+\sum_{K'\ell'_x\ell'_y} V_{K\ell_x\ell_y,K'\ell'_x\ell'_y}(\rho)\,\chi^L_{K'\ell'_x\ell'_y}(\rho)=0$$



Coordenadas de Jacobi

$$\vec{x}=\sqrt{\frac{\mu_{23}}{m}}\vec{r}_{23}$$

$$\vec{y}=\sqrt{\frac{\mu_{1,23}}{m}}\vec{r}_{1,23}$$

$$r_{23}=\sqrt{m/\mu_{23}}\,\rho\sin\alpha_{23}$$
$$r_{12}=\sqrt{m/\mu_{12}}\,\rho\sin\alpha_{12}$$
$$r_{13}=\sqrt{m/\mu_{13}}\,\rho\sin\alpha_{13}$$

Hyperspheric coordinates

$$\rho^2=x^2+y^2$$

$$\alpha=\arctan(x/y),\ \Omega_x,\ \Omega_y$$

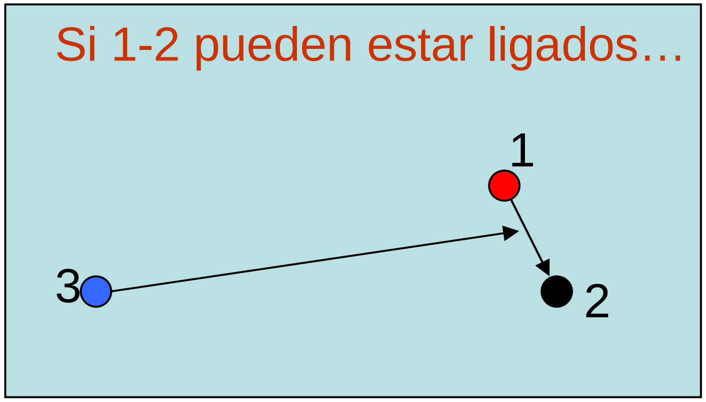
Función de onda para un sistema de tres cuerpos

Uno de los subsistemas de dos cuerpos está “privilegiado”

Three-body ⇨ $$\left[-\frac{\hbar^2}{2m}\frac{d^2}{d\rho^2}-E+\frac{\hbar^2}{2m}\frac{(K+\frac{3}{2})(K+\frac{5}{2})}{\rho^2}\right]\chi^L_{K\ell_x\ell_y}(\rho)+\sum_{K'\ell'_x\ell'_y} V_{K\ell_x\ell_y,K'\ell'_x\ell'_y}(\rho)\chi^L_{K'\ell'_x\ell'_y}(\rho)=0$$

✓La **expansión en hiperarmónicos esféricos** no funciona bien cuando al menos dos de los subsistemas de dos cuerpos están muy ligados o muy poco ligados.

✓En ambos casos no describe bien el comportamiento asintótico de la función de onda. La convergencia es muy lenta. Se necesita una base casi “infinita”.



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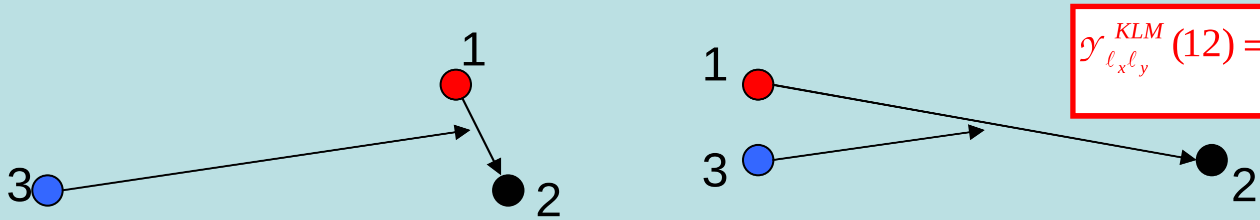
Si 1-2 y también 1-3 pueden estar ligados…

$$\left\langle \mathcal{Y}^{KLM}_{\ell_x\ell_y}(\alpha_{12},\Omega_{12},\Omega_{3,12}) \middle| V_{13}(r_{13}) \middle| \mathcal{Y}^{K'LM}_{\ell'_x\ell'_y}(\alpha_{12},\Omega_{12},\Omega_{3,12})\right\rangle$$

$$\mathcal{Y}^{KLM}_{\ell_x\ell_y}(12)=\sum_{\ell'_x\ell'_y}\left[\ell_x\ell_y,\ell'_x\ell'_y\right]_{KL}\mathcal{Y}^{KLM}_{\ell'_x\ell'_y}(13)$$



PHYSICAL REVIEW C **74**, 014603 (2006)

Continuum spectroscopy of Borromean two-neutron halo nuclei

S. N. Ershov,[*] B. V. Danilin,[†] and J. S. Vaagen

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(Received 27 March 2006; published 11 July 2006)

Energy and angular correlation distributions of the three fragments in ^{6}He breakup on ^{208}Pb at a collision energy of 240 MeV/nucleon are discussed within the microscopic four-body distorted wave model and compared with experimental data. The nuclear structure of the ground state and low-energy three-body continuum of ^{6}He is calculated by the method of hyperspherical harmonics within the three-body cluster model. Reflections of the fundamental permutation symmetry of the halo neutrons in angular and energy correlations are pointed out. The calculations describe the experimental data for fragment correlations near breakup threshold rather well, and the physics is contained in a few elementary modes; but with increasing excitation energy of ^{6}He, some striking deviations from experimental distributions are encountered. Possible reasons for this are discussed.

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I. INTRODUCTION

Halos represent a new type of nuclear structure that has been found in some light nuclei at the limits of nuclear existence. Peculiarities of halos are revealed in the specific structure of the ground state (loosely bound, abnormal spatial extention with extreme clusterization) as well as in low-energy excitations above the breakup threshold where a concentration of transition strength is observed. The nature and properties of the three-body continuum for Borromean halo nuclei present a most intriguing question.

The task of continuum spectroscopy is to determine which modes of nuclear excitation are dominant at a given excitation energy or in some region of excitation energies. For two-complete measurements when three particles—halo neutrons and core—are detected in coincidence. Then it is possible to reconstruct the spectrum of the halo nucleus and select events that correspond to low-energy excitations. For fixed excitation energy, the three fragments can still move relative to each other in a variety of ways. Thus, in parallel to the excitation spectrum, we can study many different angular and energy correlations between fragments. Three-body correlations are sensitive to different aspects of reaction dynamics. Therefore, continuum spectroscopy implies a consistent analysis of a variety of exclusive and inclusive cross sections accessible in kinematically complete experiments.

Recently, experimental data on different angular and energy correlations of the three fragments from the breakup of ^{6}He

Volvemos a empezar….

Three-body ⇨ $$\left[\frac{p_{23}^2}{2\mu_{23}}+\frac{p_{1,23}^2}{2\mu_{1,23}}+V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right]\Psi(\vec{x},\vec{y})=E\Psi(\vec{x},\vec{y})$$

$$T=\frac{p_{23}^2}{2\mu_{23}}+\frac{p_{1,23}^2}{2\mu_{1,23}}=-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}-\frac{\hat{\Lambda}^2(\alpha,\Omega_x,\Omega_y)}{\rho^2}\right]$$

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}-\frac{\hat{\Lambda}^2}{\rho^2}\right)+V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right]\Psi(\vec{x},\vec{y})=E\Psi(\vec{x},\vec{y})$$

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$$\Psi^{LM}(\vec{x},\vec{y})=\sum_{K\ell_x\ell_y}\frac{\chi_{K\ell_x\ell_y}^{L}(\rho)}{\rho^{5/2}}\mathcal{Y}_{\ell_x\ell_y}^{KLM}(\alpha,\Omega_x,\Omega_y)$$

Volvemos a empezar….

Three-body ⇨ $$\left[\frac{p_{23}^2}{2\mu_{23}}+\frac{p_{1,23}^2}{2\mu_{1,23}}+V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right]\Psi(\vec{x},\vec{y})=E\Psi(\vec{x},\vec{y})$$

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$$\Psi^{LM}(\vec{x},\vec{y})=\sum_{K\ell_x\ell_y}\frac{\chi_{K\ell_x\ell_y}(\rho)}{\rho^{5/2}}\mathcal{Y}^{KLM}_{\ell_x\ell_y}(\alpha,\Omega_x,\Omega_y)$$

¡¡Cambiamos!!

$$\Psi^{LM}(\vec{x},\vec{y})=\frac{1}{\rho^{5/2}}\sum_n f_n(\rho)\Phi_n(\rho,\Omega)$$

Volvemos a empezar….

