Non-Empirical Pairing Energy Functional for nuclei

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Outline

Theory of nuclei

Ultimate goals

- Comprehensive and unified description of all nuclei
- From basic interactions between nucleons + link to QCD
- Understand states of nuclear matter in astrophysical environments

Difficulties

- Self-bound, two-components quantum many-fermions system
- Complex interaction from low-energy regime of QCD
  - Tensor and spin-orbit components
  - Unnaturally large scattering lengths
  - NNN unavoidable
  - Repulsive core and strong tensor at short distances?
- Unified description from deuteron to SHE nuclei to NS
- Need to extrapolate to unknown regions

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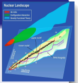
Ultimate goal

Ground state

Mass, deformation



Spectroscopy

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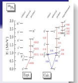
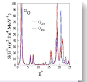


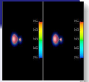
Collective modes

RPA, QRPA, GCM



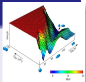
Reaction properties

Fusion, transfer...



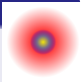
Heavy elements

Fission, fusion, SHE



Exotic behaviors

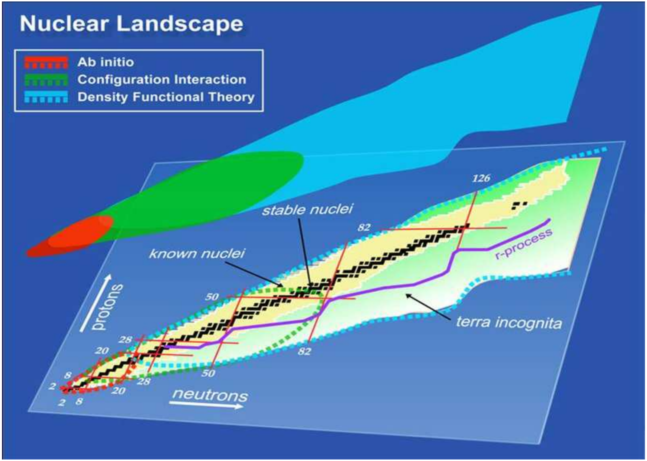
Drip-lines, halos



Astrophysics

NS, SN, r-process

Which theoretical method(s)?



- No "one size fits all" theory for nuclei
- All theoretical approaches need to be linked

Energy Density Functional (EDF) approaches

Basic elements

- Approaches not based on a correlated wave-function
- Energy is postulated to be a functional of one-body density (matrices)
- Symmetry breaking is at the heart of the method
- Two formulations (i) Single-Reference (ii) Multi-Reference

Pros

- Use of full single-particle space
- Quantal + collective picture
- Universality of EDF ($A \gtrsim 16$)
- Ground-state description
- Static (smooth) correlations

Difficulties

- No universal parametrization
- Empirical $\neq$ predictive power
- Spectroscopy / odd nuclei
- Dynamical (fluctuating) correlations
- Limited accuracy ($\sigma_{2135}^{mass} \approx 700$ keV)

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Pairing correlations in nuclei

$T = 1$ pairing in nuclei

- nn/pp superfluidity impacts all low-energy properties of nuclei
- Non-perturbative channel to be treated explicitly

Most impacted observables

- Lowest two-qp states in even-even nuclei $\approx$ measures the "gap"
- Odd-even mass staggering (OEMS) $\approx$ measures the "gap"
- Collective excitations
  - Moment of inertia of rotational bands
  - Low-lying vibrational states
  - Shape isomers from intruders
- Pair transfer
- Competition of pro- and anti-halo effects on the one-body density
- Neutron star physics
  - Glitches in the inner crust
  - Neutrino emission process
  - Heat diffusion

Single-reference EDF methods (1)

Elements of formalism

- $\mathcal{E}[\rho, \kappa^*, \kappa]$ = functional of one-body density matrices

$$\rho_{ji} = \langle \Phi | c_i^\dagger c_j | \Phi \rangle \quad ; \quad \kappa_{ji} = \langle \Phi | c_i c_j | \Phi \rangle$$

- $|\Phi\rangle$ = auxiliary/symmetry-breaking/product state of reference
- Minimizing $\mathcal{E}[\rho, \kappa^*, \kappa]$ leads to Hartree-Fock-Bogoliubov-like equations

$$\begin{pmatrix} h - \lambda & \Delta \\ -\Delta^* & -h^* + \lambda \end{pmatrix} \begin{pmatrix} U_i \\ V_i \end{pmatrix} = E_i \begin{pmatrix} U_i \\ V_i \end{pmatrix}$$

  - Effective potentials and vertices are defined through

$$h_{ij} \equiv \frac{\delta \mathcal{E}}{\delta \rho_{ji}} \equiv \sum_{kl} \overline{v}^{ph}_{ikjl} \, \rho_{lk} \quad ; \quad \Delta_{ij} \equiv \frac{\delta \mathcal{E}}{\delta \kappa^*_{ij}} \equiv \frac{1}{2} \sum_{kl} \overline{v}^{pp}_{ijkl} \, \kappa_{kl}$$

  - $\overline{v}^{ph}/\overline{v}^{pp}$ = Consistent many-body expansion in terms of NN/NNN
  - Quasiparticle w.f. (U_i, V_i), energy E_i, densities...

Single-reference EDF methods (2)

Empirical parameterizations of $\mathcal{E}[\rho,\kappa,\kappa^*]$; e.g. Skyrme or Gogny

$\mathcal{E}[\rho,\kappa,\kappa^*] \equiv$

- Tremendous successes for known nuclei
- Existing parameterizations are over/under constrained
- "Asymptotic freedom" as one enters "the next major shell"

Pairing part: example of EDF from DDDI

$$\mathcal{E}^{\kappa\kappa} = \sum_q \int d\vec{r}\, A^{\tilde{\rho}\tilde{\rho}}(\vec{r})\, |\tilde{\rho}_q(\vec{r})|^2 \equiv -\sum_q \int d\vec{r}\, \Delta_q(\vec{r})\, \tilde{\rho}^*_q(\vec{r})$$

$$\Delta^q_{ij} \equiv \frac{\delta \mathcal{E}^{\kappa\kappa}}{\delta \kappa^{q\,*}_{ij}} = \int d\vec{r}\; \Psi^{q\,*}_{ij}(\vec{r})\, \Delta_q(\vec{r})$$

- $\tilde{\rho}_q(\vec{r})$ = local pair density / $A^{\tilde{\rho}\tilde{\rho}}(\vec{r})$ = density-dependent coupling
- (Quasi-) local pairing EDF must be regularized/renormalized

Single-reference EDF methods (3)

Performance of existing pairing EDFs

- Moment of inertia of super-deformed bands = success story of the 90’
- OEMS ≈ ok but missing systematic/detailed characterization
- QP excitations = missing systematic characterization
- Divergence of predictions in the "next major shell"

Crucial undergoing works

- Enrich the analytical structure of empirical functionals
- Improve fitting protocols = data, algorithm and post-analysis

One can also propose a complementary approach...

