Crossing symmetry and phenomenological widths in effective Lagrangian models of the pion photoproduction process

C. Fernández-Ramírez a,*,1, E. Moya de Guerra a,b, J.M. Udías a

a Grupo de Física Nuclear, Departamento de Física Atómica, Molecular y Nuclear, Facultad de Ciencias Físicas, Universidad Complutense de Madrid, Avda. Complutense s/n, E-28040 Madrid, Spain
b Instituto de Estructura de la Materia, CSIC, Serrano 123, E-28006 Madrid, Spain

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Abstract

We investigate the importance of crossing symmetry in effective field models and the effects of phenomenological nucleon resonance widths on the paradigmatic case of pion photoproduction. We use reaction models containing four star resonances up to 1.8 GeV (Δ(1232), N(1440), N(1520), N(1535), Δ(1620), N(1650), Δ(1700), and N(1720)) with different prescriptions for crossed terms and widths, to fit the latest world database on pion photoproduction. We compare χ² results from selected multipoles and fits. The χ² is highly dependent on the fulfillment of crossing symmetry and the inclusion of u channels.

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In the non-perturbative regime of Quantum Chromodynamics (QCD) we have to rely on effective field models to describe physical processes governed by the strong interaction. This is particularly so in the energy region of the nucleon mass and its excitations. In an effective field model we build suitable Lagrangians to describe particle couplings compatible with the symmetries of the underlying fundamental theory (QCD) and we use a perturbative approach to calculate the physical observables. In this scheme, the reliability of any reaction model counts on the soundness of the model framework and on the fulfillment of the symmetries of the underlying theory. Following this reasoning, the reliability of a complex calculation on nuclei starting from an elementary reaction model (e.g., meson exchange currents [1] or pion photoproduction from nuclei starting from a model on pion photoproduction from the nucleon [2]) relies on how sound is the theoretical background used in the construction of the elementary reaction model.

At tree level, the invariant amplitudes we obtain from the effective Lagrangians are real. Hence, unitarity of the scattering matrix is not respected but, as long as we include all the Feynman diagrams emerging from the effective theory, crossing symmetry is fulfilled. In a perturbative effective field theory it is assumed that unitarity should be restored once we include the higher order effects. The exact calculation of higher orders is an overwhelmingly complex task, so it is customary in the development of reaction models [3–8] to take into account higher order terms effectively through form factors, final and initial state interactions (FSI and ISI), and phenomenological widths (dressing of the propagators). In doing so, one often pays the price of breaking symmetries that the theory should respect, such as crossing symmetry and consistency between widths and couplings. Since unitarity and crossing symmetry are linked together fixed-t dispersion relations are also another method to satisfy these symmetries [9].

Among all the required symmetries, crossing is a symmetry that the invariant amplitude of any well-defined effective
field theory has to fulfill [10]. Similarly to what happens with
gauge invariance [3,11], there are different options to imple-
ment crossing symmetry. In this case, one of the options is
to start from an $A^\text{bare}$ which breaks crossing symmetry, and
to build an FSI amplitude ($A^\text{FSI}$) that restores the symmetry,
so that the complete $A$ amplitude respects crossing symme-
try. The other option is to build diagrams that are explicitly
crossing symmetric: $A^\text{bare}$ is crossing symmetric and $A^\text{FSI}$ does
not break this symmetry. In this Letter we employ this sec-
ond scheme to the case of photo pion production from free
ucleons. FSI can be taken into account as a distortion of the
pion wave function [3,4,12]. $A^\text{bare}$ is just the tree-level Feyn-
man diagram but it includes the width in the propagator and
corresponding form factor in the hadronic vertex. We split the
invariant amplitude in two parts: a background given by the
Born and vector meson exchange terms, and the contribution of
the nucleon resonances. In what follows we focus on the contri-
bution of nucleon resonances to $A^\text{bare}$. We discuss the effect of
the crossed terms using different types of widths and different
prescriptions on the resonance propagators and amplitudes. We
have studied typical non-resonant multipoles because of their
reasons that will become apparent.

Focusing first on the resonant contribution to the invariant
amplitude, we recall that, at tree level, both the direct and the
crossed contributions to the invariant amplitude are real. The
inclusion of a phenomenological width changes the scenario
providing the amplitude with an imaginary part. When one con-
siders direct and crossed terms one usually includes a resonance
width in the $s$ channel but not in the $u$ channel [6,7]:

$$A^\text{res}(s,u) = h \sqrt{X_\pi(s)} \frac{A(s)}{s - M^* + i M^* \Gamma(s)} + h \frac{B(u)}{u - M^* - i M^* \Gamma(u)}.$$  

In what follows we refer to the choice of Eq. (1) as model $V$, for
reasons that will become apparent.

