Exploring Continuum Structures with a Pseudo-State Basis

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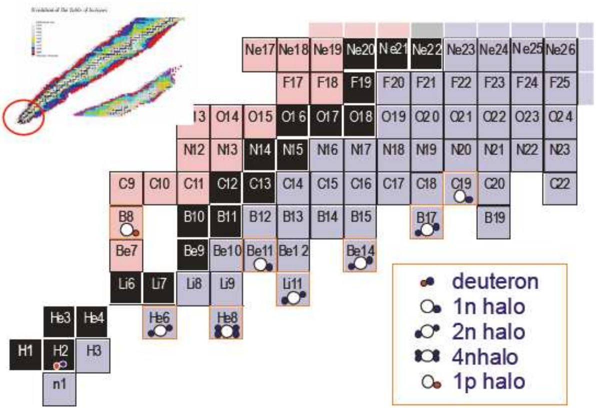
1. Motivation

2. PS discretization method
   - General PS method
   - The analytical THO basis

3. Application to reactions
   - ^{6}He+^{208}Pb @ 240 MeV/u
   - ^{6}He+^{12}C @ 240 MeV/u
   - ^{11}Be+^{12}C @ 67 MeV/u

Motivation


Ne17 Ne18 Ne19 Ne20 Ne21 Ne22 Ne23 Ne24 Ne25 Ne26
F17 F18 F19 F20 F21 F22 F23 F24 F25
13 O14 O15 O16 O17 O18 O19 O20 O21 O22 O23 O24
N12 N13 N14 N15 N16 N17 N18 N19 N20 N21 N22 N23
C9 C10 C11 C12 C13 C14 C15 C16 C17 C18 C19 C20 C22
B8 B10 B11 B12 B13 B14 B15 B17 B19
Be7 Be9 Be10 Be11 Be12 Be14
Li6 Li7 Li8 Li9 Li11
He3 He4 He6 He8
H1 H2 H3
n1
deuteron
1n halo
2n halo
4nhalo
1p halo

The role of the continuum

Coupling to BU (unbound) channels play an important role in the scattering of loosely bound nuclei

$$\Downarrow$$

Coupled-Channels (CC) method

✓ Describe the coupling to internal degrees of freedom of the projectile

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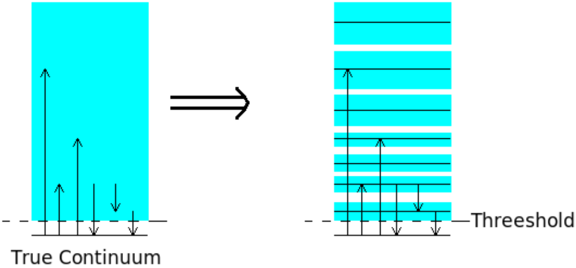
$\Downarrow$

Coupled-Channels (CC) method

- ✓ Describe the coupling to internal degrees of freedom of the projectile
- ✗ Only for a finite numbers of square-integrable states

The role of the continuum

"True" Continuum: { Infinite number of estates. No square-integrable.



The role of the continuum

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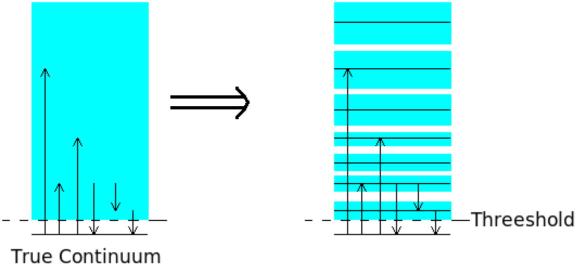


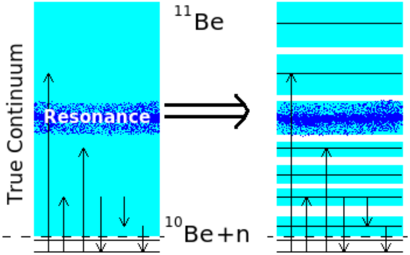
CC $\Longrightarrow$ Continuum Discretized Coupled Channels (CDCC)

The Continuum problem in CDCC

Discretization methods:

- Binning:

$$u_l^i(k) = \sqrt{\frac{2}{\pi N_i}} \int_{k_{i-1}}^{k_i} f_i(k)\phi_l(k,r)dk.$$



The Continuum problem in CDCC

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- Binning:

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- Pseudo-states.

$\Rightarrow$ The Continuum is described through a basis (HO, Sturmian) of square-integrable states.