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}-\frac{\hat{\Lambda}^2}{\rho^2}\right)+V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right]\Psi(\vec{x},\vec{y})=E\Psi(\vec{x},\vec{y})$$

$$\Psi^{LM}(\vec{x},\vec{y})=\frac{1}{\rho^{5/2}}\sum_n f_n(\rho)\Phi_n(\rho,\Omega)$$

$$\sum_n\left[-\frac{15}{4}\frac{1}{\rho^2}f_n\Phi_n+\frac{\partial^2 f_n}{\partial\rho^2}\Phi_n+2\frac{\partial f_n}{\partial\rho}\frac{\partial\Phi_n}{\partial\rho}+f_n\frac{\partial^2\Phi_n}{\partial\rho^2}+\frac{2mE}{\hbar^2}f_n\Phi_n-\frac{f_n}{\rho^2}\left(\hat{\Lambda}^2\Phi_n+\frac{2m\rho^2}{\hbar^2}\left(V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right)\Phi_n\right)\right]=0$$

Volvemos a empezar….

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}-\frac{\hat{\Lambda}^2}{\rho^2}\right)+V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right]\Psi(\vec{x},\vec{y})=E\Psi(\vec{x},\vec{y})$$

$$\Psi^{LM}(\vec{x},\vec{y})=\frac{1}{\rho^{5/2}}\sum_n f_n(\rho)\Phi_n(\rho,\Omega)$$

$$\sum_n\left[-\frac{15}{4}\frac{1}{\rho^2}f_n\Phi_n+\frac{\partial^2 f_n}{\partial\rho^2}\Phi_n+2\frac{\partial f_n}{\partial\rho}\frac{\partial\Phi_n}{\partial\rho}+f_n\frac{\partial^2\Phi_n}{\partial\rho^2}+\frac{2mE}{\hbar^2}f_n\Phi_n-\frac{f_n}{\rho^2}\left(\hat{\Lambda}^2\Phi_n+\frac{2m\rho^2}{\hbar^2}\left(V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right)\Phi_n\right)\right]=0$$

Volvemos a empezar….

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}-\frac{\hat{\Lambda}^2}{\rho^2}\right)+V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right]\Psi(\vec{x},\vec{y})=E\Psi(\vec{x},\vec{y})$$

$$\Psi^{LM}(\vec{x},\vec{y})=\frac{1}{\rho^{5/2}}\sum_n f_n(\rho)\Phi_n(\rho,\Omega)$$

$$\sum_n\left[-\frac{15}{4}\frac{1}{\rho^2}f_n\Phi_n+\frac{\partial^2 f_n}{\partial\rho^2}\Phi_n+2\frac{\partial f_n}{\partial\rho}\frac{\partial\Phi_n}{\partial\rho}+f_n\frac{\partial^2\Phi_n}{\partial\rho^2}+\frac{2mE}{\hbar^2}f_n\Phi_n-\frac{f_n}{\rho^2}\left(\hat{\Lambda}^2\Phi_n+\frac{2m\rho^2}{\hbar^2}\left(V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right)\Phi_n\right)\right]=0$$

Aproximación adiabática

ρ varía mucho más lentamente que Ω

Resolvemos primero la parte angular para valores fijos de ρ

Volvemos a empezar….

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}-\frac{\hat{\Lambda}^2}{\rho^2}\right)+V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right]\Psi(\vec{x},\vec{y})=E\Psi(\vec{x},\vec{y})$$

$$\Psi^{LM}(\vec{x},\vec{y})=\frac{1}{\rho^{5/2}}\sum_n f_n(\rho)\Phi_n(\rho,\Omega)$$

$$\sum_n\left[-\frac{15}{4}\frac{1}{\rho^2}f_n\Phi_n+\frac{\partial^2 f_n}{\partial\rho^2}\Phi_n+2\frac{\partial f_n}{\partial\rho}\frac{\partial\Phi_n}{\partial\rho}+f_n\frac{\partial^2\Phi_n}{\partial\rho^2}+\frac{2mE}{\hbar^2}f_n\Phi_n-\frac{f_n}{\rho^2}\left(\hat{\Lambda}^2\Phi_n+\frac{2m\rho^2}{\hbar^2}\left(V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right)\Phi_n\right)\right]=0$$

Aproximación adiabática

ρ varía mucho más lentamente que Ω

Resolvemos primero la parte angular para valores fijos de ρ

$$\hat{\Lambda}^2\Phi_n(\rho,\Omega)+\frac{2m\rho^2}{\hbar^2}\left(V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right)\Phi_n(\rho,\Omega)=\lambda_n(\rho)\Phi_n(\rho,\Omega)$$

Volvemos a empezar….

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}-\frac{\hat{\Lambda}^2}{\rho^2}\right)+V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right]\Psi(\vec{x},\vec{y})=E\Psi(\vec{x},\vec{y})$$

$$\Psi^{LM}(\vec{x},\vec{y})=\frac{1}{\rho^{5/2}}\sum_n f_n(\rho)\Phi_n(\rho,\Omega)$$

$$\sum_n\left[-\frac{15}{4}\frac{1}{\rho^2}f_n\Phi_n+\frac{\partial^2 f_n}{\partial\rho^2}\Phi_n+2\frac{\partial f_n}{\partial\rho}\frac{\partial\Phi_n}{\partial\rho}+f_n\frac{\partial^2\Phi_n}{\partial\rho^2}+\frac{2mE}{\hbar^2}f_n\Phi_n-\frac{f_n}{\rho^2}\left(\underbrace{\hat{\Lambda}^2\Phi_n+\frac{2m\rho^2}{\hbar^2}\left(V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right)\Phi_n}\right)\right]=0$$

Aproximación adiabática

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$$\hat{\Lambda}^2\Phi_n(\rho,\Omega)+\frac{2m\rho^2}{\hbar^2}\left(V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right)\Phi_n(\rho,\Omega)=\lambda_n(\rho)\Phi_n(\rho,\Omega)$$

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Aproximación adiabática

Las funciones Φ_n son las autofunciones de la parte angular

Las funciones Φ_n son por tanto ortogonales

$$\hat{\Lambda}^2\Phi_n(\rho,\Omega)+\frac{2m\rho^2}{\hbar^2}\left(V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right)\Phi_n(\rho,\Omega)=\lambda_n(\rho)\Phi_n(\rho,\Omega)$$

Volvemos a empezar….

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}-\frac{\hat{\Lambda}^2}{\rho^2}\right)+V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right]\Psi(\vec{x},\vec{y})=E\Psi(\vec{x},\vec{y})$$

$$\Psi^{LM}(\vec{x},\vec{y})=\frac{1}{\rho^{5/2}}\sum_n f_n(\rho)\Phi_n(\rho,\Omega)$$

$$\sum_n\left[-\frac{15}{4}\frac{1}{\rho^2}f_n\Phi_n+\frac{\partial^2 f_n}{\partial\rho^2}\Phi_n+2\frac{\partial f_n}{\partial\rho}\frac{\partial\Phi_n}{\partial\rho}+f_n\frac{\partial^2\Phi_n}{\partial\rho^2}+\frac{2mE}{\hbar^2}f_n\Phi_n-\frac{f_n}{\rho^2}\lambda_n(\rho)\Phi_n\right]=0$$

$$\left[\frac{\partial^2}{\partial\rho^2}+\frac{2mE}{\hbar^2}-\frac{1}{\rho^2}\left(\lambda_n(\rho)+\frac{15}{4}\right)\right]f_n(\rho)+\sum_{n'}\left[2P_{nn'}(\rho)\frac{\partial}{\partial\rho}+Q_{nn'}(\rho)\right]f_{n'}(\rho)=0$$

$$\hat{\Lambda}^2\Phi_n(\rho,\Omega)+\frac{2m\rho^2}{\hbar^2}\left(V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right)\Phi_n(\rho,\Omega)=\lambda_n(\rho)\Phi_n(\rho,\Omega)$$

Volvemos a empezar….