The width is included in the term $i M^* \Gamma$ in the denominator,
compatible with what is obtained by dressing the propagator with
pions [13]. $h$ stands for the strong coupling constant and $M^*$
for the mass of the resonance. The width $\Gamma$ is defined as

$$\Gamma(s) = \sum_j \Gamma_j X_j(s),$$

where $j = \pi, \pi\pi, \eta$ stands for the different decay channels,
$X_j(s)$ accounts for the energy dependence of the width, and
$\Gamma_\pi \propto h^2$.

Model $V$ breaks crossing symmetry. One may think that this
is not the case, because taking a zero width in the crossed chan-
nel is equivalent to include a width $\Gamma = \Gamma(u)$ in the $u$ channel
($\Gamma(u) = 0$ as $u < 0$). Although crossing symmetry may seem
formally respected when one includes $\Gamma = \Gamma(u)$ in the $u$ channel
there is an inconsistency between strong couplings and widths.
Consistency requires that the energy dependence that
appears in the width is taken into account in the strong vertex.
This means that the direct and the crossed terms should con-
tain the form factors $\sqrt{X_\pi(s)}$ and $\sqrt{X_\pi(u)}$ respectively. Hence,
a zero width in the crossed channel would imply a null contri-
bution from the $u$ channel. From this point of view, only the
two forthcoming choices remain consistent.

(i) One choice is to include the energy dependence of the
width as a form factor in the amplitude removing completely
the $u$ channel,

$$A^\text{res}(s,u) = h \sqrt{X_\pi(s)} \frac{A(s)}{s - M^* + i M^* \Gamma(s)} + 0.$$  

We call to this choice, Eq. (3), model IV. In the limiting case
where the width is a constant ($\Gamma(s) = \Gamma_0$; $X_\pi(s) = 1$) Eq. (3)
transforms into

$$A^\text{res}(s,u) = h \sqrt{X_\pi(s,u)} \frac{A(s)}{s - M^* + i M^* \Gamma_0} + 0,$$

which we call model III.

From an effective field theory point of view the complete dis-
appearance of the $u$ channel does not seem sensible but it cannot
a priori be discarded.

(ii) The other consistent choice is to include an energy-
dependent width which depends on both $s$ and $u$ Mandelstam
variables and contributes to both direct and crossed terms [3]:

$$A^\text{res}(s,u) = h \sqrt{X_\pi(s,u)} \frac{A(s)}{s - M^* + i M^* \Gamma(s,u)} + h \sqrt{X_\pi(s,u)} \frac{B(u)}{u - M^* + i M^* \Gamma(s,u)}.$$  

The width $\Gamma(s,u)$ in Eq. (5) is defined as

$$\Gamma(s,u) = \sum_j \Gamma_j X_j(s,u),$$

with

$$X_j(s,u) \equiv X_j(s) + X_j(u) - X_j(s) X_j(u).$$

This is the choice that we took in Refs. [3,4,18] and that we
call here model I.

In the limiting case where the width is a constant ($\Gamma_0$) we get
from (5):

$$A^\text{res}(s,u) = h \sqrt{X_\pi(s,u)} \frac{A(s)}{s - M^* + i M^* \Gamma_0} + h \sqrt{X_\pi(s,u)} \frac{B(u)}{u - M^* + i M^* \Gamma_0},$$

which we call model II.

With choice (ii), the $u$ channel also contributes to the imagi-
ary part of the electromagnetic multipoles. However, the imagi-
ary part of the $u$ channel contributes differently to the multi-
pole amplitudes than the direct term, acting as a background.
This can be seen in Fig. 1 where we show the $u$ channel con-
ditions to every multipole.

In Fig. 1 we show examples, extracted from our extensive
analysis, of the imaginary parts of five bare electromagnetic
multipole amplitudes for pion photoproduction using the same parameter
set for models I and II. The parameter set used is the one ob-
tained by fitting data with model I. We focus on the imaginary


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Fig. 1. Contribution of $s$ and $u$ channels to bare electromagnetic multipoles. Curve conventions: Thick solid: Complete calculation with energy-dependent width model I; Thin solid: Contribution of $u$ channel to model I; Thick dashed: Complete calculation with a constant width model II; Thin dashed: Contribution of $u$ channel to model II; Short dashed: Contribution of $s$ channel to model I. All the curves have been obtained with the same set of coupling constants, that is the ones obtained fitting data with model I.

