Chaos in Baryons

César Fernández-Ramírez

European Centre for Theoretical Studies in Nuclear Physics and Related Areas, ECT*

&

Fondazione Bruno Kessler, FBK

Baryons: Historical Perspective

- $\Delta(1232)$

Anderson, Fermi, Long, Nagle, Phys. Rev. 85 (1952) 936

- Proliferation of baryons
- Non-relativistic quark models

Gell-Mann, Zweig, Greenberg, Dalitz, Karl, Koniuk, Isgur, (60's and 70's)

- Relativistic quark models

Capstick, Isgur, PRD 34 (1986) 2809

Bonn model, EPJA 10 (2001) 309; 395; 447

- Effective QCD-inspired models

Page, Swanson, Szczepaniak, PRD 59 (1999) 034016

Llanes-Estrada, Cotanch, PLB 504 (2001) 15; NPA 697 (2002) 303

- Lattice QCD

Bernard et al., PRD 64 (2001) 054506

The Problem of Missing Resonances

- Experiments vs quark models $\Longrightarrow$ missing states
- Experimental effort

  v.g. at JLab and Mainz

- We apply spectral statistic techniques to test quark models and survey the problem of missing resonances

Spectral Statistics

- It studies how an ordered sequence of numbers (v.g. an energy spectrum) matches an statistical theory
- Two kinds of statistics
  - Nearest neighbors
  - Long distance correlations
- Statistical methods are a powerful tool to study the energy spectrum of quantum systems
- Methods have improved over the last years: Analysis of systems with low number of levels are presently reliable and problems such as the hadron spectrum can be faced

Spectral Fluctuations

- Spectra can be split in a smooth and a fluctuating part
  $\rho(E) = \rho_s(E) + \rho_f(E)$
- Universality of fluctuations in chaotic and integrable systems
- This allows to consider the system as a black-box without considering the underlaying interaction
- Fluctuations are extracted from the spectrum through an unfolding procedure
- Nearest Neighbors Spacings Distribution is the most utilized

  $\longrightarrow$ Distance between two consecutive levels with the same symmetries (quantum numbers)

Integrability and Chaoticity in Quantum Systems

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- Statistical properties of the energy-level fluctuations are universal and determine whether a system is chaotic or integrable
- Integrable and chaotic systems display different fluctuation pattern
- The sequence of spacings $\{s_i\}$ for an integrable system can be considered as a sequence of independent random variables (non-correlated sequence of levels)
- Chaotic systems are characterized by a correlation structure described by RMT (standard set)
- Paradigms of integrable and chaotic systems

$\longrightarrow$ Quantum billiards

Fluctuations and Integrable Spectra

→ Integrable systems (uncorrelated)

- Fluctuations follow Poisson distribution

Berry, Tabor, Proc. R. Soc. London A 356 (1977) 375

- Uncorrelated systems
- Example: Random noise

Fluctuations and Chaotic Spectra

$\longrightarrow$ Chaotic systems (correlated)

- Flutuations follow Wigner surmise

Bohigas, Giannoni, Schmit, PRL 52 (1984) 1

- Example: Nuclei
- Standard for chaotic systems: Random Matrix Theory (useful for statistical studies)

