Exploring Continuum Structures with a Pseudo-State Basis

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2 PS discretization method
   - General PS method
   - The analytical THO basis

3 Application to reactions
   - $^6\text{He}+^{208}\text{Pb} @ 240 \text{ MeV/u}$
   - $^6\text{He}+^{12}\text{C} @ 240 \text{ MeV/u}$
   - $^{11}\text{Be}+^{12}\text{C} @ 67 \text{ MeV/u}$
Motivation

PS discretization method
Application to reactions

Continuum structures with a PS basis
The role of the continuum

Coupling to BU (unbound) channels play an important role in the scattering of loosely bound nuclei

Coupled-Channels (CC) method

✓ Describe the coupling to internal degrees of freedom of the projectile
Coupling to BU (unbound) channels play an important role in the scattering of loosely bound nuclei.

Coupled-Channels (CC) method

- Describe the coupling to internal degrees of freedom of the projectile
- Only for a finite numbers of square-integrable states
The role of the continuum

“True” Continuum: \{ Infinite number of estates.
No square-integrable. \}

True Continuum

- Threshold
The role of the continuum

“True” Continuum: { Infinite number of estates.
No square-integrable.

CC $\Rightarrow$ Continuum Discretized Coupled Channels (CDCC)
The Continuum problem in CDCC

Discretization methods:

- Binning:

\[ u_i^j(k) = \sqrt{\frac{2}{\pi N_i}} \int_{k_{i-1}}^{k_i} f_i(k) \phi_l(k, r) dk. \]
Discretization methods:

- **Binning:**
  \[
  u^i_l(k) = \sqrt{\frac{2}{\pi N_i}} \int_{k_{i-1}}^{k_i} f_i(k) \phi_l(k, r) dk.
  \]

- **Pseudo-states.**

  ⇒ The Continuum is described through a basis (HO, Sturmian) of square-integrable states.
Pseudo-states (PS) discretization method

- Discrete set of $L^2$ functions: $|\phi_n\rangle$

Completeness condition:

$$\sum_{i=1}^{N} |\phi_i\rangle\langle\phi_i| \approx I$$

- To diagonalize the internal hamiltonian of a projectile $\mathcal{H}_p$

Matrix elements:

$$\mathcal{H}_p \rightarrow \sum_{n,n'} |\phi_n\rangle\langle\phi_n| \mathcal{H}_p |\phi_{n'}\rangle\langle\phi_{n'}|$$
Pseudo-states (PS) discretization method

Eigenstates of the matrix \( N \times N \):

\[
|\varphi_n^{(N)}\rangle = \sum_{i=1}^{N} C_i^n |\phi_i\rangle
\]

- \( n_b \) states with \( \varepsilon_n < 0 \) representing the bound states.
- \( N-n_b, \varepsilon_n > 0 \) \( \Rightarrow \) discrete representation of the Cont.
- Orthogonal and normalizable.

Which basis may I use? Sturmian, Harmonic Oscillator?
Harmonic Oscillator basis

**HO vs THO:**

\[
\phi(s) \mapsto e^{-\left(\frac{s}{b}\right)^2} \quad \Rightarrow \quad \phi[s(r)] \mapsto e^{-\frac{\gamma^2}{2b^2}r}
\]

- Correct asymptotic behaviour for bound states.
- Range controlled by the parameters of the LST.

**Analytic LST from Karataglidis et al., PRC71, 064601 (2005)**

\[
s(r) = \frac{1}{\sqrt{2b}} \left[ \frac{1}{(\frac{1}{r})^m + \left(\frac{1}{\gamma r}\right)^m} \right]^{\frac{1}{m}}
\]
**THO parameters**

- $b$ is treated as a variational parameter to minimize g.s. energy.
- Then $\frac{\gamma}{b}$ is related to the $k_{\text{max}}$:

$$\frac{\gamma}{b} = \sqrt{2k_{\text{max}}}$$

![Graph showing energy levels for different $\gamma$ values](image-url)
Convergence in HO vs. THO with deuteron