- Known data hardly constrain non-trivial characteristics of pairing EDF
- Interesting not to rely entirely on fitting data

Outline

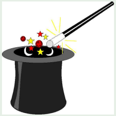
1. Introduction
   - Nuclear theory: goals and methods
   - Energy Density Functional methods
2. Non-empirical energy functional
   - The pairing part of the EDF as a first step
   - Low momentum interactions
   - Separable operator representation of $V_{\mathrm{low\ k}} + V_{\mathrm{Coul}}$
3. Pairing gaps in finite nuclei
   - Implementation for finite nuclei calculations
   - Results including nuclear, Coulomb and CSB terms
4. Results for soft versus hard NN interactions
   - Dependence of pairing gaps on the RG scale Λ
   - Fully microscopic calculations in infinite nuclear matter
5. Summary and outlook

Constructing non-empirical EDFs for nuclei

Long term objective

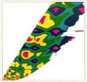
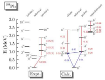
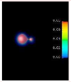
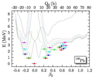
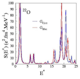
Build non-empirical EDF in place of existing models

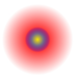
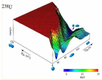
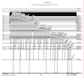
Empirical



Predictive?

Finite nuclei

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Long term objective

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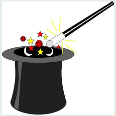
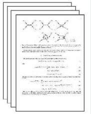
Empirical



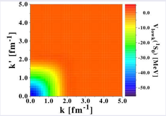
Predictive?

Non-empirical

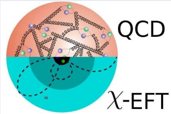


Predictive...

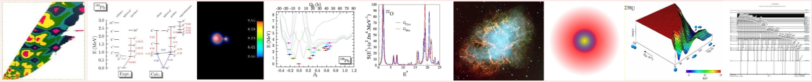
Low-k NN+NNN



QCD / χ-EFT



Finite nuclei



Long term project and collaboration

Design non-empirical Energy Density Functionals

- Bridge with "ab-initio" many-body techniques
- Calculate properties of heavy/complex nuclei from NN+NNN
- Controlled calculations with theoretical error bars

SPhN	T. Duguet, J. Sadoudi
IPNL	K. Bennaceur, J. Meyer
TRIUMF	A. Schwenk, K. Hebeler
NSCL	S. K. Bogner, B. Gebremariam
OSU	R. J. Furnstahl, L. Platter
ORNL	T. Lesinski

First step: pairing part of the EDF

Motivations for a non-empirical approach

- Empirical schemes lack predictive power
- Microscopic origin of superfluidity in finite nuclei?
  - Contribution from the direct term of V_{NN} ($^1S_0$, $^1D_2$, $^3PF_2$)?
  - Coupling to density/spin/isospin fluctuations: 40%?
  - Absolute value/isotopic trend is of great interest

Start with v^{pp} built at 1st order in V_{NN} (nuclear + Coulomb)

- Single channel ($^1S_0$) dominates at sub-nuclear densities
- Virtual state at $E \simeq 0$ makes V_{NN} almost separable in $^1S_0$

Nuclear interactions and the Renormalization Group

Approach

- $V(\vec{k},\vec{k}',\Lambda=\infty) = V^{\text{hard}}(\vec{k},\vec{k}')$
- Run down Λ
- Keep $\delta^{{}^SL_J}(k)$ and $E_{Deuteron}$

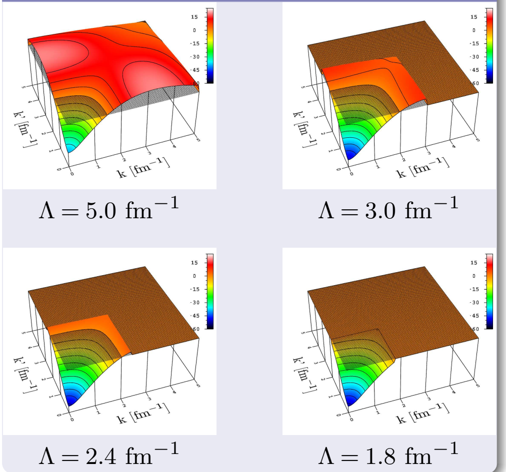
General Properties

- Vacuum interaction
- Universal $V_{NN}(\Lambda \approx 2) \equiv V_{\text{low k}}$
- $V_{\text{low k}}$ is perturbative

Crucial points

- $H = V_{NN}(\Lambda) + V_{NNN}(\Lambda) + \ldots$
- $\partial_\Lambda A \neq 0 \Rightarrow$ missing pieces
- Ex: omitted NNN(Λ)

Convergence of the RG flow



Finite nuclei calculations

Low-momentum interactions for finite nuclei calculations

- High-precision bare interactions with regularized hard-core
- Good starting point for structure calculations through EDF method?

$V_{\text{low k}}$ is given as tables of numbers

Produce analytical operatorial representation

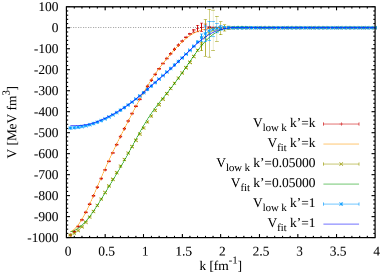
- Why?
  - Interest to understand encoded operatorial structure
  - Perform integrals analytically in codes
- Which representation?
  - Gaussian/Gogny-like (V. Rotival)
  - Sum of separable terms (T. Lesinski)

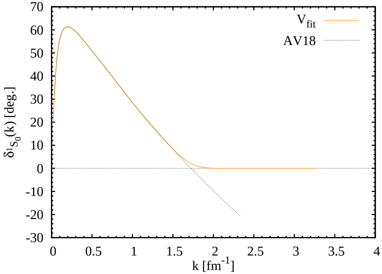
Separable representation of $V_{\text{low k}}(\Lambda) + V_{\text{Coul}}$

- High precision separable representation of rank n

$$V_n^{^1S_0}(k,k',\Lambda) \quad = \quad \sum_{\alpha,\beta=1}^{n} g_\alpha(k)\ \lambda_{\alpha\beta}\ g_\beta(k')$$