Missing Levels in a Spectrum

Statistical tools allow to identify the existence of missing states

Bohigas, Pato, PLB 595 (2004) 25; PRE 74 (2006) 036212

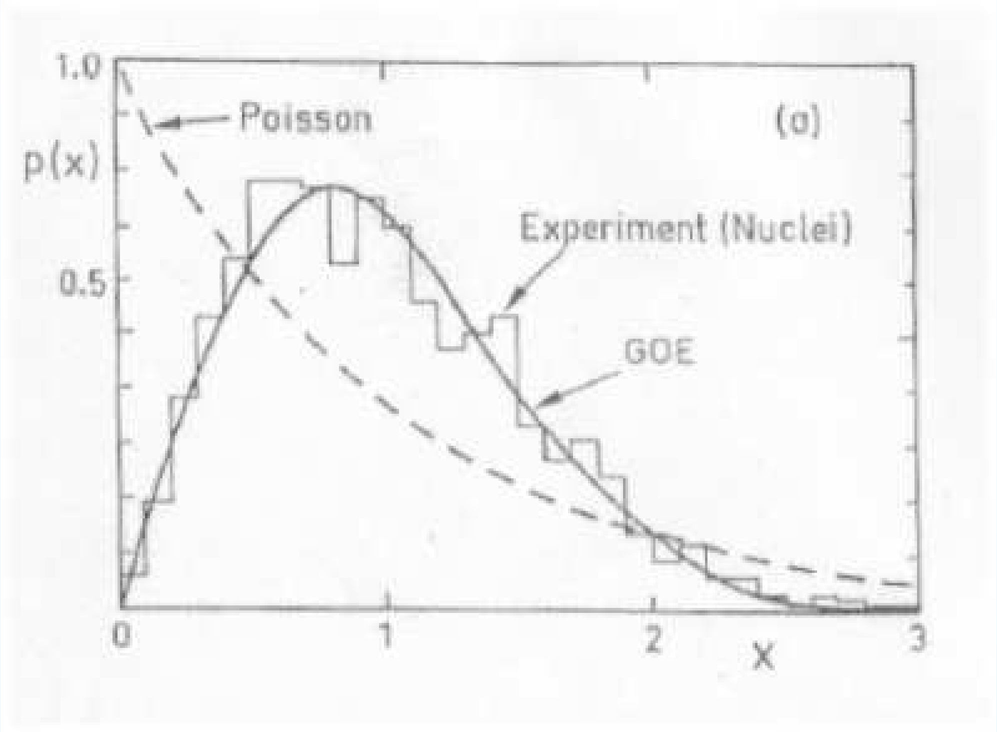
Molina, Retamosa, Muñoz, Relaño, Faleiro, PLB 644 (2007) 25

Missing levels cause the spectral fluctuations of a spectrum with Wigner distribution look more like a Poisson distribution

We can use this property to identify missing levels in a spectrum

Spectral Fluctuations and Nuclei

Experimental spectrum in nuclei follows RMT (Wigner surmise)



Bohigas, NPA 751 (2005) 343c

Symmetries in the Baryon Spectrum

- I isospin
- J spin
- π parity
- We drop strangeness due to $SU(3)$ invariance
- From the full spectrum we extract sequences of levels with given $I\,(J^\pi)$

Unfolding Procedure

- $\rho(E) = \rho_s(E) + \rho_f(E)$
- We choose the simplest unfolding prescription
- $S_i = E_{i+1} - E_i$
- We rescale using its average value

$$s_i = S_i / \langle S \rangle$$

- Nearest Neighbor Spacings (NNS)
- This procedure assumes an energy independent behavior of the smooth part of the density $\rho_s(E) = 1/\langle S \rangle$

Pascalutsa, EPJA 16 (2003) 149

Nearest Neighbor Spacing Distribution (NNSD)

- Poisson: integrable / uncorrelated

$$P(s) = \exp(-s)$$

- Wigner: chaotic / correlated

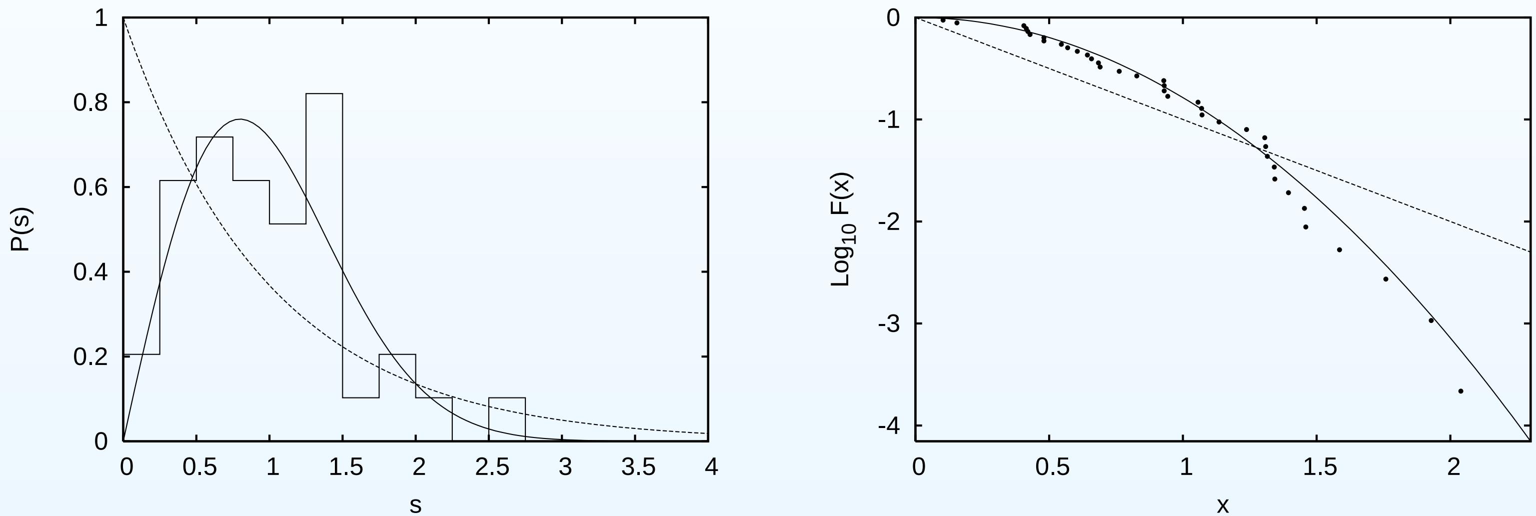
$$P(s) = \frac{\pi s}{2} \exp\left(-\frac{\pi s^2}{4}\right)$$