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}-\frac{\hat{\Lambda}^2}{\rho^2}\right)+V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right]\Psi(\vec{x},\vec{y})=E\Psi(\vec{x},\vec{y})$$

$$\Psi^{LM}(\vec{x},\vec{y})=\frac{1}{\rho^{5/2}}\sum_n f_n(\rho)\Phi_n(\rho,\Omega)$$

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$$P_{nn'}(\rho)=\left\langle\Phi_n(\rho,\Omega)\left|\frac{\partial}{\partial\rho}\right|\Phi_{n'}(\rho,\Omega)\right\rangle$$

$$Q_{nn'}(\rho)=\left\langle\Phi_n(\rho,\Omega)\left|\frac{\partial^2}{\partial\rho^2}\right|\Phi_{n'}(\rho,\Omega)\right\rangle$$

$$\left[\frac{\partial^2}{\partial\rho^2}+\frac{2mE}{\hbar^2}-\frac{1}{\rho^2}\left(\lambda_n(\rho)+\frac{15}{4}\right)\right]f_n(\rho)+\sum_{n'}\left[2P_{nn'}(\rho)\frac{\partial}{\partial\rho}+Q_{nn'}(\rho)\right]f_{n'}(\rho)=0$$

$$\hat{\Lambda}^2\Phi_n(\rho,\Omega)+\frac{2m\rho^2}{\hbar^2}\left(V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right)\Phi_n(\rho,\Omega)=\lambda_n(\rho)\Phi_n(\rho,\Omega)$$

Volvemos a empezar….

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}-\frac{\hat{\Lambda}^2}{\rho^2}\right)+V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right]\Psi(\vec{x},\vec{y})=E\Psi(\vec{x},\vec{y})$$

$$\Psi^{LM}(\vec{x},\vec{y})=\frac{1}{\rho^{5/2}}\sum_n f_n(\rho)\Phi_n(\rho,\Omega)$$

Aproximación adiabática

Las funciones Φ_n son las autofunciones de la parte angular

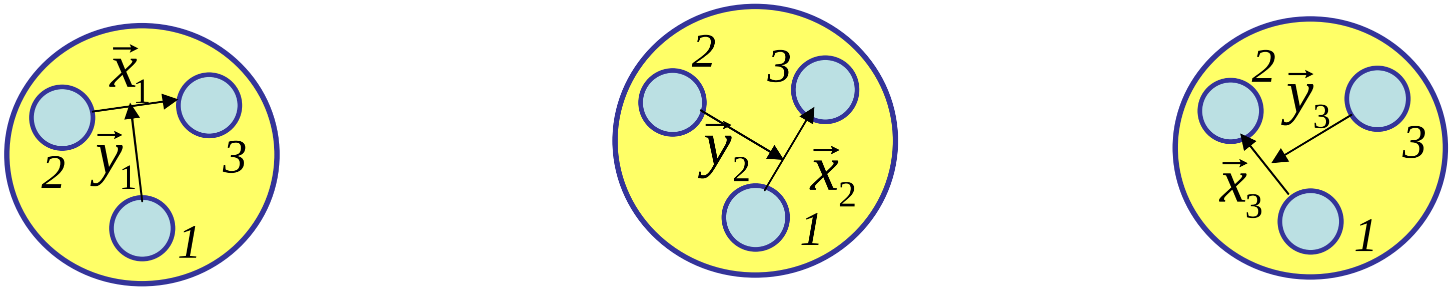
Los autovalores λ_n entran como un potencial efectivo en la parte radial

Las funciones $P_{nn'}$ y $Q_{nn'}$ acoplan las distintas funciones radiales f_n

$$\left[\frac{\partial^2}{\partial\rho^2}+\frac{2mE}{\hbar^2}-\frac{1}{\rho^2}\left(\lambda_n(\rho)+\frac{15}{4}\right)\right]f_n(\rho)+\sum_{n'}\left[2P_{nn'}(\rho)\frac{\partial}{\partial\rho}+Q_{nn'}(\rho)\right]f_{n'}(\rho)=0$$

$$\hat{\Lambda}^2\Phi_n(\rho,\Omega)+\frac{2m\rho^2}{\hbar^2}\left(V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right)\Phi_n(\rho,\Omega)=\lambda_n(\rho)\Phi_n(\rho,\Omega)$$

Ecuaciones de Faddeev



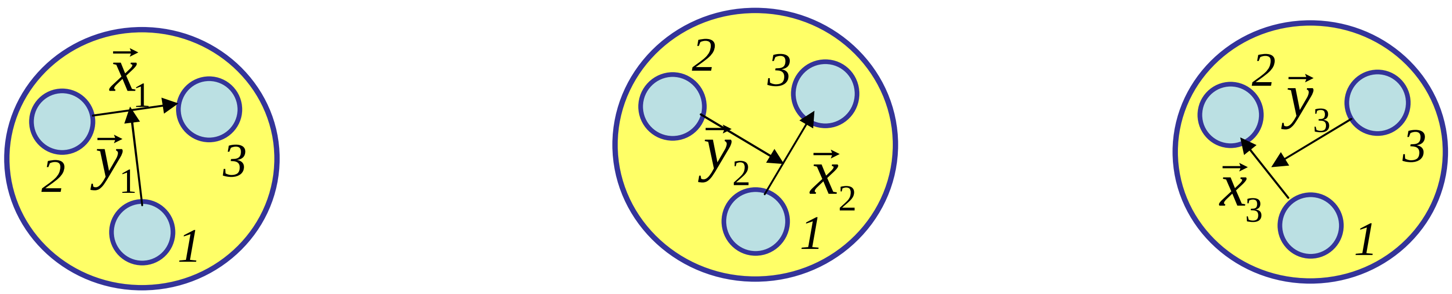
$$\Psi = \psi^{(1)}(\vec{x}_1, \vec{y}_1) + \psi^{(2)}(\vec{x}_2, \vec{y}_2) + \psi^{(3)}(\vec{x}_3, \vec{y}_3)$$

$$(T-E)\psi^{(1)} + V_{23}(x_1)(\psi^{(1)} + \psi^{(2)} + \psi^{(3)}) = 0$$

$$(T-E)\psi^{(2)} + V_{31}(x_2)(\psi^{(1)} + \psi^{(2)} + \psi^{(3)}) = 0$$

$$(T-E)\psi^{(3)} + V_{12}(x_3)(\psi^{(1)} + \psi^{(2)} + \psi^{(3)}) = 0$$

Ecuaciones de Faddeev



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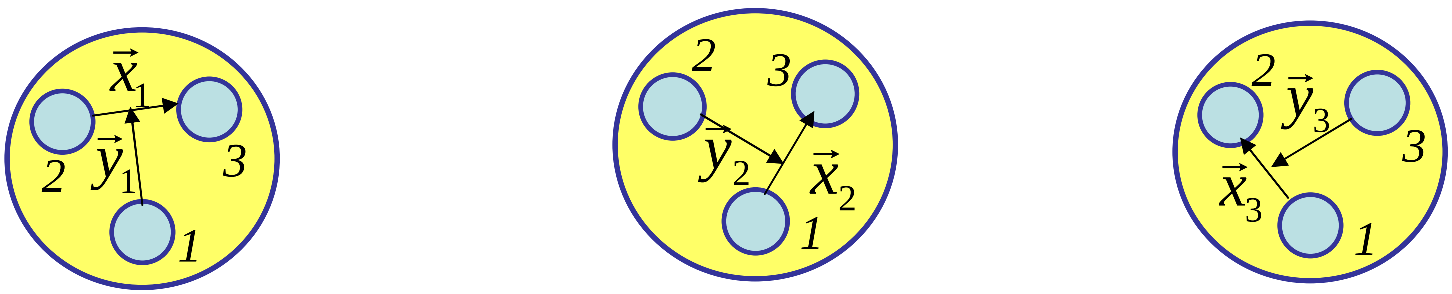
$$(T-E)\psi^{(3)} + V_{12}(x_3)(\psi^{(1)} + \psi^{(2)} + \psi^{(3)}) = 0$$

$$(T-E)(\psi^{(1)} + \psi^{(2)} + \psi^{(3)}) + (V_{23} + V_{31} + V_{12})(\psi^{(1)} + \psi^{(2)} + \psi^{(3)}) = 0$$

Ecuaciones de Faddeev



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$$(T-E)\psi^{(3)} + V_{12}(x_3)(\psi^{(1)} + \psi^{(2)} + \psi^{(3)}) = 0$$