- Fit of $g_\alpha(k)$ and $\lambda_{\alpha\beta}$ to $V_{\text{low k}}^{^1S_0}(k,k',\Lambda)$ and $\delta^{^1S_0}(k)$
- For $\Lambda = 1.8/4.0/\text{"}\infty\text{"}\ \text{fm}^{-1}$ (rank $2/4/15$) and smooth cutoff





- $\exists$ separable representation of $V^a_{\text{Coul},\ell=0}(k,k')$

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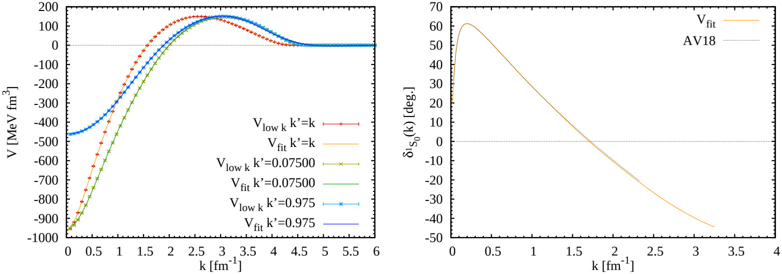

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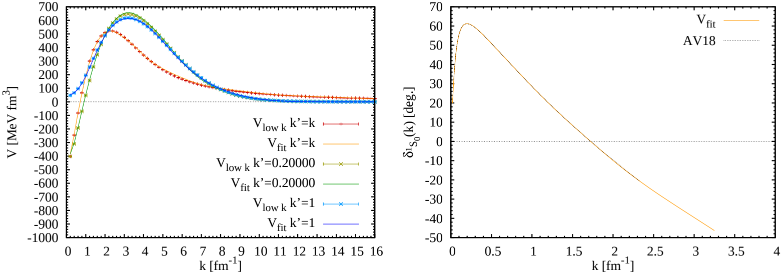

- $\exists$ separable representation of $V^a_{{\rm Coul},\ell=0}(k,k')$

Coulomb interaction

Need to incorporate Coulomb effects on proton gaps

- Only one such published calculation so far: Madrid group (Gogny)
- Simplified treatment of e.m. interaction (Coulomb)

Truncate the Coulomb interaction at $r = a > 2R_{\text{nucleus}}$

- A separable expansion exists (keep ${}^1S_0$ part here)

$$
\begin{aligned}
V^a_{\text{Coul},\ell=0}(k,k') &= 4\pi e^2 a^2 \sum_{n=0}^{\infty} (2n+1)\, j_n^2\left(\frac{ak}{2}\right) j_n^2\left(\frac{ak'}{2}\right), \\
\lambda_{\alpha\beta} &= e^2 a^2 (2\alpha+1)\,\delta_{\alpha\beta} \\
g_\alpha(k) &= \sqrt{4\pi}\, j_\alpha^2\left(\frac{ak}{2}\right) \\
G_\alpha(r) &= \frac{1}{\sqrt{\pi}\, a^2 r} P_\alpha\left(1 - 2(\tfrac{r}{a})^2\right) \text{ for } r \leq a
\end{aligned}
$$

- $\sim$ 15 terms needed (peanuts !)

Outline

EDF calculations in spherical nuclei (1)

- Separable force in coordinate-space $[\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \quad \mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2]$

$$\langle \mathbf{r}'_1 \mathbf{r}'_2 | V_n^{{}^1S_0} | \mathbf{r}_1 \mathbf{r}_2 \rangle = \sum_{\alpha,\beta}^{n} G_\alpha(r') \, \lambda_{\alpha\beta} \, G_\beta(r) \, \delta(\mathbf{R}' - \mathbf{R}),$$

- Coordinate-space form factor $G_\alpha(r)$ = fourier transform of $g_\alpha(k)$
- Pairing functional

$$\mathcal{E}^{\kappa\kappa} = \sum_q \frac{1}{2} \int d^3\mathbf{R} \sum_{\alpha,\beta=1}^{n} \breve{\tilde{\rho}}_\alpha^{q\,*}(\mathbf{R}) \, \lambda_{\alpha\beta} \, \breve{\tilde{\rho}}_\beta^{q}(\mathbf{R})$$

- One defines effective pair densities $\breve{\tilde{\rho}}_\alpha^{q}(\mathbf{R})$ through

$$\breve{\tilde{\rho}}_\alpha^{q}(\mathbf{R}) = \int d^3\mathbf{r} \, G_\alpha(r) \sum_\sigma (-)^{\frac{1}{2}-\sigma} \, \kappa^q(\mathbf{R} + \mathbf{r}/2, \sigma; \mathbf{R} - \mathbf{r}/2, -\sigma)$$

➡ Incorporate the finite range/non-locality of the interaction

➡ Induce non-local pairing field and density

➡ BUT the functional depends only on *effective* pair densities *locally* !

EDF calculations in spherical nuclei (2)

- Define reduced two-body wave-functions (spin-singlet part)

$$\breve{\Psi}^{q\,\alpha}_{ij}(\mathbf{R}) \equiv \int d^3\mathbf{r}\;\, G_\alpha(r)\,\Psi^q_{ij}(\mathbf{R}+\mathbf{r}/2,\mathbf{R}-\mathbf{r}/2)$$

$$\Psi^q_{ij}(\mathbf{r},\mathbf{r}') \equiv \sum_\sigma (-)^{s-\sigma}\phi_i(\mathbf{r},\sigma,q)\phi_j(\mathbf{r}',-\sigma,q).$$

- The ϕ_i are basis functions : the $\breve{\Psi}^{q\,\alpha}_{ij}(\mathbf{R})$ are computed once
- Build densities and pairing field matrix elements

$$\breve{\Delta}^q_\alpha(\mathbf{R}) \equiv \frac{1}{2}\sum_\beta^n \lambda_{\alpha\beta}\,\breve{\tilde{\rho}}^q_\beta(\mathbf{R}) \equiv \frac{1}{2}\sum_\beta^n \lambda_{\alpha\beta}\sum_{ij}\breve{\Psi}^{q\,\beta}_{ij}(\mathbf{R})\,\kappa^q_{ij}$$

$$\Delta^q_{ij} = \sum_\alpha^n \int d^3\mathbf{R}\;\breve{\Psi}^{q,\alpha}_{ij}(\mathbf{R})\,\breve{\Delta}^q_\alpha(\mathbf{R})$$

➡ Pseudo-local pairing problem!