- Accumulated NNSD,

$$F(x) = 1 - \int_0^x ds\, P(s),$$

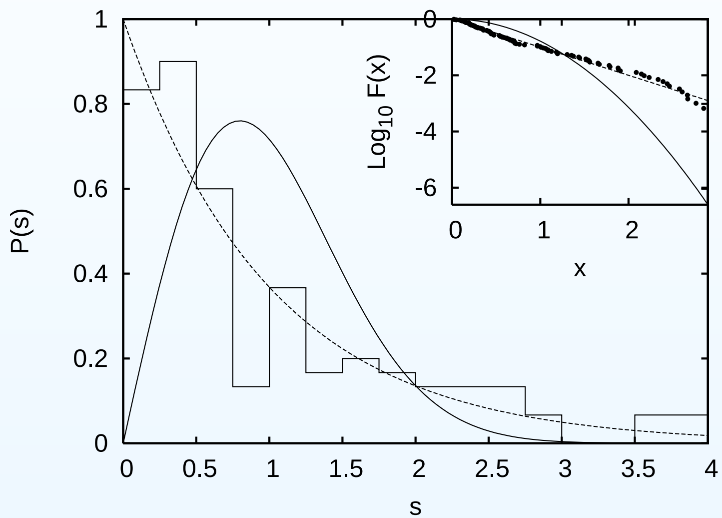
allows a better study of the tail of the distribution

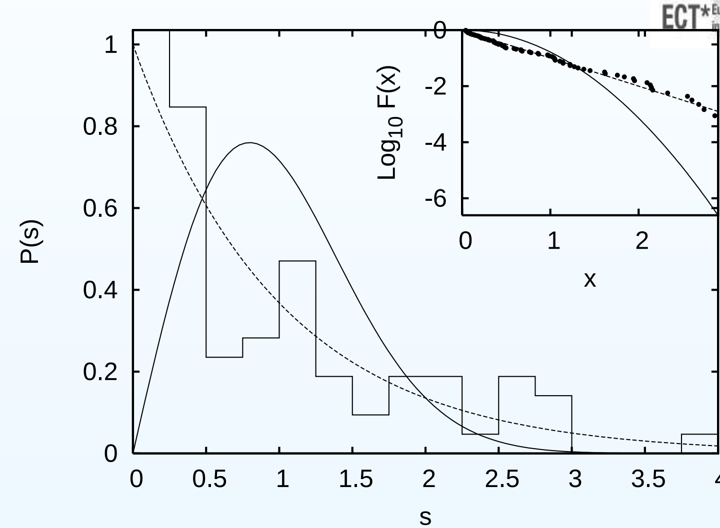
Experimental Baryon Spectrum up to 2.2 GeV