Pseudo-states (PS) discretization method

- Discrete set of $\mathcal{L}^2$ functions: $|\phi_n\rangle$

Completeness condition:

$$\sum^{N} |\phi_i\rangle\langle\phi_i| \approx \mathrm{I}$$

- To diagonalize the internal hamiltonian of a projectile $\mathcal{H}_p$

Matrix elements:

$$\mathcal{H}_p \longmapsto \sum_{n,n'} |\phi_n\rangle\langle\phi_n|\mathcal{H}_p|\phi_{n'}\rangle\langle\phi_{n'}|$$

Pseudo-states (PS) discretization method

Eigenstates of the matrix NxN:

$$|\varphi_n^{(N)}\rangle = \sum^{N} C_i^n |\phi_i\rangle$$

- $\begin{cases} \text{n}_b \text{ states with } \varepsilon_n < 0 \text{ representing the bound states.} \\ \text{N-n}_b, \varepsilon_n > 0 \Rightarrow \text{discrete representation of the Cont.} \end{cases}$
- Orthogonal and normalizable.

Which basis may I use? Sturmian, Harmonic Oscillator?

Harmonic Oscillator basis

HO vs THO:

$$\phi(s) \longmapsto e^{-\left(\frac{s}{b}\right)^2} \quad \Longrightarrow \quad \phi[s(r)] \longmapsto e^{-\frac{\gamma^2}{2b^2} r}$$

- Correct asymptotic behaviour for bound states.
- Range controlled by the parameters of the LST.

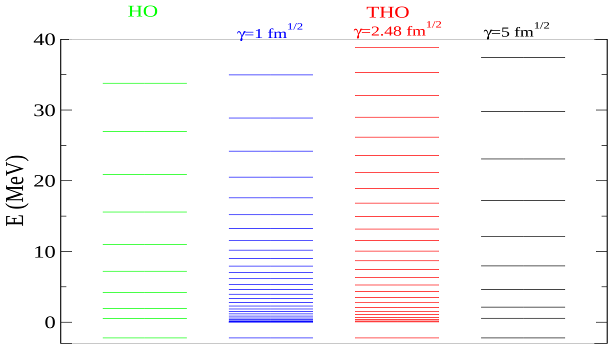
Analytic LST from Karataglidis *et al.*,PRC71,064601(2005)

$$s(r) = \frac{1}{\sqrt{2}b} \left[\frac{1}{\left(\frac{1}{r}\right)^m + \left(\frac{1}{\gamma\sqrt{r}}\right)^m} \right]^{\frac{1}{m}}$$

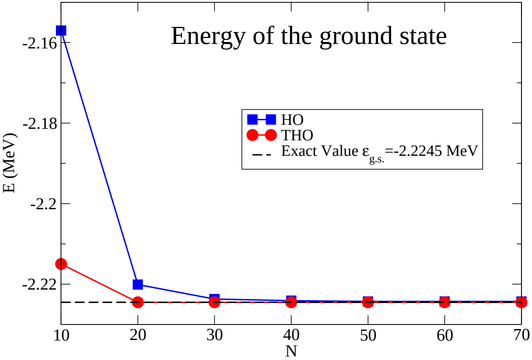
THO parameters

- b is treated as a variational parameter to minimize g.s. energy
- Then $\frac{\gamma}{b}$ is related to the k_{max}:

$$\frac{\gamma}{b} = \sqrt{2k_{max}}$$



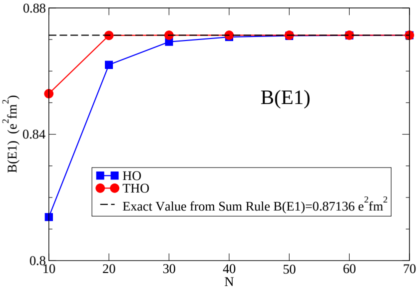
Convergence in HO vs. THO with deuteron

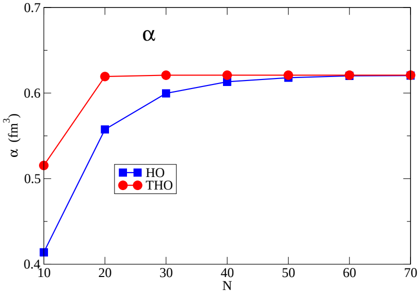


!! Not influenced by the continuum

Convergence in HO vs. THO with deuteron

$$B(E1) \propto \sum_{n=1}^{N} \left| \langle \varphi_{n,\ell}^{(N)} || \mathcal{M}(E1) || \varphi_{g.s.} \rangle \right|^2 \quad \alpha \equiv \frac{8\pi}{9} \int d\varepsilon \frac{1}{(\varepsilon - \varepsilon_{\mathrm{g.s.}})} \frac{dB(E1)}{d\varepsilon}$$