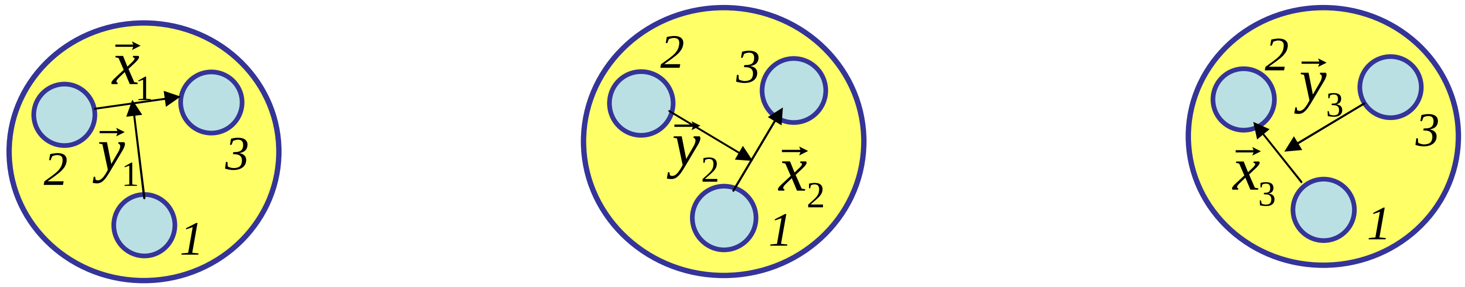
$$(T-E)\Psi + (V_{23} + V_{31} + V_{12})\Psi = 0$$

⬇

Equivalente a resolver Schrödinger

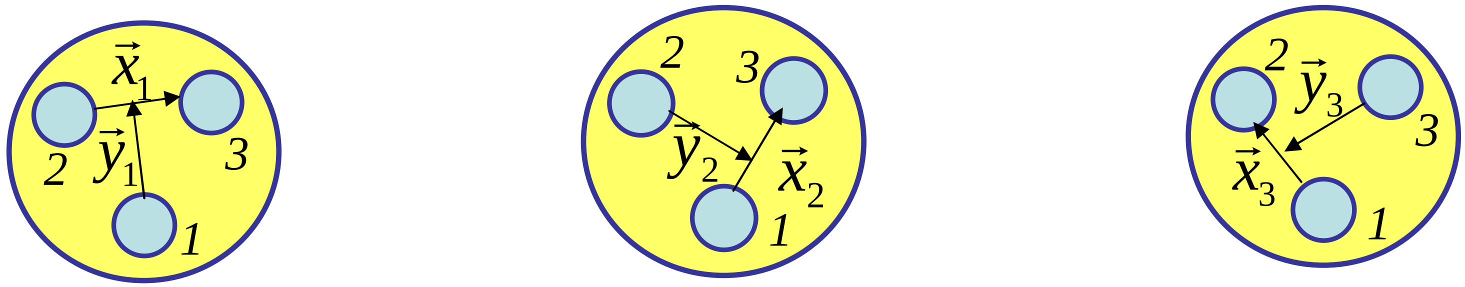
Aproximación adiabática

$$\hat{\Lambda}^2\Phi_n(\rho,\Omega)+\frac{2m\rho^2}{\hbar^2}\left(V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right)\Phi_n(\rho,\Omega)=\lambda_n(\rho)\Phi_n(\rho,\Omega)$$



Aproximación adiabática

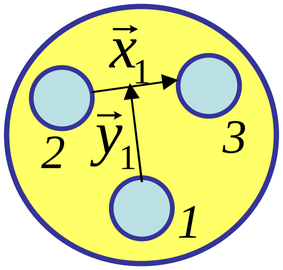
$$\hat{\Lambda}^2\Phi_n(\rho,\Omega)+\frac{2m\rho^2}{\hbar^2}\left(V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right)\Phi_n(\rho,\Omega)=\lambda_n(\rho)\Phi_n(\rho,\Omega)$$

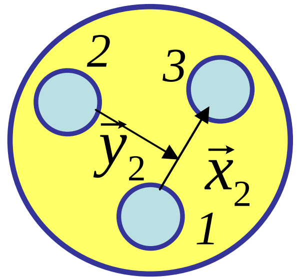


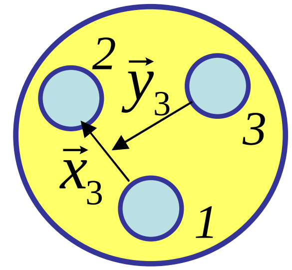
$$\Phi_n = \phi_n^{(1)}(\rho,\Omega_1) + \phi_n^{(2)}(\rho,\Omega_2) + \phi_n^{(3)}(\rho,\Omega_3)$$

Aproximación adiabática

$$\hat{\Lambda}^2\Phi_n(\rho,\Omega)+\frac{2m\rho^2}{\hbar^2}\left(V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right)\Phi_n(\rho,\Omega)=\lambda_n(\rho)\Phi_n(\rho,\Omega)$$







$$\Phi_n = \phi_n^{(1)}(\rho,\Omega_1) + \phi_n^{(2)}(\rho,\Omega_2) + \phi_n^{(3)}(\rho,\Omega_3)$$

$$\hat{\Lambda}^2\phi_n^{(1)}+\frac{2m\rho^2}{\hbar^2}V_{23}(x_1)(\phi_n^{(1)}+\phi_n^{(2)}+\phi_n^{(3)})=\lambda_n(\rho)\phi_n^{(1)}$$

$$\hat{\Lambda}^2\phi_n^{(2)}+\frac{2m\rho^2}{\hbar^2}V_{31}(x_2)(\phi_n^{(1)}+\phi_n^{(2)}+\phi_n^{(3)})=\lambda_n(\rho)\phi_n^{(2)}$$

$$\hat{\Lambda}^2\phi_n^{(3)}+\frac{2m\rho^2}{\hbar^2}V_{12}(x_3)(\phi_n^{(1)}+\phi_n^{(2)}+\phi_n^{(3)})=\lambda_n(\rho)\phi_n^{(3)}$$

Expansión adiabática en hiperharmónicos esféricos

Separa las ecuaciones en parte angular y radial

⬇

Se resuelve la parte angular de las ecuaciones de Faddeev

$$\hat{\Lambda}^2\phi_n^{(i)} + \frac{2m\rho^2}{\hbar^2}V_{jk}(x_i)(\phi_n^{(i)} + \phi_n^{(j)} + \phi_n^{(k)}) = \lambda_n(\rho)\phi_n^{(i)}$$

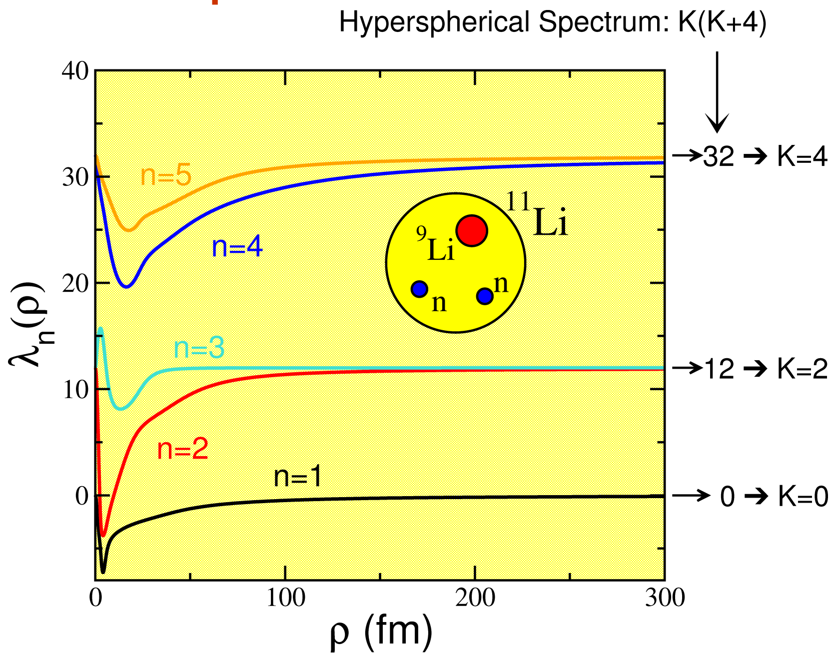
Los autovalores entran en la parte radial como un potencial efectivo