EDF calculations in spherical nuclei (3)

- New spherical code BSLHFB (T. Lesinski, unpublished)
  - Handle finite-range/non-local forces for systematic calculations
  - Calculations almost as cheap as for a local EDF
  - Basis of spherical bessel functions $j_\ell(kr)$
  - Well suited for drip-line physics
- Calculations
  - Results for 470 nuclei predicted spherical (Gogny-D1S)
  - Pairing complemented with (SLy5) Skyrme EDF ; $m_0^* = 0.7m$
  - $k_{\max} \sim 4.0$ fm^{-1}, $R_{box} = 20$ fm, $j_{max} = 45/2$
- Comparison of theoretical and experimental pairing gaps

✔ Reminder: nothing in the pairing channel is adjusted in nuclei

[T. Duguet and T. Lesinski, Eur. Phys. J. Special Topics **156** (2008) 207]

[T. Lesinski, T. Duguet, K. Bennaceur, J. Meyer, arXiv:0809.2895]

Gaps from $v^{pp} = V_{NN}(\Lambda = 1.8)$

[T. Duguet and T. Lesinski, Eur. Phys. J. Special Topics **156** (2008) 207]

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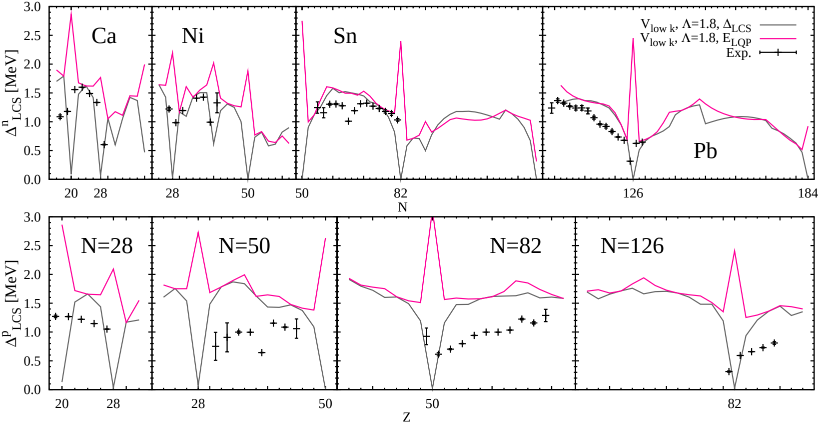

- Neutron gaps Δ^n are consistently close to experimental data
- Proton gaps Δ^p overestimates experimental data systematically

Gaps from $v^{pp} = V_{NN}(\Lambda = 1.8) + V^{a}_{\mathrm{Coul},\ell=0}$

[T. Duguet and T. Lesinski, Eur. Phys. J. Special Topics **156** (2008) 207]

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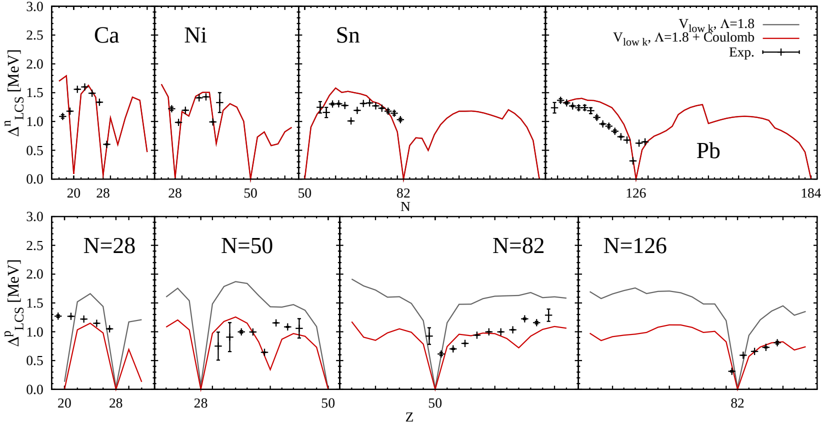
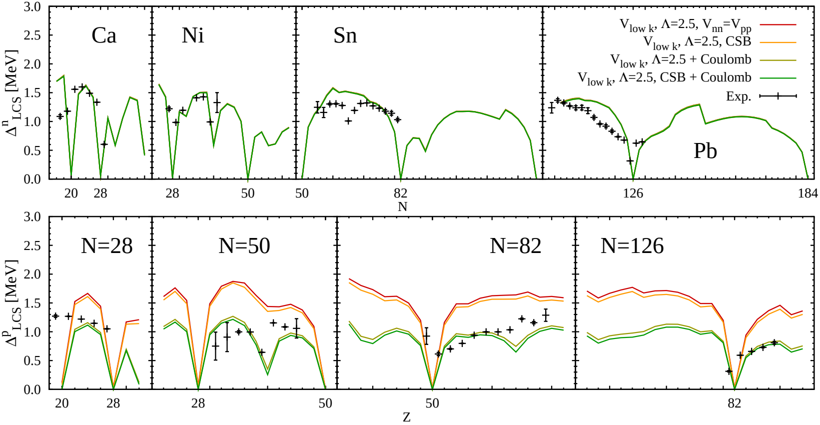


- Coulomb decreases Δ^{p} by $\sim 40\%$ to bring them close to experiment
- Few masses in the next major shell will be extremely valuable

Gaps from $v^{pp} = V_{NN}(\Lambda = 1.8) + V^{a}_{\mathrm{Coul},\ell=0} + \mathrm{CSB}$

[T. Duguet and T. Lesinski, Eur. Phys. J. Special Topics **156** (2008) 207]

[T. Lesinski, T. Duguet, K. Bennaceur, J. Meyer, arXiv:0809.2895]



- NN is different in $T_z = \pm 1 \Longleftrightarrow$ CSB
- Effect of CSB on Δ^{p} negligible compared to Coulomb

Gaps from $v^{pp} = V_{NN}(\Lambda = 1.8) + V^{a}_{\mathrm{Coul},\ell=0} + \mathrm{CSB}$

[T. Duguet and T. Lesinski, Eur. Phys. J. Special Topics **156** (2008) 207]