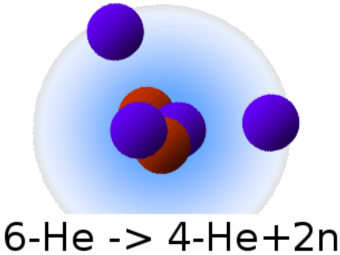




THO with ^{6}He

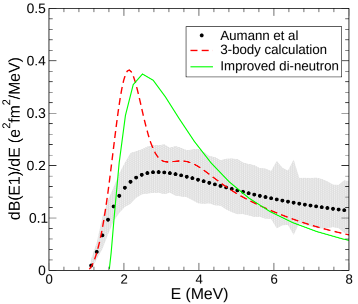
Improved Di-neutron model

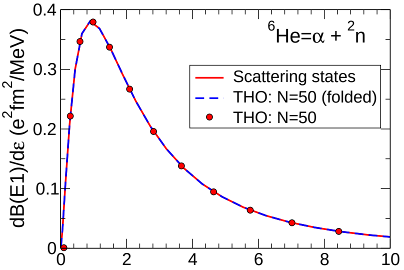
- ^{6}He $\approx$ 2n-α
  Moro *et al.*, PRC75, 064607 (2007)



Electric Transition Probabilities

$$\frac{dB(E\lambda)}{d\varepsilon} \propto \left| \sum_{n=1}^{N} \langle \varphi_\ell(k) | \varphi_{n,\ell}^{(N)} \rangle \langle \varphi_{n,\ell}^{(N)} || \mathcal{M}(E\lambda) || \varphi_{g.s.} \rangle \right|^2$$





M. Rodríguez-Gallardo *et al.*, PRC77(2008)064609

Electric Transition Probabilities

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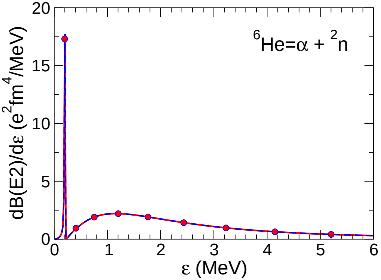
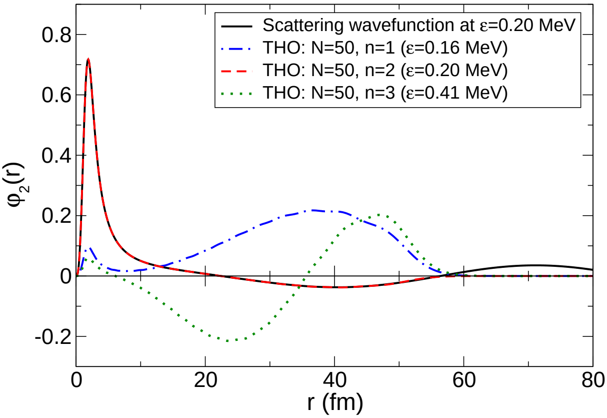
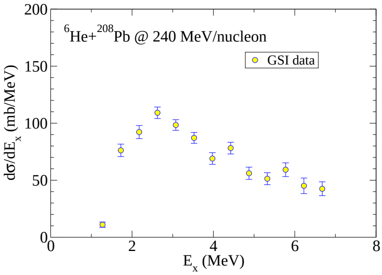