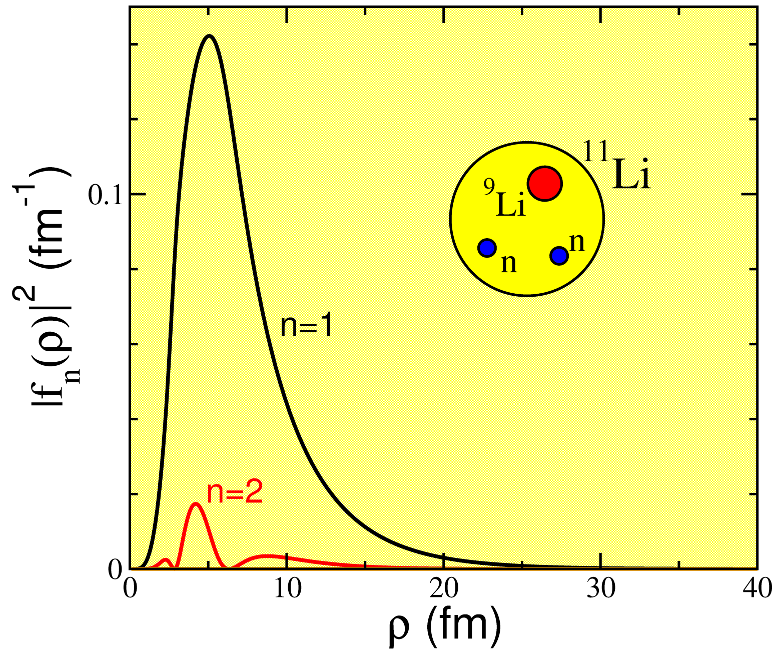
⬇

$$\left[\frac{\partial^2}{\partial\rho^2} + \frac{2mE}{\hbar^2} - \frac{1}{\rho^2}\left(\lambda_n(\rho) + \frac{15}{4}\right)\right] f_n(\rho) + \sum_{n'}\left[2P_{nn'}(\rho)\frac{\partial}{\partial\rho} + Q_{nn'}(\rho)\right] f_{n'}(\rho) = 0$$

$$\Psi^{LM}(\vec{x},\vec{y}) = \frac{1}{\rho^{5/2}}\sum_n f_n(\rho)\Phi_n(\rho,\Omega)$$

Expansión adiabática en hiperharmónicos esféricos



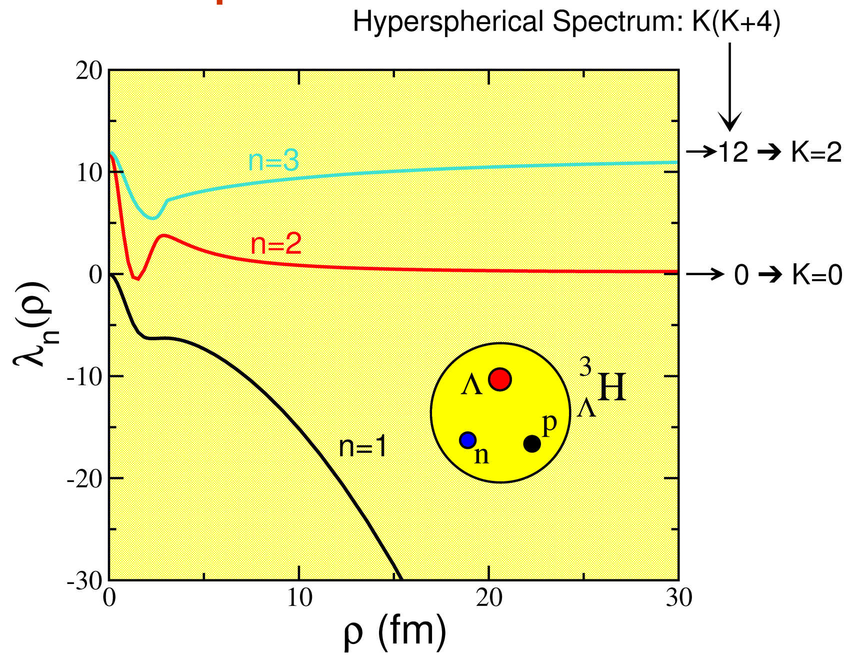


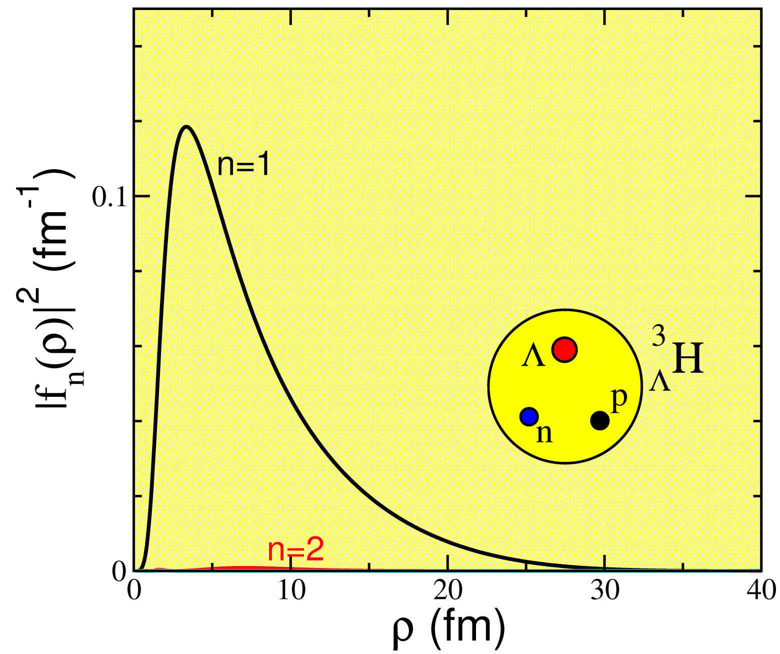
$$\hat{\Lambda}^2\phi_n^{(i)}+\frac{2m\rho^2}{\hbar^2}V_{jk}(x_i)(\phi_n^{(i)}+\phi_n^{(j)}+\phi_n^{(k)})=\lambda_n(\rho)\phi_n^{(i)}$$

$$\left[\frac{\partial^2}{\partial\rho^2}+\frac{2mE}{\hbar^2}-\frac{1}{\rho^2}\left(\lambda_n(\rho)+\frac{15}{4}\right)\right]f_n(\rho)+\sum_{n'}\left[2P_{nn'}(\rho)\frac{\partial}{\partial\rho}+Q_{nn'}(\rho)\right]f_{n'}(\rho)=0$$

$$\Psi^{LM}(\vec{x},\vec{y})=\frac{1}{\rho^{5/2}}\sum_n f_n(\rho)\Phi_n(\rho,\Omega)$$

Expansión adiabática en hiperharmónicos esféricos

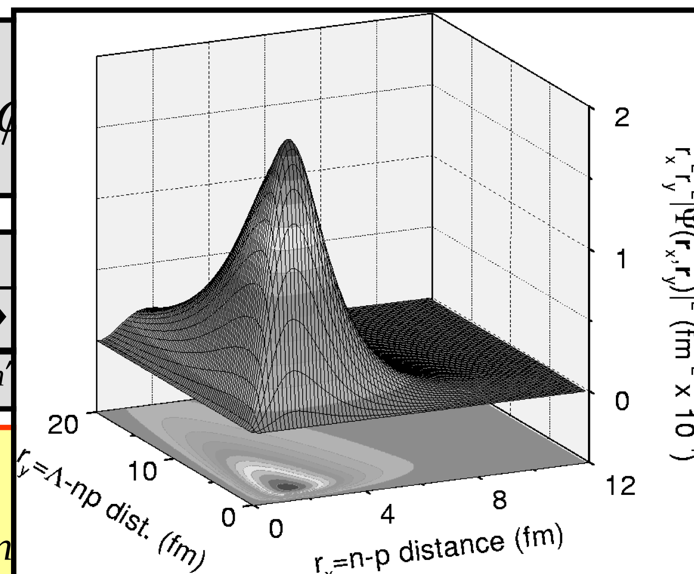




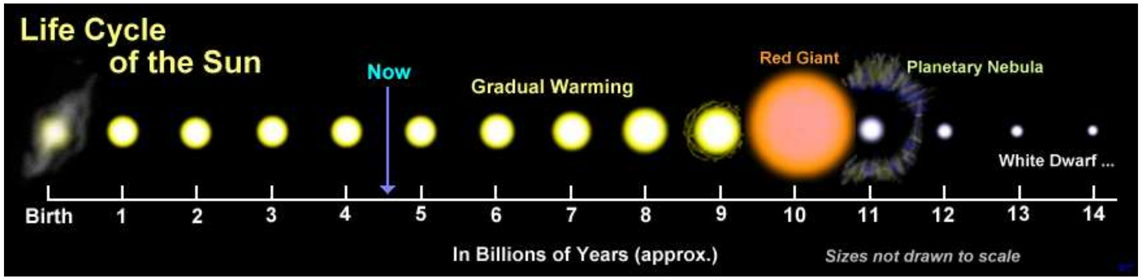
$$\hat{\Lambda}^2\phi_n^{(i)} + \frac{2m\rho^2}{\hbar^2}V_{jk}(x_i)(\phi_n^{(i)} + \phi$$