part of the multipoles because it is there where the contribution of the phenomenological widths shows up more clearly. The contribution of Born and meson-exchange terms to the bare amplitudes is real. Hence, the imaginary part of the bare multipoles in pion photoproduction [16]: $E$ and $M$ stand for electric and magnetic multipoles; the superindex stands for the isospin: $3/2$ for isospin-$3/2$, $p$ for proton isospin-$1/2$, and $n$ for neutron isospin-$1/2$; the first subindex is related to the relative orbital angular momentum ($L$) of the outgoing pion and nucleon: it takes the value $L$ for the electric multipole and $L+1$ for magnetic one; and the second subindex stands for the parity: $+$ or $-$. The same behavior is observed for all the multipoles with the exception of the $M_{1+}^{3/2}$ multipole. The $M_{1+}^{3/2}$ is dominated by the $u$ channel and shows a stronger dependence on the character of the width (constant or energy-dependent). The $s$ channel contribution to this multipole (short-dashed curve) comes basically from the $\Delta(1620)$ resonance. For completeness we also provide results on the imaginary part of the two resonant multipoles of the $\Delta(1232)$ ($M_{1+}^{3/2}$ and $E_{1+}^{3/2}$) where it can be seen that the $u$ channel contribution to the $M_{1+}^{3/2}$ multipole is zero as expected.

From this analysis we may conclude that for most of the multipoles the overall behavior can be obtained with a constant width. Aside from the $M_{1+}^{3/2}$ multipole, the energy dependence of the width becomes important to account for fine details of some multipoles, such as the cusp peak that appears in the $E_{0+}$ electromagnetic multipole [3,16], which is due to the opening of the $\eta$ decay channel of the $N(1535)$ resonance.

Let us now discuss the results obtained with the models I to V. In order to treat each model on its own foot we have fitted the parameters to the data independently for each model. We fit the calculated electromagnetic multipoles to data provided by the energy-independent solution of SAID [16], up to spin-3/2 and up to 1.2 GeV photon energy in the laboratory frame, using masses and widths from [17]. We use the optimization technique described in [3]. For further details on the fitting procedure we refer the reader to [3,18]. In these fits, the intrinsic $E2/M1$ ratio (EMR) of the $\Delta(1232)$ [4] is an output of the fit. In all the fits it is consistent with the latest results from lattice QCD [19] within the error bars, that is EMR = ($-1.93 \pm 0.94$)% for $Q^2 = 0.1$ GeV$^2$ and $m_\pi = 0$; and EMR = ($-1.40 \pm 0.60$)% for $Q^2 = 0$ GeV$^2$ and $m_\pi = 370$ MeV. To summarize, the five models considered are:

I: Eq. (5), $s$ and $u$ channels with $\Gamma = \Gamma(s, u)$ (model in Ref. [3]);
II: Eq. (8), $s$ and $u$ channels with constant width $\Gamma = \Gamma_0$;
III: Eq. (4), only $s$ channel with constant width $\Gamma = \Gamma_0$;
IV: Eq. (3), only $s$ channel with $\Gamma = \Gamma(s)$;
V: Eq. (1), $s$ channel with $\Gamma = \Gamma(s)$ and $u$ channel with $\Gamma = 0$.

Apart from the treatment of the resonance crossed terms and widths, the five models are constructed in the same way. FSI are included through the inclusion of a phase to the electromagnetic multipoles which matches the total phase as discussed in Refs. [3,4,18] and, in particular, the same spin-3/2 couplings are also used in models I to V. Expressions for the Lagrangians and electromagnetic multipoles can be found in the same references. As remarked in [3] the choice of the spin-3/2 couplings is very important. For many years it has been customary to choose for the spin-3/2 Lagrangians the coupling scheme of Ref. [14] that presents pathologies such as [3,15]: spin-1/2 pollution, quantization anomalies, non-positive definite commutators, accidental fields, as well as bad threshold and high energy behaviors. In that scheme, the $u$ channel provides a too large contribution in the high energy region. To regularize their contribution one has to include an extra cutoff in the crossed terms [7], which explicitly breaks crossing symmetry. In our calculations, to avoid all these problems we use the spin-3/2 coupling scheme suggested by Pascalutsa [15] that avoids...
Table 1

Comparison of $\chi^2$ values obtained with the different choices for widths and crossed terms

<table>
<thead>
<tr>
<th>Model</th>
<th>Eq.</th>
<th>$\chi^2/\chi^2_{\text{Model I}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I: $\Gamma = \Gamma(s, u)$, model in Ref. [3]</td>
<td>(5)</td>
<td>1</td>
</tr>
<tr>
<td>II: $\Gamma = \Gamma_0$, constant width</td>
<td>(8)</td>
<td>1.35</td>
</tr>
<tr>
<td>III: $\Gamma = \Gamma_0$, constant width</td>
<td>(4)</td>
<td>1.85</td>
</tr>
<tr>
<td>IV: $\Gamma = \Gamma(s)$, $s$ channel</td>
<td>(3)</td>
<td>1.51</td>
</tr>
<tr>
<td>V: $\Gamma = \Gamma(s)$, traditional width scheme</td>
<td>(1)</td>
<td>1.18</td>
</tr>
</tbody>
</table>

all these pathologies and provides amplitudes that behave properly in both the low and high-energy regions [3].