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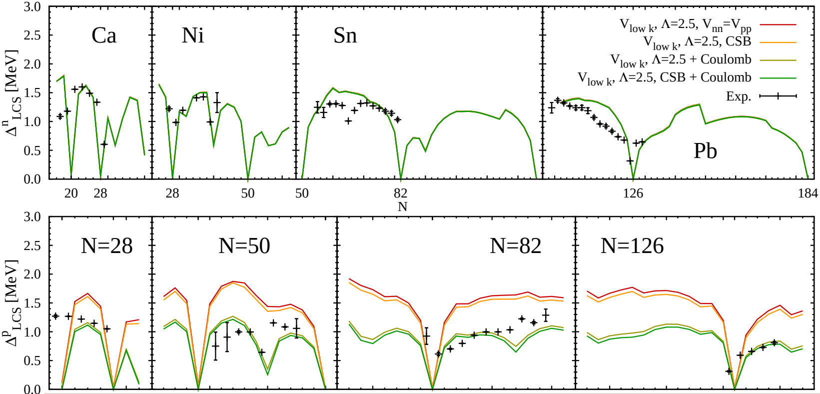


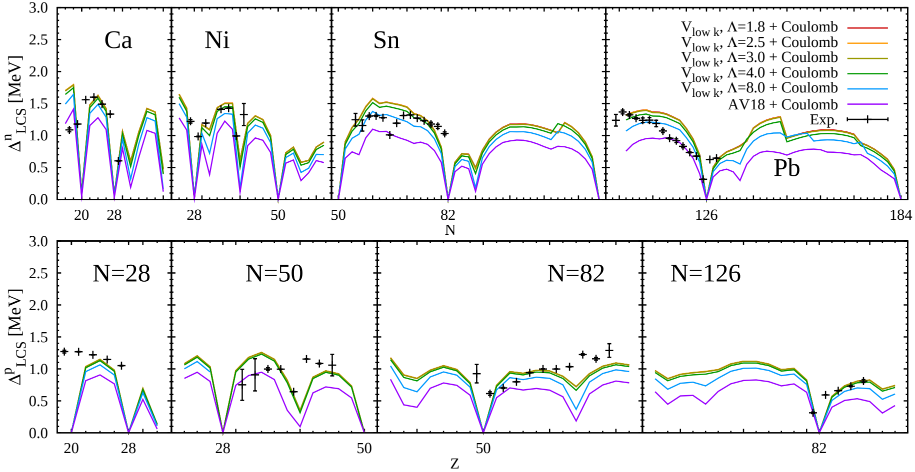
Neutron gap in ^{120}Sn from SLy4 + AV18 - Milan group

- $\Delta^{\mathbf{n}}_{\mathbf{AV18}}/\Delta^{\mathbf{n}}_{\mathbf{low\ k}} \approx 2/3$**!?**
- Is there an inconsistency? What are the reasons for such a difference?

Outline

1. Introduction
   - Nuclear theory: goals and methods
   - Energy Density Functional methods
2. Non-empirical energy functional
   - The pairing part of the EDF as a first step
   - Low momentum interactions
   - Separable operator representation of $V_{\text{low k}} + V_{\text{Coul}}$
3. Pairing gaps in finite nuclei
   - Implementation for finite nuclei calculations
   - Results including nuclear, Coulomb and CSB terms
4. Results for soft versus hard NN interactions
   - Dependence of pairing gaps on the RG scale Λ
   - Fully microscopic calculations in infinite nuclear matter
5. Summary and outlook

Dependence of Δ^q on the RG cut-off Λ



- Calculations with SLy4 + $V_{NN}(\Lambda)$ for varying Λ
- No variation for $\Lambda \in [1.8, 4]$ fm^{-1} but arises for $\Lambda > 4$ fm^{-1} with $\Delta^q \searrow$

Consistency of ph and pp self-energies

The Λ dependence of physical observable characterizes

- Missing pieces in the Hamiltonian one keeps at each Λ
- Correlations missing in the many-body calculation
- Effects of bad approximations at the level one is working at

Fully microscopic calculations in infinite nuclear matter - K. Hebeler et al.

- Different many-body expansions for large and small Λ
  - Results at large and small Λ likely to be different at a given order
  - v^{ph} and v^{pp} must be produced at same Λ
- Impact of using empirical v^{ph} such that $m^*_\tau(k^\tau_F) \cong m^*_\tau(k \approx k^\tau_F, k_F)$

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Outline

Summary

Pairing gaps in finite nuclei from vacuum NN + Coul.

- Based on low-momentum interactions from RG methods
- First systematic calculations in finite nuclei

First set of results

- Lowest order accounts for the magnitude of experimental gaps
- Coulomb essential for proton gaps ($\sim 40\%$)
- Effects beyond lowest-order seems negligible or cancel each other

Microscopic calculations in SNM and PNM

- Soft and hard interactions rely on different many-body expansions
- Lowest and higher-order contributions differ in each scheme
- Results for soft interaction in finite nuclei confirmed
- Momentum averaging of $m^*(k, k_{\mathrm{F}})$ reliable for soft interaction
- Fine tuning needed to design $m^*_{Sk}(k^{\tau}_{\mathrm{F}})$ for hard interaction

Outlook

Works in progress or envisioned

- Extensive study including other observables (T. Lesinski)
- Extension to deformed nuclei (T. Lesinski)
- Fine-tuned Skyrme appropriate to hard NN (T. Lesinski, A. Pastore)
- Equivalent semi-empirical DDDI functional (J. Margueron)
- Incorporate NNN (T. Lesinski)
- Construct ph part (B. Gebremariam)

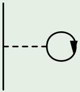
Thank you !

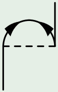
Self energies from many-body expansion

Expansion for soft V_{NN} = small Λ

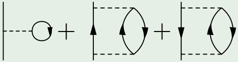
Perturbative

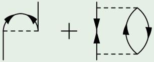
1st order

$\Sigma^{(1)} =$ 

$\Delta^{(1)} =$ 

2nd order $(\Delta^2/\varepsilon_F \to 0)$

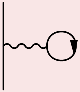
$\Sigma^{(2)} =$ 

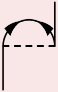
$\Delta^{(2)} =$ 

Expansion for hard V_{NN} = large Λ

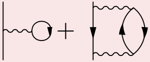
Hole lines/pp-irreducible vertex

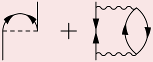
1st order

$\Sigma^{(1)} =$ 

$\Delta^{(1)} =$ 

2nd order $(\Delta^2/\varepsilon_F \to 0)$

$\Sigma^{(2)} =$ 

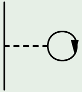
$\Delta^{(2)} =$ 

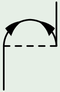
Self energies from many-body expansion

Expansion for soft V_{NN} = small Λ

Perturbative

1st order

$\Sigma^{(1)} =$ 

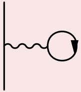
$\Delta^{(1)} =$ 

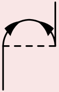
2nd order ($\Delta^2/\varepsilon_F \to 0$)