$$\left[\frac{\partial^2}{\partial\rho^2}+\frac{2m\color{red}{E}}{\hbar^2}-\frac{1}{\rho^2}\left(\color{red}{\lambda_n(\rho)}+\frac{15}{4}\right)\right] f_n(\rho)+\sum_{n'} \quad 0$$

$$\Psi^{LM}(\vec{x},\vec{y}) = \frac{1}{\rho^{5/2}}\sum_n f_n$$



The triple alpha reaction



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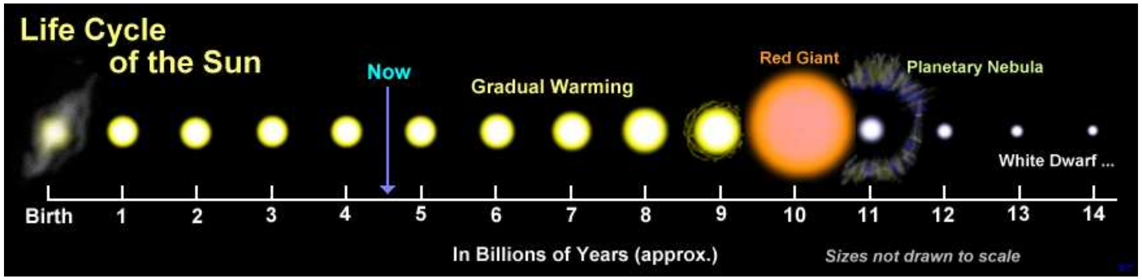
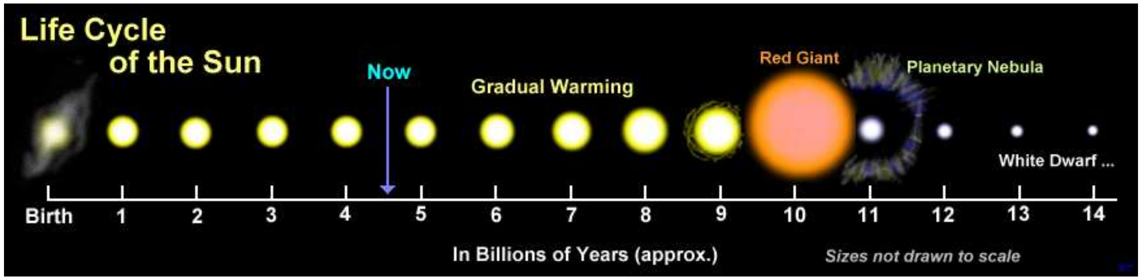


In the early stages of the life cycle the source of energy is the hydrogen nuclei

The **pp-chain** transforms four protons into ^{4}He

The triple alpha reaction



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$$ppI - \text{chain}$$

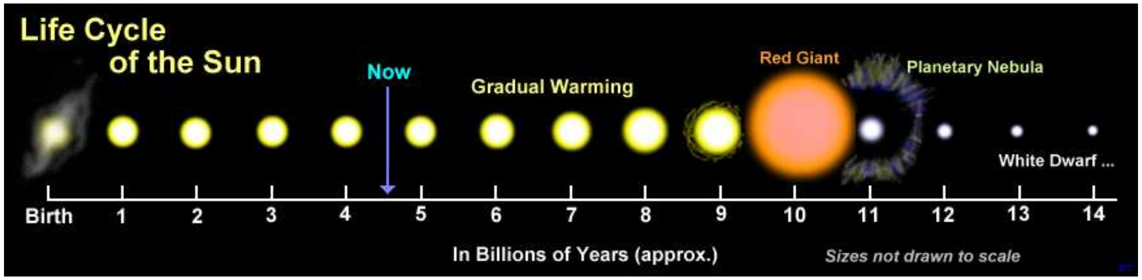
$$(T \sim 10^7\,\text{K})$$

$$p + p \rightarrow d + e^+ + \nu_e$$

$$p + d \rightarrow {}^3\text{He} + \gamma$$

$${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + p + p$$

The triple alpha reaction



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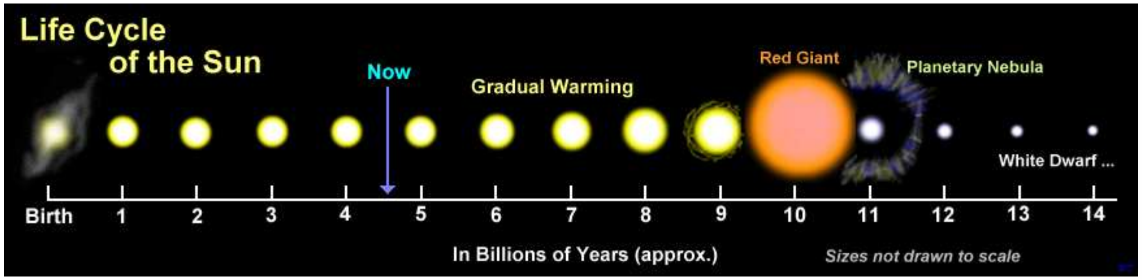
$${}^3\mathrm{He}+{}^3\mathrm{He} \rightarrow {}^4\mathrm{He}+p+p$$

The core of the star begins to accumulate ^{4}He

The triple alpha reaction



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$$p+p \rightarrow d+e^+ +\nu_e$$

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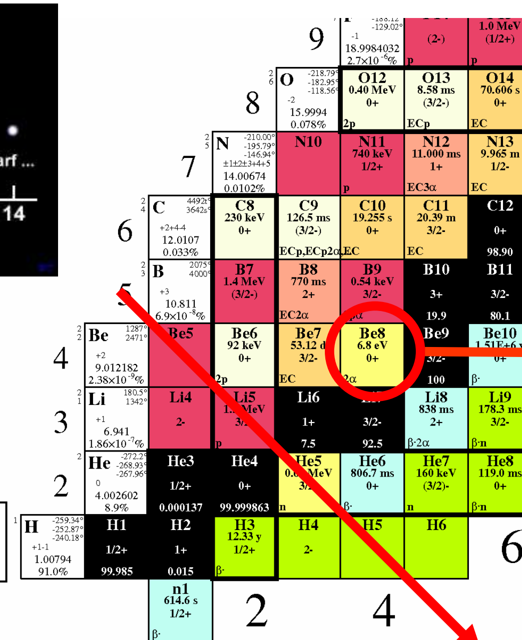