The energy-dependent widths have been parametrized as in Ref. [3] so that they fulfill the following physical requirements:

(a) $\Gamma = \Gamma_0$ at $\sqrt{s} = M^*$;
(b) $\Gamma \rightarrow 0$ when $k_\pi \rightarrow 0$, where $k$ is the three-momentum of the outgoing pion in the center of mass reference system;
(c) $\Gamma$ has the correct angular momentum barrier at threshold, $k_\pi^{2L+1}$, with $L$ the angular momentum of the resonance.

$X_j(l)$ in Eq. (7) is given by

$$X_j(l) = \frac{\gamma_j}{M} \frac{\Gamma_j^{2L+1}}{1 + \frac{\Gamma_j^{2L+1}}{\gamma_j}} \Theta(l - (M + m_j)^2),$$

where $M$ is the mass of the nucleon, $m_j$ stands for the mass of the meson of the corresponding decay channel $j = \pi, \pi \pi, \eta$, and $k_j = \sqrt{(l - M^2 - m_j^2)^2 - 4m_j^2M^2/(2\sqrt{l})}$, (10)

with $k_{j0} = k_j$ when $l = M^{*2}$.

In Table 1 we compare the $\chi^2$ obtained from the fits with models I to V.

Notably, models that take into account $u$ channels provide the best $\chi^2$ (models I, II, and V), with better $\chi^2$ for those which include an energy dependence in the widths (I and V). This is due to the large energy range covered by all the models, where a constant width is less reliable. Between the two remaining fits, the best fit is obtained by the model which exhibits crossing symmetry. The other models (III and IV) provide $\chi^2$ more than a 50% larger, and have the same number of parameters. Hence, it can be concluded that $u$ channels make a difference when it comes to describe the experimental data.

In Fig. 2 we show the comparison to the data [16] of our results for a few electromagnetic multipoles ($\text{Im}[M^{3/2}_{1+}]$, $\text{Im}[E^{3/2}_{1+}]$, $\text{Im}[M^{1/2}_{1-}]$, $\text{Im}[E^{1/2}_{1+}]$, and $\text{Im}[M^{1/2}_{1+}]$) including FSI. For the well-established resonant $M^{3/2}_{1+}$ and $E^{3/2}_{1+}$ multipoles of the $\Delta(1232)$ all the models provide similar results (upper panels in Fig. 2). For most multipoles, the overall behavior is reproduced with the constant-width model (model II) that also respects crossing symmetry, but the $\chi^2$ is smaller when energy dependent widths are considered (model I). As observed in Figs. 1 and 2 the multipole $\text{Im}[M^{3/2}_{1+}]$, is more sensitive to the choice of the width, which only is well described by models I and IV (crossing symmetric with energy dependent widths). In the higher energy region ($E_\gamma > 1$ GeV), the description of data on $\text{Im}[M^{3/2}_{1+}]$ multipole is not satisfactory for any model. This shows that high-lying resonances may not be well accounted for. Three star resonances, which have not been included, may play a role in the improvement of the data description in the high-energy region.

In Fig. 3 we compare the bare electromagnetic multipoles obtained using models I to V and their corresponding coupling constants. For each multipole the results obtained with models I and II are similar. However, the latter differ substantially from the other models III, IV, and V. The comparison of the bare multipoles in Fig. 3 to their corresponding dressed multipoles in Fig. 2 show that FSI play an important role in neutral pion production and are essential to describe properly the imaginary part of the electromagnetic multipoles.

We conclude that the inclusion of the width in the $u$ channel in a crossing symmetric way is not merely academic but makes a significant difference as it stems from results in Table 1. Certain observables such as the $M^{3/2}_{1+}$ and $M^{1/2}_{1+}$ multipoles are particularly sensitive to the $u$ channel contribution. The in-
fluence of the imaginary part of the resonance amplitude is not important in most of the multipoles, but it makes a difference in the imaginary parts of $M_{3/2}^{-1}$ and $M_{1}^{n}$. Actually, in Im$M_{3/2}^{-1}$ multipole, the $u$ channels of the resonances play a more important role than the direct channel contributions, which are dominated by the $\Delta(1620)$. This multipole is highly interesting from the theoretical point of view as it offers the possibility to study the effects of crossing symmetry and the energy dependence of the widths. It will be very interesting to test the different models also in pion photoproduction from nuclei where the bare amplitudes should, in principle, be used.

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References