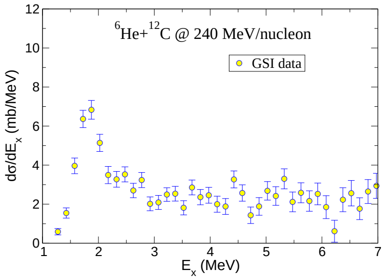
Treatment of Resonances

for N not too large, resonances tend to concentrate 1-2 eigenstates



^{6}He+^{208}Pb,^{12}C @ 240 MeV/u





- Coulomb Excitation
- Nuclear Interaction

T. Aumann *et al.* PRC 59, 1252 (1999)

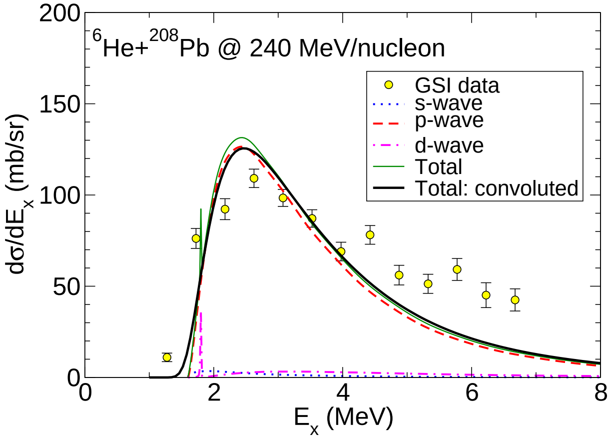
^{6}He+^{208}Pb @ 240 MeV/u

CDCC calculations

Potentials involved

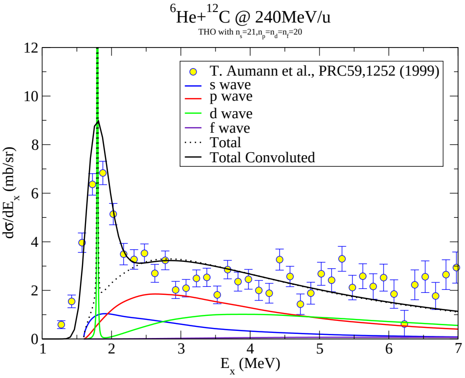
1. 2n-α potential from di-neutron model
   Moro *et al.*, PRC75, 064607 (2007)
2. α-^{208}Pb
   Khoa *et al.*, PRC59, 1252 (2002)
3. 2n-^{208}Pb folding potential from n-^{208}Pb potential

From T. Aumann *et al.*

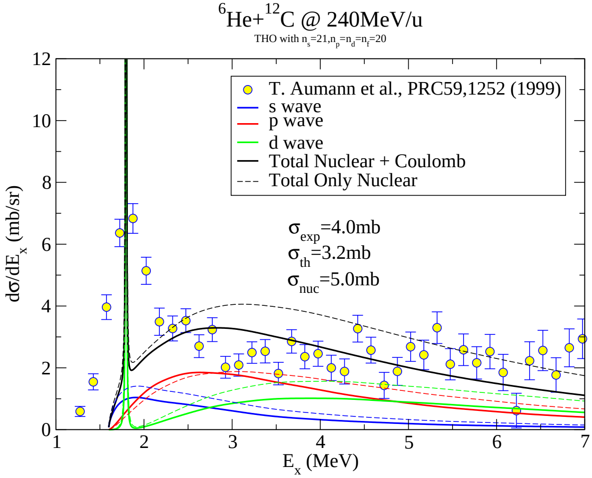


Energy distribution dominated by (Coulomb) coupling to dipole states

^{6}He+^{12}C @ 240 MeV/u



- Low energy dominated by coupling to 2^+ resonance
- High energy *s*,*p*,*d* background

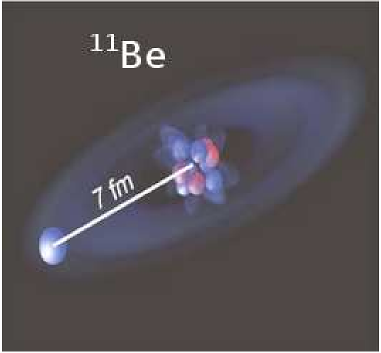


- Destructive interference with the Coulomb part

^{11}Be+^{12}C @ 67 MeV/nucleon

Potentials involved

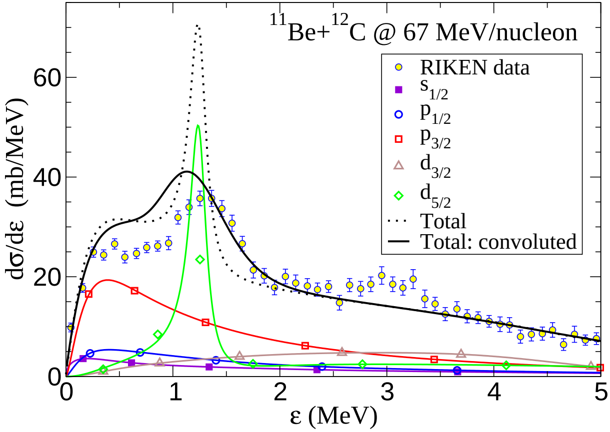
1. n-^{10}Be with ^{10}Be in its g.s. 0^+