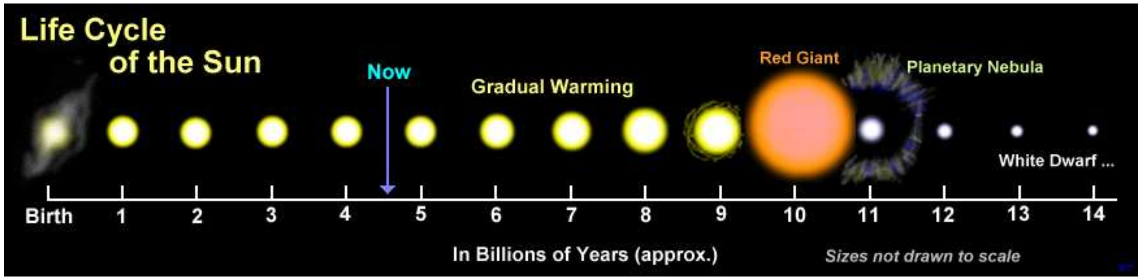
^{8}Be is unbound

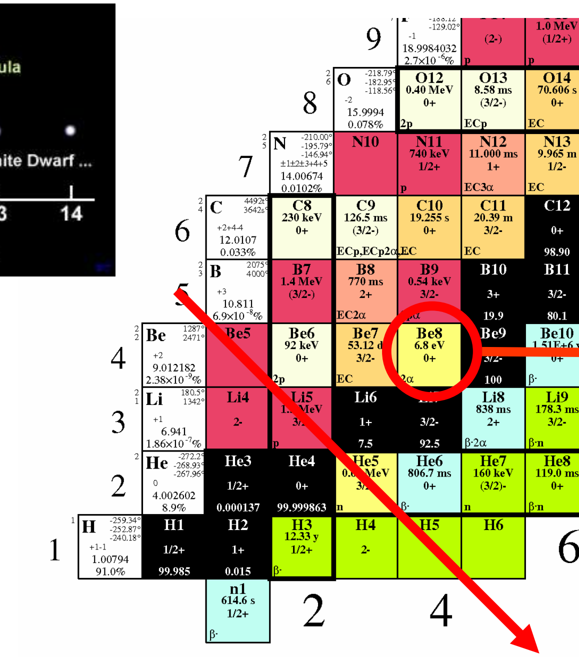
All the A=5 nuclei are particle unstable

Production of heavier nuclei requires to skip the A=5 and A=8 gaps

The core of the star begins to accumulate ^{4}He

The triple alpha reaction





^{8}Be is unbound

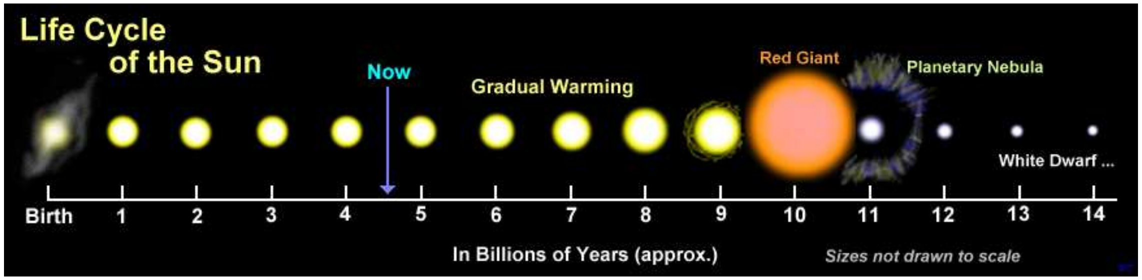
All the A=5 nuclei are particle unstable

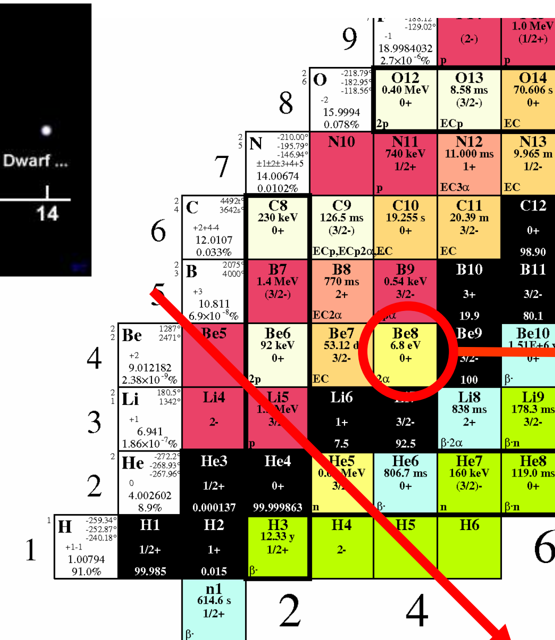
Production of heavier nuclei requires to skip the A=5 and A=8 gaps

When the hydrogen fuel is exhausted the nuclear reactions in the core stop

The core of the star begins to accumulate ^{4}He

The triple alpha reaction





^{8}Be is unbound

All the A=5 nuclei are particle unstable

Production of heavier nuclei requires to skip the A=5 and A=8 gaps

The gravitational collapse of the core raises the temperature

When the hydrogen fuel is exhausted the nuclear reactions in the core stop

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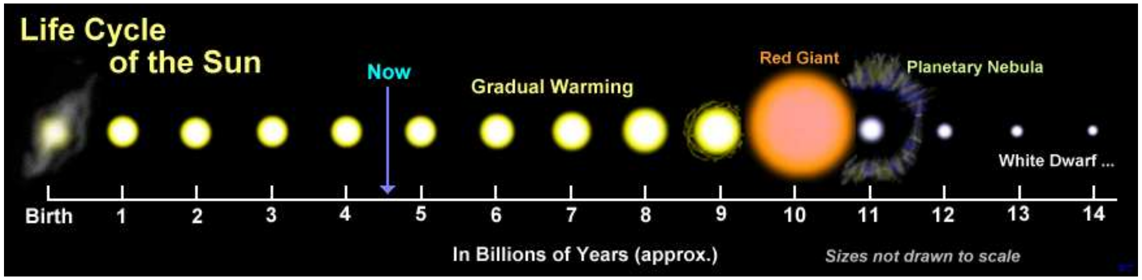

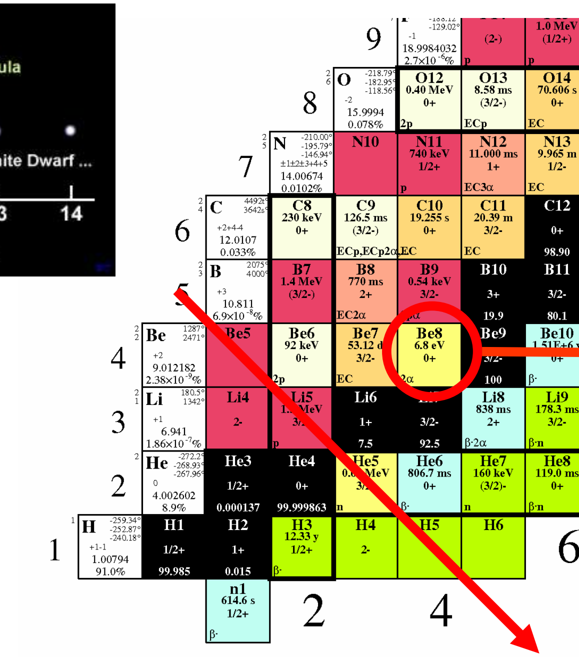

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The fusion of the external layers begin: Red giant phase

⬆

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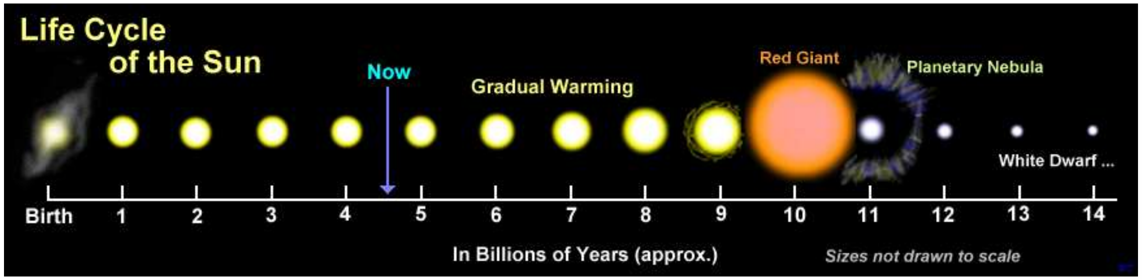


When T~10^8 K…
$\alpha+\alpha+\alpha \rightarrow {}^{12}C+\gamma$ is relevant!!!!