Energy of the ground state

!! Not influenced by the continuum
Motivation
PS discretization method
Application to reactions

General PS method
The analytical THO basis

Convergence in HO vs. THO with deuteron

\[ B(E1) \propto \sum_{n=1}^{N} \left| \langle \phi_{n,\ell}^{(N)} | \mathcal{M}(E1) | \phi_{g.s.} \rangle \right|^2 \]

\[ \alpha \equiv \frac{8\pi}{9} \int d\varepsilon \frac{1}{(\varepsilon - \varepsilon_{g.s.})} \frac{dB(E1)}{d\varepsilon} \]

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Continuum structures with a PS basis
THO with $^6\text{He}$

Improved Di-neutron model

$^6\text{He} \approx 2n-\alpha$

Moro et al., PRC75, 064607 (2007)

6-He -> 4-He+2n
Electric Transition Probabilities

\[ \frac{dB(E\lambda)}{d\varepsilon} \propto \sum_{n=1}^{N} \langle \varphi_{\ell}(k) | \varphi^{(N)}_{n,\ell} \rangle \langle \varphi^{(N)}_{n,\ell} | M(E\lambda) | \varphi_{g.s.} \rangle \]

M. Rodríguez-Gallardo et al., PRC77(2008)064609
Motivation
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Electric Transition Probabilities

\[
\frac{dB(E\lambda)}{d\varepsilon} \propto \left| \sum_{n=1}^{N} \langle \phi_{\ell}(k) | \phi_{n,\ell}^{(N)} \rangle \langle \phi_{n,\ell}^{(N)} | M(E\lambda) | \phi_{g.s.} \rangle \right|^2
\]

\[6 \text{He} = \alpha + ^2n\]

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Continuum structures with a PS basis
for $N$ not too large, resonances tend to concentrate 1-2 eigenstates
**Motivation**

PS discretization method

**Application to reactions**

\[ ^6\text{He} + ^{208}\text{Pb}, ^{12}\text{C} @ 240 \text{ MeV/u} \]

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**Coulomb Excitation**

T. Aumann *et al.* PRC 59, 1252 (1999)

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**Nuclear Interaction**

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J.A. Lay *et al.*

Continuum structures with a PS basis
CDCC calculations

Potentials involved

1. 2n-α potential from di-neutron model
   Moro et al., PRC75, 064607 (2007)

2. α-208Pb
   Khoa et al., PRC59, 1252 (2002)

3. 2n-208Pb folding potential from n-208Pb potential
Energy distribution dominated by (Coulomb) coupling to dipole states
Motivation
PS discretization method
Application to reactions

${}_6\text{He} + {}^{12}\text{C} @ 240 \text{ MeV/u}$

- Low energy dominated by coupling to $2^+$ resonance
- High energy $s,p,d$ background

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T. Aumann et al., PRC59,1252 (1999)

s wave
p wave
d wave
f wave
Total
Total Convoluted

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Continuum structures with a PS basis
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$^6\text{He}^+\^{12}\text{C} @ 240\text{MeV/u}$

THO with $n_s=21$, $n_p=n_d=n_f=20$

$\frac{d\sigma}{dE_x}$ (mb/sr)

$T. \text{ Aumann et al., PRC59,1252 (1999)}$

$s$ wave

$p$ wave

$d$ wave

Total Nuclear + Coulomb

Total Only Nuclear

$\sigma_{\text{exp}}=4.0\text{mb}$

$\sigma_{\text{th}}=3.2\text{mb}$

$\sigma_{\text{nuc}}=5.0\text{mb}$

Destructive interference with the Coulomb part

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Continuum structures with a PS basis
Motivation
PS discretization method
Application to reactions

$^{11}\text{Be} + ^{12}\text{C} @ 67 \text{ MeV/nucleon}$

Potentials involved

1. n-$^{10}\text{Be}$ with $^{10}\text{Be}$ in its g.s. 0$^+$

!! contribution of $^{10}\text{Be}$ in its 1$^{\text{st}}$ excited state 2$^+$
Motivation
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$\theta_{\text{c.m.}}$ (deg)