Expansion for hard V_{NN} = large Λ

Hole lines/pp-irreducible vertex

1st order

$\Sigma^{(1)} =$ 

$\Delta^{(1)} =$ 

2nd order ($\Delta^2/\varepsilon_F \to 0$)

- Second order extendable to all orders in screening bubbles
- Category of diagrams different depending on the starting $V_{NN}(\Lambda)$
- Work here at **1st order** for both soft and hard V_{NN}

First order - soft interaction - Hartree-Fock (HF)

Self energy

$$\text{Basic vertex} = \left\langle k' \right| V^{\tau\tau'}{}_{lS}^{J} \left| q \right\rangle$$

$$\Sigma_\tau^{(1)}(p,\omega,k_F) = 2\sum_{\mathbf{q},\tau'} n(q,k_F^{\tau'}) \left\langle \frac{\mathbf{p}-\mathbf{q}}{2} \right| V^{\tau\tau'} \left| \frac{\mathbf{p}-\mathbf{q}}{2} \right\rangle$$

Single-particle energy

$$\varepsilon_p^\tau \equiv \frac{p^2}{2} + Re\,\Sigma_\tau^{(1)}(p)$$

- Non self-consistent problem

First order - hard interaction - Brueckner-Hartree-Fock (BHF)

Self energy

$$\left\langle k' \right| G^{\tau\tau'}{}^{J}_{lS}(P,\omega,k_{\rm F}) \left| k \right\rangle = \left\langle k' \right| V^{\tau\tau'}{}^{J}_{lS} \left| k \right\rangle + \frac{2}{\pi} \int q^2 \, dq \left\langle k' \right| V^{\tau\tau'}{}^{J}_{lS} \left| q \right\rangle \cdot \cdot \frac{\left\langle Q^{\tau\tau'}(P,q) \right\rangle}{\omega - \left\langle \varepsilon^{\tau\tau'}(P,q) \right\rangle + i\delta} \left\langle q \right| G^{\tau\tau'}{}^{J}_{lS}(P,\omega) \left| k \right\rangle$$

$$\Sigma^{(1)}_{\tau}(p,\omega,k_{\rm F}) = 2 \sum_{\mathbf{q},\tau'}^{|q|<k_{\rm F}^{\tau'}} \left\langle \frac{\mathbf{p}-\mathbf{q}}{2} \right| G^{\tau\tau'}(|\mathbf{p}+\mathbf{q}|, \omega + \varepsilon^{\tau'}_{q}) \left| \frac{\mathbf{p}-\mathbf{q}}{2} \right\rangle$$

Single-particle energy

$$\varepsilon^{\tau}_{p} \equiv \frac{p^2}{2} + Re\,\Sigma^{(1)}_{\tau}(p, \varepsilon^{\tau}_{p})$$

- Self-consistent problem

Effective masses

Effective k-mass and e-mass

$$\frac{m^{*(1)}_{\tau,k}(p,k_{\rm F})}{m} \equiv \left[1+\frac{1}{p}\,\frac{\partial Re\Sigma^{(1)}_{\tau}(\omega,p,k_{\rm F})}{\partial p}\bigg|_{\omega=\varepsilon^{\tau}_{p}}\right]^{-1}$$

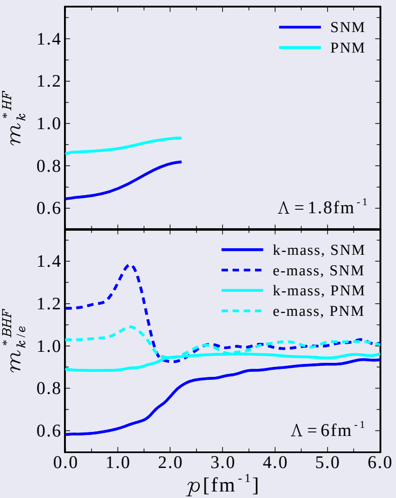
$$\frac{m^{*(1)}_{\tau,e}(p,k_{\rm F})}{m} \equiv 1-\frac{1}{m}\,\frac{\partial Re\Sigma^{(1)}_{\tau}(\omega,p,k_{\rm F})}{\partial \omega}\bigg|_{\omega=\varepsilon^{\tau}_{p}}$$

Total effective mass from $\Sigma_{\Lambda}(\mathbf{k},\varepsilon_{\mathbf{k}})$

$$\frac{m^{*(1)}_{\tau}(p,k_{\rm F})}{m} \equiv \frac{m^{*(1)}_{\tau,k}(p,k_{\rm F})}{m}\,\frac{m^{*(1)}_{\tau,e}(p,k_{\rm F})}{m}$$

- $\Sigma^{(1)}_{\rm soft}$ from soft V_{NN} provides k-mass only
- $\Sigma^{(1)}_{\rm hard}$ from hard V_{NN} provides both k-mass and e-mass

Momentum dependence of the effective mass, $k_F^\tau = 1.2\ \text{fm}^{-1}$



Soft - HF

- Mild p-dependence
- Smaller in SNM than in PNM
- Limited to $p \lesssim \Lambda = 1.8\ \text{fm}^{-1}$

Hard - BHF

- e-mass enhancement at k_F^τ
- Stronger p-dependence
- Extend to $p \lesssim \Lambda = 6\ \text{fm}^{-1}$
- Larger overall

Reducing the momentum dependence

Remember

Skyrme EDF provides at best $m^{*\,\tau}_{Sk}(k^\tau_{\mathrm{F}})$ **independent of momentum**

Averaging procedure of $X = m^*_\tau(p, k^\tau_{\mathrm{F}})$ or $Z_\tau(p, k^\tau_{\mathrm{F}})$

- Evaluation on the Fermi surface

$$X_{pe}(k^\tau_{\mathrm{F}}) \equiv X(p = k^\tau_{\mathrm{F}})$$

- Averaging around the Fermi surface

$$X_{av}(k^\tau_{\mathrm{F}}) \equiv \frac{\int f(q,\Lambda)\, q^2\, dq\, X(q)\, u(q,k^\tau_{\mathrm{F}})\, v(q,k^\tau_{\mathrm{F}})}{\int f(q,\Lambda)\, q^2\, dq\, u(q,k^\tau_{\mathrm{F}})\, v(q,k^\tau_{\mathrm{F}})}$$

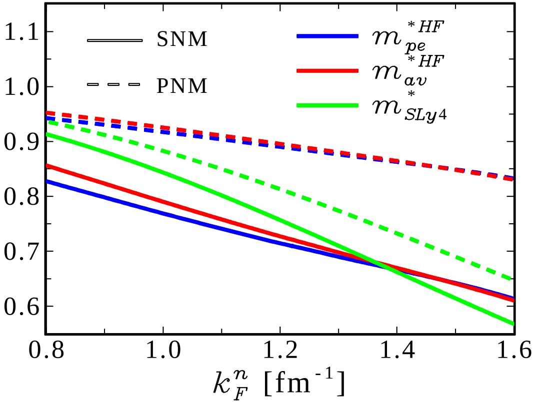
- Other variants

Questions

- Are the results sensitive to the averaging procedure?
- Is there a qualitative difference between hard and soft interactions?