The fusion of the external layers begin: Red giant phase

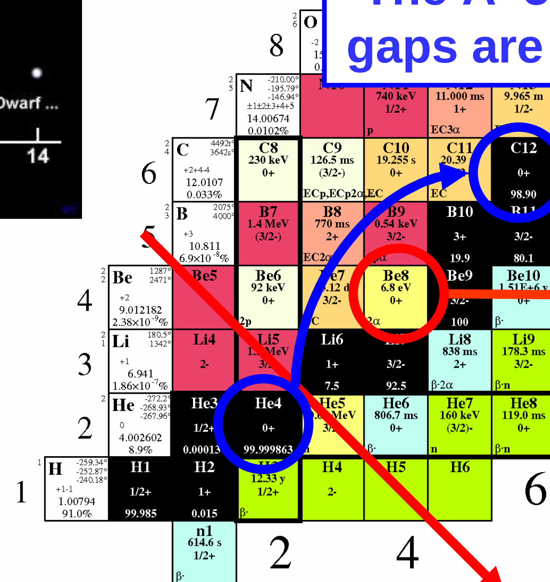
⬆

The gravitational collapse of the core raises the temperature

⬆

When the hydrogen fuel is exhausted the nuclear reactions in the core stop

The A=5 and A=8 gaps are bridged!!!



^{8}Be is unbound

All the A=5 nuclei are particle unstable

Production of heavier nuclei requires to skip the A=5 and A=8 gaps

The core of the star begins to accumulate ^{4}He

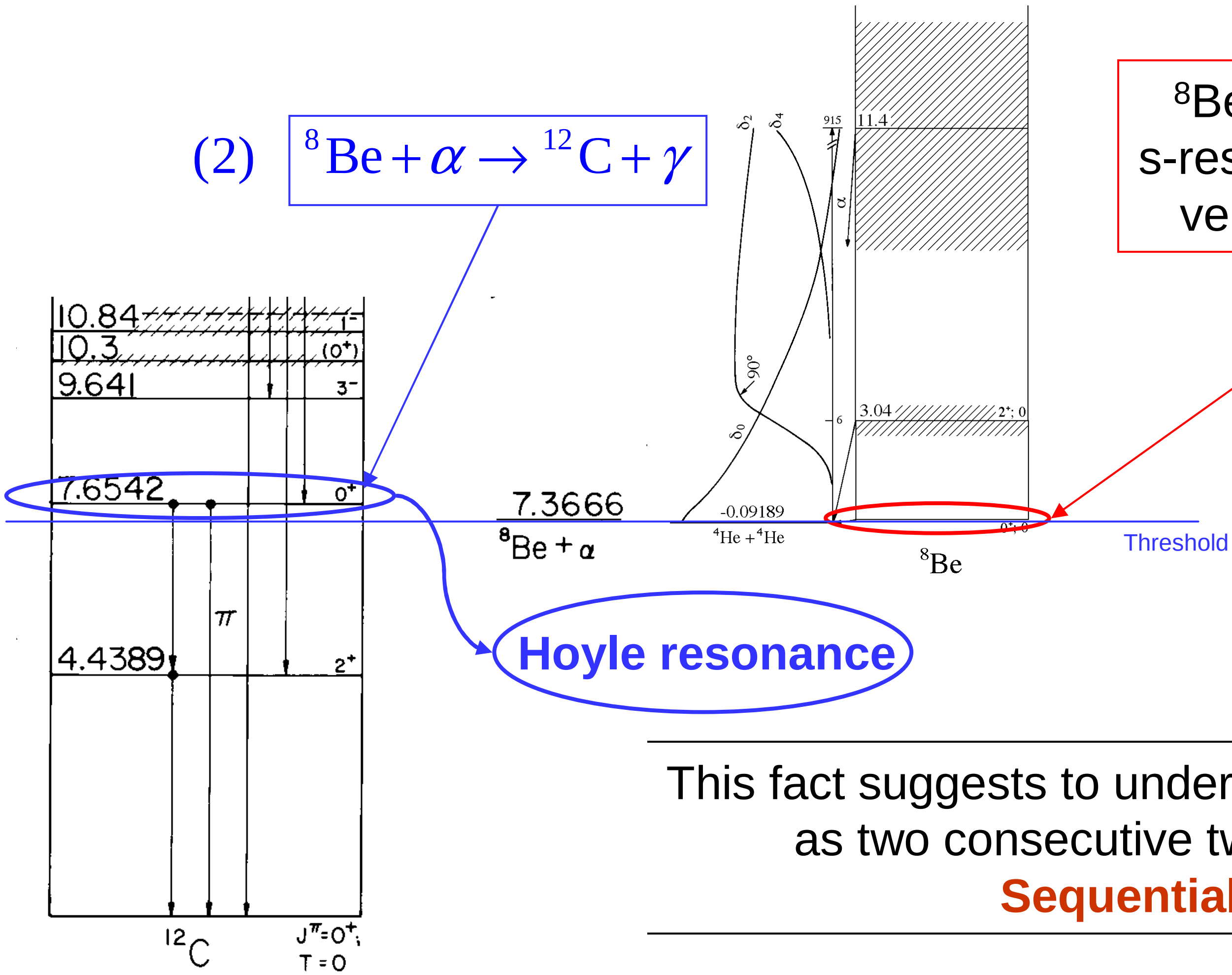
The triple alpha reaction

The $\alpha+\alpha+\alpha \rightarrow {}^{12}C+\gamma$ reaction

(2) $^{8}Be + \alpha \rightarrow {}^{12}C + \gamma$

^{8}Be has a very low lying s-resonance (~92 keV) and very narrow (a few eV).

(1) $\alpha + \alpha \rightarrow {}^{8}Be + \gamma$



Hoyle resonance

This fact suggests to understand the triple α reaction as two consecutive two-body processes.

Sequential process

The triple alpha reaction

What is the production rate for the different reactions in the stellar medium??

$$a+b+c \rightarrow d+\gamma$$

Radiative capture process

$$P_{abc}(\rho,T)=n_a n_b n_c \frac{\hbar^3}{c^2}\left(\frac{m_a+m_b+m_c}{m_a m_b m_c}\right)^{3/2}\frac{2\pi}{(K_B T)^3}e^{-\frac{Q}{K_B T}}\int_{|Q|}^{\infty}E^2\sigma_{\gamma,d}(E)e^{-\frac{E}{K_B T}}dE$$

$$\sigma_{\gamma,d}=\sum_{\lambda}\left(\sigma_{\gamma,d}^{E\lambda}+\sigma_{\gamma,d}^{M\lambda}\right)$$

$$\sigma_{\gamma,d}^{E\lambda}(E_\gamma)=\frac{\alpha(2\pi)^3\hbar c(\lambda+1)}{\lambda\left[(2\lambda+1)!!\right]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda-1}\frac{dB(E\lambda)}{dE_\gamma}$$

$$B(E\lambda, I_i \rightarrow n\, I_f)=\sum_{\mu,M_f}\left|\left\langle n\, I_f M_f \left|\mathcal{M}_\mu(E\lambda)\right| I_i M_i\right\rangle\right|^2; \quad \mathcal{M}_\mu(E\lambda)=e\sum_i Z_i r_i^\lambda Y_{\lambda,\mu}(\hat{r}_i)$$

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Three-body continuum wave function

Three-body bound state wave function

$$B(E\lambda, I_i \rightarrow n\, I_f) = \sum_{\mu, M_f} \left| \left\langle n\, I_f M_f \right| \mathcal{M}_\mu(E\lambda) \left| I_i M_i \right\rangle \right|^2; \quad \mathcal{M}_\mu(E\lambda) = e \sum_i Z_i r_i^\lambda Y_{\lambda,\mu}(\hat{r}_i)$$

The triple alpha reaction

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$$a + b + c \rightarrow d + \gamma$$

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Given a temperature *T*, only values of $E \sim K_B T$ are relevant

$$T = 10\ \text{GK} \Rightarrow K_B T \approx 0.9\ \text{MeV}$$

In a standard star, like the sun, $T \sim 10^7$ K=*0.01* GK ⇨ $K_B T \approx 0.001$ *MeV*

The triple alpha reaction

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In the stellar medium only very low relative energies are relevant !!!

Resumiendo….

- ✓La ecuación de Schrödinger puede resolverse para tres cuerpos de forma análoga a como se hace con dos partículas, pero desarrollando la función de onda en hiperarmónicos esféricos.

- ✓La expansión en H.H. no describe bien el comportamiento asintótico cuando dos o más subsistemas de dos cuerpos están muy ligados, o muy poco ligados.

- ✓La resolución de las Ecuaciones de Faddeev trata del mismo modo todos los subsistemas de dos cuerpos. La aproximación adiabática permite una convergencia rápida de la función de onda, y describe bien los comportamientos asintóticos (reproduce los estados de Efimov).

- ✓Cualquiera de los métodos permite obtener estados del continuo y resonancias, para lo cual es recomendable emplear el método de rotación compleja.