$\frac{d\sigma}{d\Omega}$ (mb/sr)

RIKEN data
THO: $n=2$ ($\varepsilon=0.86\text{ MeV}$)
THO: $n=3$ ($\varepsilon=1.25\text{ MeV}$)
THO: $n=4$ ($\varepsilon=1.75\text{ MeV}$)
THO: $n=3$ (convoluted)

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Continuum structures with a PS basis

Only 1 eigenstate vs. 15 bins

Conclusions

THO PS method
Provides a suitable discrete description of the continuum.

Dealing with Resonances
Natural and accurate treatment of narrow resonances.

Next Step
Generalise the PS method to (halo) two-body nuclei with deformed core: $^{11}\text{Be}, ^{19}\text{C}$...
Phase-shifts integral formula

Hazi and Taylor Formula, PRA1,1109 (1970)

\[ \tan \delta_{\ell}(k) = -\frac{\int_0^\infty u_{\ell}(k, r)[E - H]f(r)F_{\ell}(kr)dr}{\int_0^\infty u_{\ell}(k, r)[E - H]f(r)G_{\ell}(kr)dr} \] (1)
Calculating energy distribution of $B(E\lambda)$ with PS

For $B(E\lambda)$:

\[
\frac{dB(E\lambda)}{d\varepsilon} \bigg|_{\varepsilon=\varepsilon_n} \simeq \frac{1}{\Delta_n} \left| \langle \varphi^{(N)}_{n,\ell} \| M(E\lambda) \| \varphi_{g.s.} \rangle \right|^2
\]

\[
\frac{dB(E\lambda)}{d\varepsilon} \simeq \frac{\mu_{bc} k}{(2\pi)^3 \hbar^2} \left| \sum_{n=1}^{N} \langle \varphi_{\ell}(k) \| \varphi^{(N)}_{n,\ell} \rangle \langle \varphi^{(N)}_{n,\ell} \| M(E\lambda) \| \varphi_{g.s.} \rangle \right|^2
\]

\[
\Delta_n = \frac{\varepsilon_{n+1} - \varepsilon_{n-1}}{2}
\]
For $B(E\lambda)$:

$$B(E\lambda) = \int d\varepsilon \frac{dB}{d\varepsilon} = \frac{2J_f + 1}{2J_i + 1} (D^{(\lambda)}_{J_i,J_f})^2 \langle \varphi_{g.s.} | r^{2\lambda} | \varphi_{g.s.} \rangle$$  (2)
$^6\text{He} + ^{208}\text{Pb} @ 240 \text{ MeV/u}$

2n-$^{208}\text{Pb}$ folding potential

\[
U(\vec{R}) = \int d\vec{r} \rho_{nn}(r) \left( V_{n-^{208}\text{Pb}}(\vec{R} + \frac{1}{2}\vec{r}) + V_{n-^{208}\text{Pb}}(\vec{R} - \frac{1}{2}\vec{r}) \right)
\]

$\rho_{nn}$ obtained from Three-Body model of $^6\text{He}$
THO with $^6\text{He}$

Phase-shifts

* For $\varphi_\ell(k)$ $\Rightarrow$ Asymptotic behavior
* For $\varphi_{n,\ell}^{THO}$ $\Rightarrow$ Integral formula

Graphs showing phase-shifts $\delta_0(\epsilon)$, $\delta_1(\epsilon)$, and $\delta_2(\epsilon)$ for $^6\text{He}=\alpha+^2\text{n}$ with different values of $l$ (0, 1, 2) and energy $\epsilon$ (MeV).
Energy distribution of pseudo-states

Density of states

$$\rho^{(N)}(k) = \sum_{n=1}^{N} \langle \varphi_\ell(k) | \varphi_{n,\ell}^{(N)} \rangle$$

![Graph showing the density of states for different values of $\gamma$.](image-url)