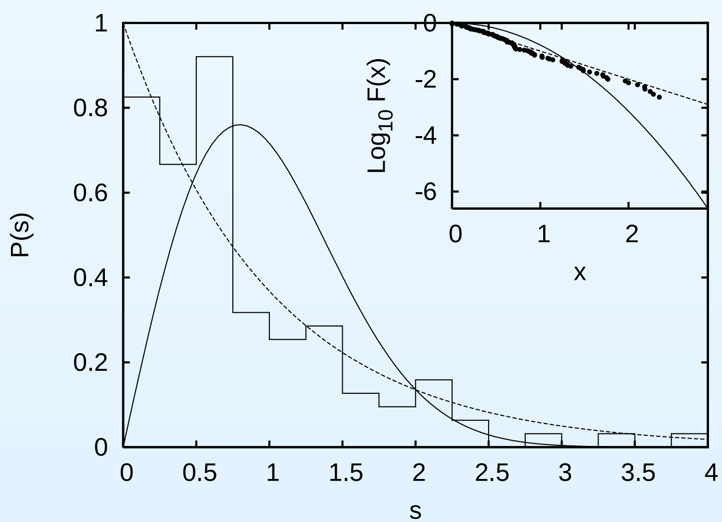


70 energy levels distributed in 15 sequences

Relativistic Quark Models







Models by

Löring *et al.*, (sets L1 & L2)

EPJA 10 (2001) 309; 395; 447

Capstick and Isgur, (set CI)

PRD 34 (1986) 2809

Unfolding. Problems?

- Sometimes, very short sequences of levels
- In such cases, unfolding can provide misleading results, making spacings spuriously closer and bringing the NNSD tend to the Wigner surmise
- Unfolding can yield different effects in different spectra: We avoid a direct comparison of the spectral fluctuations
- Complementary analysis: Kolmogorov-Smirnov goodness-of-fit tests

Kolmogorov-Smirnov Goodness-of-Fit Test

- To determine whether two datasets differ significantly
- No assumption about the distribution of data (non-parametric and distribution free)
- Based on the maximum distance between cumulative probabilities

Kolmogorov, Giornale dell'Istituto Italiano degli Attuari 4 (1933) 83

Smirnov, Bull. Moscow Univ. 2 (1933) 3; Ann. Math. Stat. 19 (1948) 279

Feller, Ann. Math. Stat. 19 (1948) 177

NAG Libraries, http://www.nag.co.uk

Applied Procedure

- We build synthetic spectra (Wigner-like and Poisson-like) optimized to each set
- Each synthetic spectrum has the same size and is distorted by the unfolding in the same way as sets EXP, CI, L1, and L2 are
- We do this many times (500 realizations) so we can have statistical significance

Kolmogorov-Smirnov Test

Probability to obtain, under the null hypothesis, a value of the Kolmogorov-Smirnov test statistic as the one observed

Spectrum	EXP	CI	L1	L2
Poisson	0.51	0.49	0.25	0.53
Wigner	0.80	0.18	0.05	0.01

Null hypothesis: Both distributions display equal spectral fluctuations

Estimation of Missing Levels

If we assume that the *real* distribution is 100% Wigner we can speculate on the maximum amount of missing states

- We remove levels randomly from a Wigner distribution until we get values for the K-S test closer to what is observed
- Very rough estimation: <20% of missing levels

Conclusions (I)

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- The statistical techniques developed to study the chaotic character of quantum systems have evolved into powerful and reliable techniques that can provide new insight in hadron physics
- From the spectral fluctuations of the *experimental* baryon spectrum one can conclude the importance of correlations in the underlying physics
- From the analysis of *theoretical* spectra from constituent quark models, one can conclude that, as presently built, they do not describe the basic statistical properties of the low-lying baryon spectrum and they need to include more correlations

Conclusions (II)

- The result is model-independent
- There is room for missing resonances at the 20% level
- Predictions of missing states derived from constituent quark models are not reliable

Fernández-Ramírez, Relaño, PRL98 (2007) 062001

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Fernández-Ramírez, Relaño, PRL98 (2007) 062001

Questions:

Is the spectrum really Wigner?

What is missing in the quark models?

What does this tell us on confinement/QCD/hadrons?