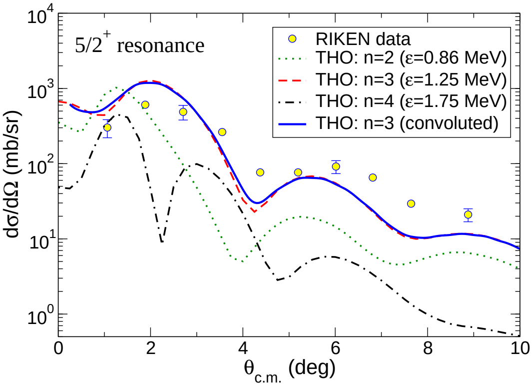


!! contribution of ^{10}Be in its 1st excited state 2^+

^{11}Be+^{12}C @ 67 MeV/nucleon

RIKEN: N. Fukuda *et al.*, Phys. Rev. C70, 054606 (2004)





Only 1 eigenstate vs. 15 bins

⇒ J. Phys. (London) G31, S1881 (2005)

⇒ Phys. Rev. C82, 024605 (2010)

Conclusions

THO PS method

Provides a suitable discrete description of the continuum.

Dealing with Resonances

Natural and accurate treatment of narrow resonances.

Next Step

Generalise the PS method to (halo) two-body nuclei with deformed core: ^{11}Be, ^{19}C...

Phase-shifts integral formula

Hazi and Taylor Formula, PRA1,1109 (1970)

$$\tan \delta_\ell(k) = -\frac{\int_0^\infty u_\ell(k,r)[E-H]f(r)F_\ell(kr)dr}{\int_0^\infty u_\ell(k,r)[E-H]f(r)G_\ell(kr)dr} \tag{1}$$

Calculating energy distribution of B($E\lambda$) with PS

For B($E\lambda$):

- $$\left.\frac{dB(E\lambda)}{d\varepsilon}\right|_{\varepsilon=\varepsilon_n} \simeq \frac{1}{\Delta_n}\left|\langle\varphi_{n,\ell}^{(N)}||\mathcal{M}(E\lambda)||\varphi_{g.s.}\rangle\right|^2$$
- $$\frac{dB(E\lambda)}{d\varepsilon} \simeq \frac{\mu_{bc}k}{(2\pi)^3\hbar^2}\left|\sum_{n=1}^{N}\langle\varphi_\ell(k)|\varphi_{n,\ell}^{(N)}\rangle\langle\varphi_{n,\ell}^{(N)}||\mathcal{M}(E\lambda)||\varphi_{g.s.}\rangle\right|^2$$

$$\Delta_n = \frac{\varepsilon_{n+1}-\varepsilon_{n-1}}{2}$$

Sum Rules

For B($E\lambda$):

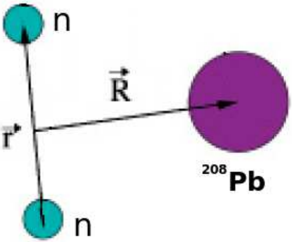
$$B(E\lambda) = \int d\varepsilon \frac{dB}{d\varepsilon} = \frac{2J_f+1}{2J_i+1}(D^{(\lambda)}_{J_i,J_f})^2 \langle \varphi_{\mathrm{g.s.}} | r^{2\lambda} | \varphi_{\mathrm{g.s.}} \rangle \qquad (2)$$

^{6}He+^{208}Pb @ 240 MeV/u

2n-^{208}Pb folding potential

$$U(\vec{R}) = \int d\vec{r}\rho_{nn}(r)\left(V_{n-^{208}Pb}(\vec{R}+\frac{1}{2}\vec{r}) + V_{n-^{208}Pb}(\vec{R}-\frac{1}{2}\vec{r})\right)$$

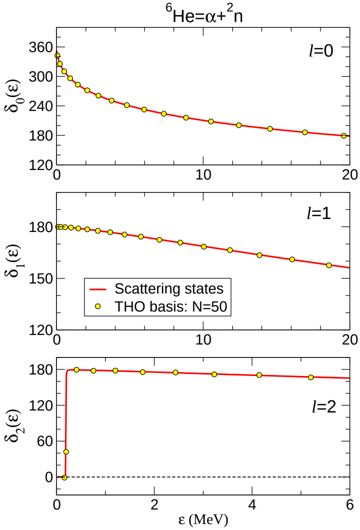
ρ_{nn} obtained from Three-Body model of ^{6}He



THO with ^{6}He

Phase-shifts

* For $\varphi_\ell(k) \Rightarrow$ Asymptotic behavior
* For $\varphi_{n,\ell}^{THO} \Rightarrow$ Integral formula



Energy distribution of pseudo-states

Density of states

$$\rho^{(N)}(k) = \sum_{n=1}^{N} \langle \varphi_\ell(k) | \varphi_{n,\ell}^{(N)} \rangle$$