Momentum independent effective masses for soft interaction

[K. Hebeler, T. Duguet, T. Lesinski, A. Schwenk, in preparation)]

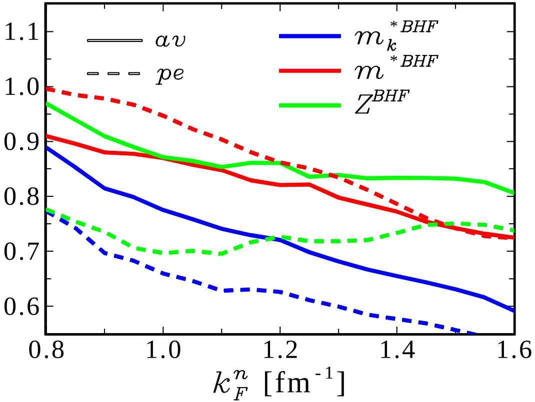


Only slight difference between av and pe at HF level

- SLy4 gives reasonable account of $m^{*HF}_{k}(k^{\tau}_{\mathrm{F}} \approx 1.36\ \mathrm{fm}^{-1})$ in SNM
- Wrong isovector m^{*}_{1} of SLy4 [T. Lesinski et al. 2006)]

Momentum independent effective masses in SNM for hard interaction

[K. Hebeler, T. Duguet, T. Lesinski, A. Schwenk, in preparation)]



Difference between av/pe much larger than for soft cutoff interaction

- Momentum dependence stronger
- Larger momentum-space for averaging procedure

Pairing gaps

Gap equation

- After pole approximation

$$\hat{\Delta}(\mathbf{k}) = -\int \frac{d^3\mathbf{k}'}{(2\pi)^3} \frac{Z(\mathbf{k})\, V_{NN}(\mathbf{k},\mathbf{k}';\Lambda)\, Z(\mathbf{k}')\, \hat{\Delta}(\mathbf{k}')}{2\sqrt{(\varepsilon_{\mathbf{k}'} - \mu)^2 + \hat{\Delta}^2(\mathbf{k}')}}$$

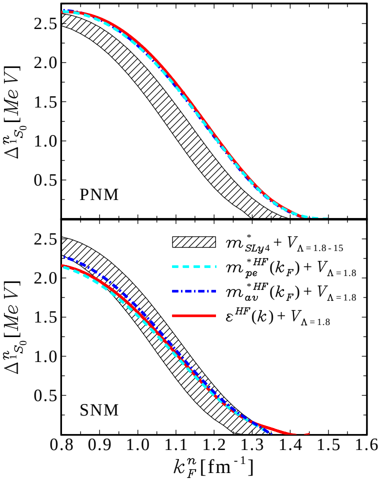
- $\hat{\Delta}(\mathbf{k}) = Z(\mathbf{k})\,\Delta(\mathbf{k}) =$ physical gap of the excitation spectrum
- Effective mass approximation relates to $\varepsilon_{\mathbf{k}'} - \mu$ in the denominator

Questions of interest regarding results in finite nuclei

- Impact of using $m^*_\tau(k, k_F) \approx m^*_\tau(k^\tau_F)$?
- Is there a qualitative difference between hard and soft interactions?

Pairing gaps from soft interactions - PNM and SNM

[K. Hebeler, T. Duguet, T. Lesinski, A. Schwenk, in preparation]



Gaps from full $m^{*HF}_\tau(k, k_F)$

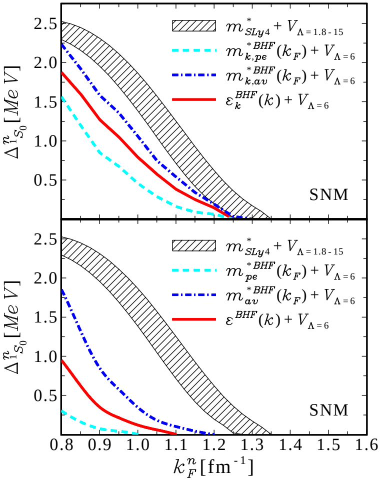
- Close to SLy4 + $V_{\Lambda=1.8}$ for $k^n_{\rm F} \in [1.1, 1.4]$
- Trace of wrong m^*_1 of SLy4 in PNM

Gaps from $m^{*HF}_\tau(k, k_F) \approx m^*_\tau(k^\tau_F)$

- Reproduce well gaps from $m^{*HF}_\tau(k, k_F)$
- No sensitivity to averaging procedure
- Variation $\ll$ bandwidth of SLy4 calc.
- Could optimize $m^*_{Sk}(k^\tau_{\rm F})$

Pairing gaps from hard interactions - SNM

[K. Hebeler, T. Duguet, T. Lesinski, A. Schwenk, in preparation]



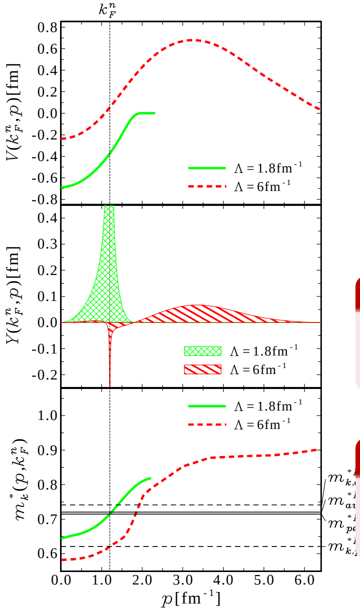
Gaps from full $m^{*BHF}_{\tau,e/k}(k, k_F)$

- Lower than SLy4 + $V_{\Lambda=15}$ for all k^n_{F}
- Very small if considering $Z_\tau(k, k_F)$

Gaps from $m^{*BHF}_{\tau,e/k}(k, k_F) \approx m^*_\tau(k^\tau_F)$

- Strong sensitivity to averaging procedure
- Variation $\gtrsim$ bandwidth of SLy4 calc.
- Fine tuning needed to design $m^*_{Sk}(k^\tau_{\mathrm{F}})$
  $\rightarrow$ Collaboration with Milan group