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Fernández-Ramírez, Relaño, PRL98 (2007) 062001

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Backstage

Direct Comparison by means of a Goodness-of-Fit Test

Wilcoxon Rank-Sum Test

Wilcoxon, Biometrics Bull. 1 (1945) 80
Mann, Whitney, Ann. Math. Stat. 18 (1947) 50

	CI	L1	L2
EXP	0.0487	0.1067	0.1036

Allows to test whether two populations of different size are statistically alike

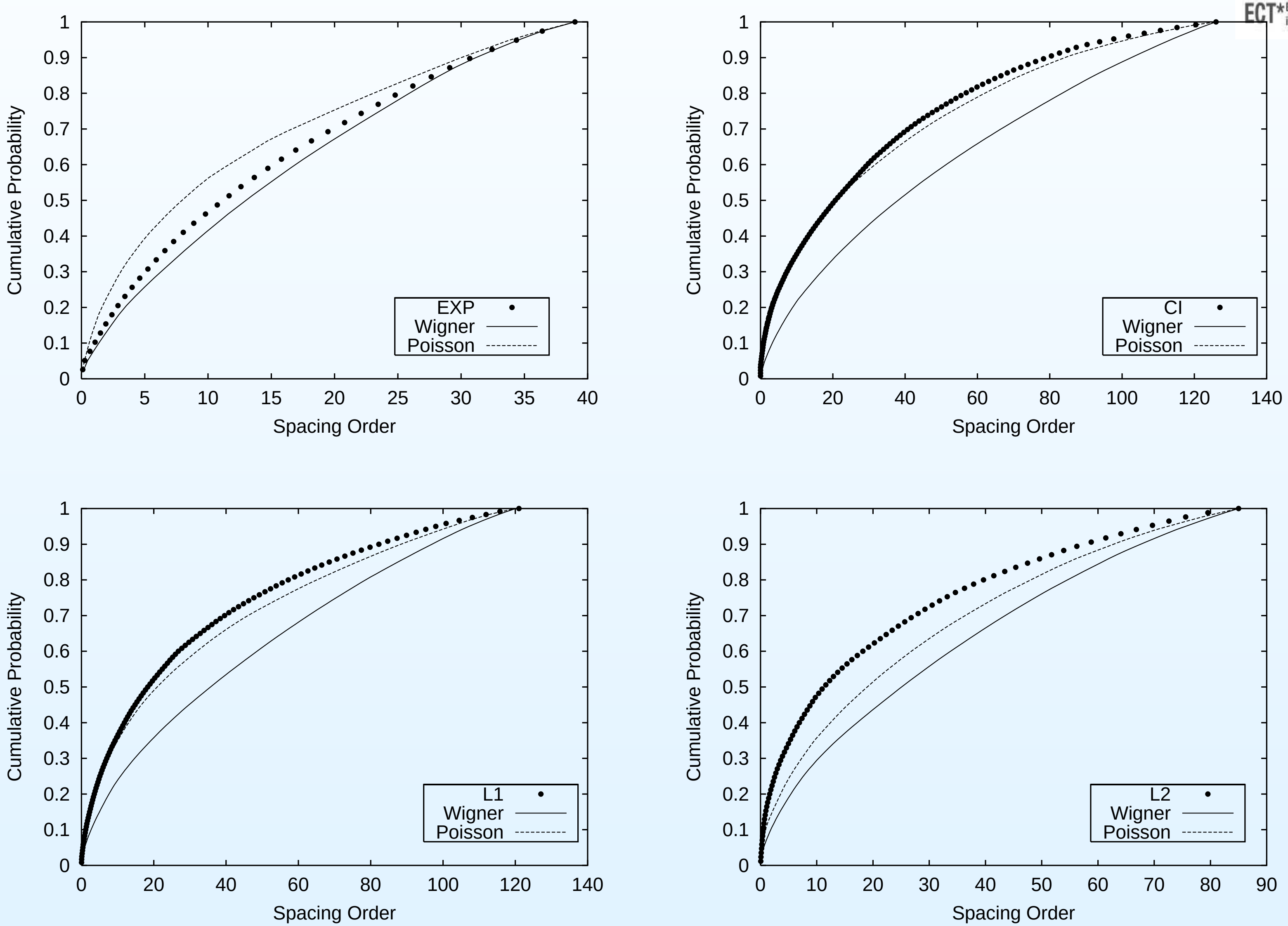
Wilcoxon Rank-Sum Test

Probability to obtain, under the null hypothesis, a value of the Wilcoxon Rank-Sum test statistic as extreme as that observed

Spectrum	EXP	CI	L1	L2
Poisson	0.67	0.64	0.55	0.22
Wigner	0.92	0.0015	0.019	0.059

Null hypothesis: Both distributions are statistically equal
Only one realization of the "experiment"