Analysis



- Consider matrix elements $V_{\Lambda=1.8/6}(k_{\rm F}^n, p)$
- Write gap equation schematically as

$$\hat{\Delta}(k_{\rm F}^n) \equiv \int dq \, Y(k_{\rm F}^n, q)$$

Gap generated

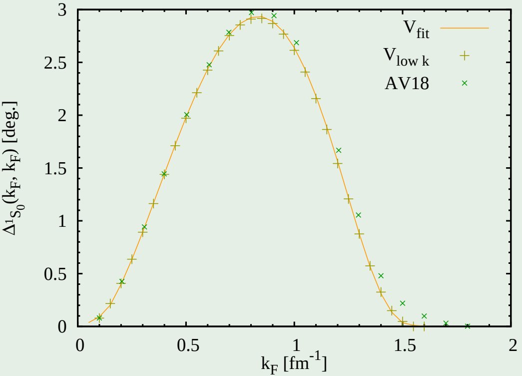
- Around the Fermi surface for soft Λ
- Mainly at large momenta for hard Λ

Effect of $m_\tau^*(k_F^\tau) =$ constant - 'pe' values here

- Good approx for soft Λ around $k_{\rm F}^n$
- Bad approx for hard Λ at relevant $p \gtrsim 2\,{\rm fm}^{-1}$

$\Delta^n_{^1S_0}(k_F, k_F, \Lambda)$ in INM from $V_{\mathrm{low\ k}}(\Lambda)$

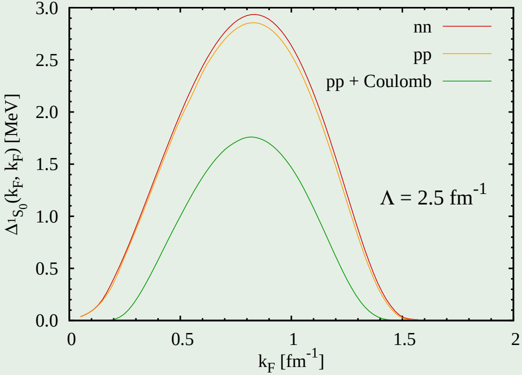
Neutron gaps with $\epsilon_k = \hbar^2 k^2/2m \quad (m^* = m)$



- Gaps of numerical $V_{\mathrm{low\ k}}$ reproduced perfectly without direct fitting
- True for $\Delta^n_{^1S_0}(k, k_F, \Lambda)$ $\forall\, k \lesssim \Lambda$
- $\partial_\Lambda \Delta^n_{^1S_0}(k_F, k_F, \Lambda) = 0$ [K. Hebeler, A. Schwenk, B. Friman, 2007]

$\Delta^q_{^1S_0}(k_F, k_F, \Lambda)$ in INM $V_{\text{low k}}(\Lambda) + V_{\text{Coul}}$

Neutron/proton gaps with $\epsilon_k = \hbar^2 k^2/2m$ $(m^* = m)$



- Coulomb reduces proton gaps by $\sim 40\%$
- Comparatively CSB effects are negligible

Effective masses

Effective k-mass, e-mass and total mass from $\Sigma_\Lambda(\mathbf{k}, \varepsilon_{\mathbf{k}})$

$$\frac{m_\tau^{*(1)}(p, k_\mathrm{F})}{m} \equiv \frac{m_{\tau,k}^{*(1)}(p, k_\mathrm{F})}{m} \frac{m_{\tau,e}^{*(1)}(p, k_\mathrm{F})}{m}$$

- $\Sigma_{\mathrm{soft}}^{(1)}$ from soft V_{NN} provides k-mass only
- $\Sigma_{\mathrm{hard}}^{(1)}$ from hard V_{NN} provides both k-mass and e-mass

Remember

Skyrme EDF provides at best $m_{Sk}^{*\tau}(k_\mathrm{F}^\tau)$ **independent of momentum**

Momentum-dependence averaging $m_\tau^*(k, k_F) \approx m_\tau^*(k_F^\tau)$

- Result insensitive to procedure for soft V_{NN}
- Result very sensitive to procedure for hard V_{NN}

Reducing the momentum dependence

Remember

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- Other variants

Results

- Provides exact same $m^*_\tau(k^\tau_F)$ for soft interaction
- Result very sensitive to procedure for hard interaction

Pole approximation to the gap equation

General gap equation with $\Gamma^{irr} = V$

$$\Delta(\mathbf{k}) = \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \int \frac{d\omega'}{2\pi i} V(\mathbf{k},\mathbf{k}') F(\mathbf{k}',\omega')$$

- Neglect imaginary part of Σ
- Find pole $E_{\mathbf{k}}$ by solving $F^{-1}(\mathbf{k},\omega) = 0$
- Expand propagator around these poles
- Perform the energy integral analytically in the limit $\Delta \ll \varepsilon_F$

$$\Delta(\mathbf{k}) = -\int \frac{d^3\mathbf{k}'}{(2\pi)^3} \frac{V(\mathbf{k},\mathbf{k}') Z(\mathbf{k}') \Delta(\mathbf{k}')}{2\sqrt{\left[\varepsilon^0_{\mathbf{k}'} - \mu + \frac{1}{2}\left[\Sigma(\mathbf{k}',\varepsilon_{\mathbf{k}'}) + \Sigma(\mathbf{k}', 2\mu - \varepsilon_{\mathbf{k}'})\right]\right]^2 + \Delta^2(\mathbf{k}')}}$$

with $Z(\mathbf{k}) = m_e^{-1}(\mathbf{k})$.

Reference: Definition of Fermi momenta

$$
\begin{aligned}
k_{\mathrm{F}}^{p} &= \left[\frac{3\pi^2}{2}(1-\beta)\rho\right]^{1/3} \\
k_{\mathrm{F}}^{n} &= \left[\frac{3\pi^2}{2}(1+\beta)\rho\right]^{1/3} \\
\beta &= (\rho_n - \rho_p)/\rho \\
\rho &= \rho_n + \rho_p = \frac{1}{3\pi^2}\left[(k_{\mathrm{F}}^{n})^3 + (k_{\mathrm{F}}^{p})^3\right]
\end{aligned}
$$

In SNM we have $k_{\mathrm{F}}^{p} = k_{\mathrm{F}}^{n} \equiv k_{\mathrm{F}} = \left[3\pi^2\rho_q\right]^{1/3}$.