Cumulative Probabilities: The K-S Test in more Detail

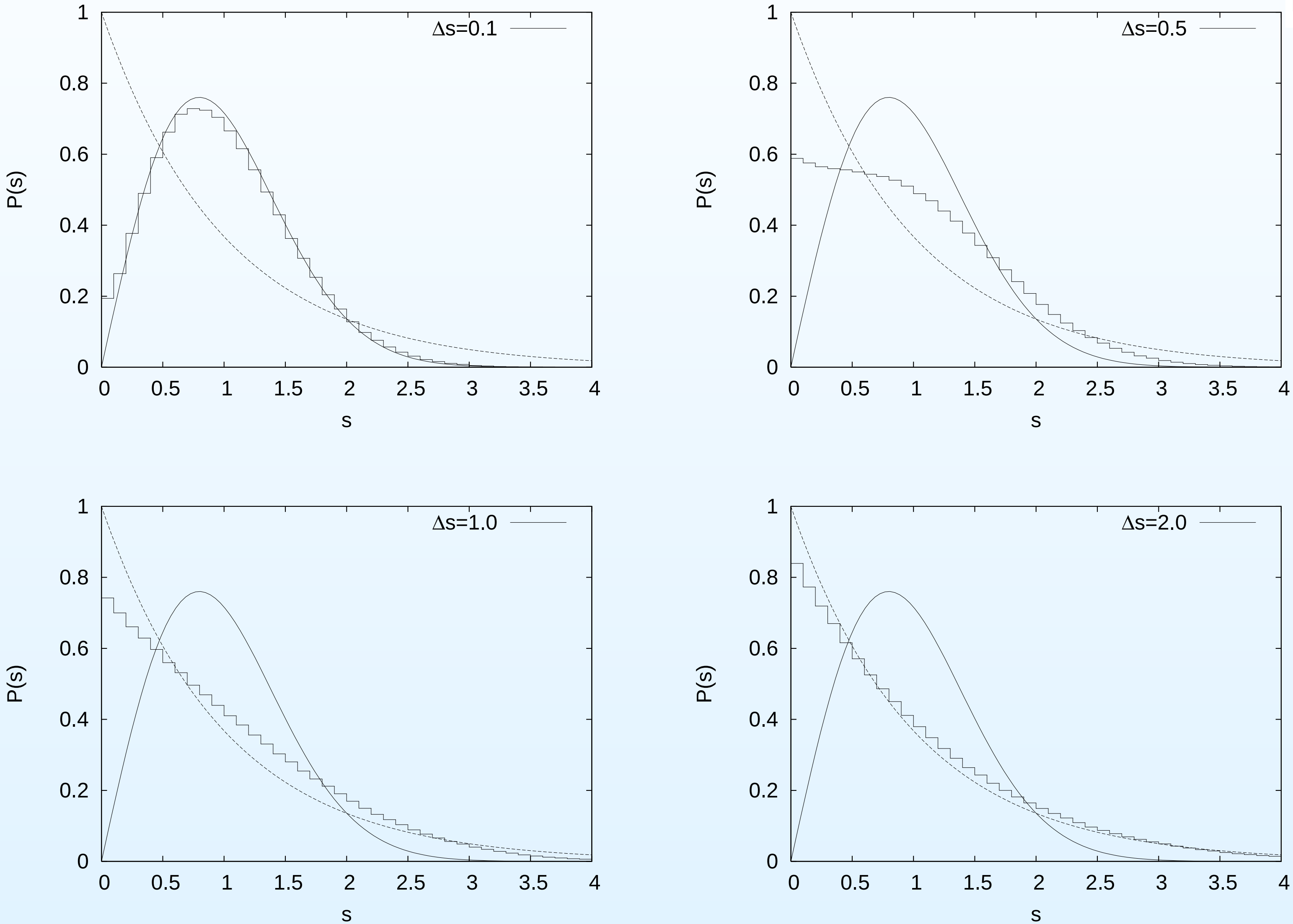


Deviations from Wigner Surmise

Due to

- Uncertainties in the masses (error bars)
  $\longrightarrow$ Errors mean random noise, which brings the NNSD closer to a Poisson distribution
- Existence of missing states
  $\longrightarrow$ Missing resonances

Error Bars (Toy Model Simulation): NNSD



Error Bars (Toy Model Simulation): Accumulated NNSD